Bayesian Stackelberg Game for Cache Allocation in Wireless Network with Incomplete Information

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Abstract
Caching at base stations is a promising technique to meet the rapid surge of mobile traffic. In this paper, we consider the cache allocation in the commercial caching system consists of one Infrastructure Provider (InP) and several Content Providers (CPs) under incomplete information circumstance. We model the problem based on Bayesian Stackelberg game in which InP is leader and CPs are followers. InP is assumed not have fully knowledge of CPs but knows the distribution of their weight parameters. Apply backward induction, we investigate and prove the existence of Stackelberg Equilibrium. Simulation result gives the comparison of the utility of InP in the complete and incomplete information scenarios.

1. Introduction
The mobile traffic has been predicted to get 10 fold increasing in 2019 compared with 2014 and video traffic will constitute for 72% of mobile data traffic by 2019. This requires new technique to distribute the high quantity of video content over the world. Cache the content at the base stations (BSs) or access points (APs) in radio access network is a promising approach. Compared with the content server located in core network, BSs or APs are much nearer to end users. In addition, local caching at BSs also reduce the traffic through limited backhaul link [1].

RAN caching is an interesting topic, which attracts thousands of researchers to do research. Most existing works focus on cache placement, cooperative caching, and coding caching. However, cache allocation also affects the performance of caching system. A few work have done in the cache allocation direction [2]. In another hand, considering the commercial caching which consists of Infrastructure Provider (InP) and Content Providers (CPs), both InP and CPs are beneficial from leasing and renting the cache. While InP can get income from leasing cache resource, SPs can provide faster service to their users. However, both of InP and CPs are selfish and want to maximize their utilities. Not only CPs compete with InP to get optimal utilities, but CPs also compete with each other because of limited the cache space at BSs. This problem can be handled effectively by game theory. In [3], the authors considered model caching problem in D2D network as a Stackelberg game in which the SBSs are the leaders and the users are the followers. In this model, the SBSs start by predicting users’ reactions and determine the optimal price that maximizes their offloaded traffic from to the users’ devices. With the rewards received from SBSs, users can then decide whether to help the SBSs by caching the files and participating D2D transmission or not. In [4], the authors also used Stackelberg game to propose a new cache incentive mechanism between one MNO and multiple CPs.

The above-mentioned game model did not consider the scenario of incomplete information, in which, InP may not have full information of CPs, so InP may get difficult to determine the optimal strategy. To deal with incomplete information, Bayesian Stackelberg game framework is applied in many areas in communications such as security [5], anti-jamming transmission [6], resource allocation [7]...

Motivated by the above, in this paper, we consider commercial caching system include one, several CPs. We consider cache allocation with incomplete information by applying the Bayesian Stackelberg game and prove the existence of Stackelberg equilibrium. In addition, we give a numerical result in comparing between the utility of InP in complete information and incomplete information.

The rest of this paper is organized as follows. Section 2 describes the system model. Section 3 presents Bayesian Stackelberg game as an approach for one InP and many CPs. Section 4 provides the simulation results and Section 5 concludes the paper.

2. System Model

a. Network Model
We consider commercial caching system consists
of one InP and the set of $K=\{1,2,\ldots,K\}$ CPs. InP owns SBSs. Each SBS can cache at most $Q$ files. We assume that each CP has local content catalog $F_m$ with $F_m$ files. The global files catalog is denoted $F=UF_m$. All files assumed to have the same size.

b. Proactive Caching

One feature of video content is that there is a portion of contents more popular than others, which means these contents are more frequently requested. Based on this feature, the idea is duplicating and caching the popular contents. We assume the distribution of local popular of file of CP $k$ is known. According to the paper [8], we assume that the popularity distribution follows the power law as follows:

$$f(n_k) = \begin{cases} n_k^{-(1-n_k)} & f \geq 1 \\ 0, & f < 1 \end{cases}$$

Where $f$ is a point in the support of the corresponding content and $n_k >1$ is the steepness of distribution curve of local popular of file of CP $k$.

