Interference-limited Resource Allocation in Cognitive Radio Networks with Primary User Protection.

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Abstract

The performance of multihop cognitive radio networks (CRN) can be improved significantly by using multiple channels in spectrum underlay fashion. However, interference due to the sharing of common radio channel and congestion due to the contention among those flows that share the same links become an obstacle to meet this challenge. How to control efficiently congestion and allocate power optimally to obtain a high end-to-end throughput is a key objective in this work. We reexamined the Network Utility Maximum (NUM) problem with a new primary outage constraint and proposed a novel resource allocation strategy to solve it effectively and efficiently.

I. INTRODUCTION

Cognitive Radio has been realized to be a new communication paradigm for more efficient utilization of radio spectrum. In fact, CRNs are based on the principles of spectrum sensing dynamic spectrum access. However, recently as proposed by many researchers, secondary transmission may be done with the primary users (PUs) over the same spectrum band simultaneously on the condition that the harmful interference introduced by the secondary users (SUs) to the PU receiver is below an acceptable threshold, known as spectrum underlay [1].

In multi-hop CRNs, the harmful interference emitted by the SUs can make the PU's reception unsuccessful and the link capacity strongly depends on the mutual interference among the links. Hence, the end-to-end utility calls for an efficient power allocation strategy. Most existing works [2], [3], [4] proposed the different solutions to optimally allocate power for the SUs in spectrum under fashion. In [4], Hasan *at al.* proposed the suboptimal and optimal algorithms to allocate power under a fixed power budget with a risk return model which considers the reliability and availability of licensed spectrum bands. The authors in [3] introduced the band transmit power constraints and interference power constraints for the SUs. Son *at al.* in [2] introduced a new interference power outage constraint to protect the PUs along with a transmit power constraint for the SUs' power budget. the optimal and suboptimal algorithms to maximize the capacity of the SUs are derived in [2].

However, most above works focus on the CRNs with infrastructure where the secondary transmission only occurs in the single hop between the SUs and CR base station. Hence, the mutual interference among the SUs has not taken into consideration yet. In this paper, we investigate the resource allocation for multi-hop CRNs. Accordingly, we consider both the mutual interference among the SUs due to concurrent transmission and the harmful interference due to simultaneously transmit on the same spectrum band with the PUs in a power allocation strategy. Moreover, the congestion due to the contention of sources traversing the same link in multihop CRNs is investigated via a congestion control policy which tightly coupled with the power allocation strategy.

II. SYSTEM MODEL AND PROBLEM FORMULATION

We consider a spectrum underlay multi-hop CRN comprising a set of N secondary nodes[1] allowed