We assume that CP rent the same storage capacity of each SBS. We denote that CP $k$ rent cache capacity of $f_k$ files of each SBS of InP, thus the hit ratio is given by

$$h_k = \int_0^{f_k} p_k(f,n_k)df$$

$$= \int_0^{f_k} (n_k^{-(1-n_k)}df$$

$$= 1-f_k^{1-n_k}$$

3. Bayesian Stackelberg Game with one InP, many CPs with complete information

When the requested content are cached at BS, it can be served directly from BSs. Otherwise, this content is fetched from content server first, and then transmit to users. Therefore, caching at BSs reduces transmission latency. To valuate renting cache of CP, we denote CP $k$’s profit is a function of hit ratio $U(h_k)$. We choose linear function $U(h_k)=w_k h_k$ where $w_k$ is the weight of CP k.

Additionally, we denote the payment of CP $k$ for renting cache is

$$s_k \log f_k$$

Where $s_k$ is the price that InP offer for CP $k$. We choose function log as a way to incentive renting because the payment decreases when the amount of demanded renting increases. This function is used in [9]. The utility of CP is defined as the difference between the profit it get from hit contents and the payment for InP. Therefore, depending on specific price that InP charge to CP for renting, CP will decide how much cache to rent to maximize its total utility

$$\max: w_k (1-f_k^{1-n_k}) - s_k \log f_k$$

$$s.t. \quad f_k \geq 0$$

The Nash equilibrium of follower is obtained by setting the first derivative of the utility equal 0

$$f_k(s_k,w_k) = n_k^{-1} \sqrt{\frac{w_k(n_k-1)}{s_k}}$$

InP (as leader of the game) do not know about the weight that each CP uses to validate the utility of renting cache space from InP. The weight is considered as the type of CPs. However, InP knows the distribution of weight of each CP $k$. As the types of CPs are unknown, the InP (leader) aims to maximize its expected utility, which is given as follows:

$$\max \sum_k \left( \int_{w_k} \log \left(f_k(s_k,w_k)\right) p_k(w_k)dw_k \right) s_k$$

$$s.t. \sum_k \left(\int_{w_k} \log \left(f_k(s_k,w_k)\right) p_k(w_k)dw_k \right) \leq Q$$

Proposition 1: The optimization problem defined in (4) has at least one solution.

Proof: Replace (2) into (3) and denote

$$I_k = \int_{w_k} \log \left(w_k\right) p_k(w_k)dw_k, I'_k = \int_{w_k} \frac{1}{w_k^{n_k-1}} p_k(w_k)dw_k$$

which is a constant regarding $s_k$. We can rewrite (3) as follows:

$$\max \sum_k \left( I_k + \log \left(\frac{n_k-1}{s_k}\right) \right) \frac{s_k}{n_k-1}$$

$$s.t. \sum_k \left( I'_k \left(\frac{n_k-1}{s_k}\right)^{n_k-1} \right) \leq Q$$

$$s_k \geq 0$$
By getting the Hessian matrix of objective function of (4). We have (4) is a concave optimization problem, so (3) have at least one solution. Nash equilibrium always exists in the CP (follower) problem. (4) has at least one solution. Thus, based on the definition of Stackelberg equilibrium [10], we have the SE always exists.

6. Numerical Results

Figure 1 demonstrates the utility of InP with complete/incomplete information. It’s clear that the first one is better, because InP know all information so that it can partition its cache space and design better pricing scheme. Moreover, when Q increases, the utility of InP also increases in both scenarios.

![Figure 1: The utility of InP when InP have complete / incomplete information](image)

5. Conclusions

In this paper, we considered a commercial caching system consisting of an InP and multiple CPs, where the InP leases its SBSs’ cache space to the CPs for gaining profits while the CPs, after storing popular videos to the rented SBSs’ cache, can increase the hit ratio. We propose Bayesian Stackelberg game theoretic framework and investigate the equilibrium. Finally, we show numerical result of comparison utility of InP in complete and incomplete information.

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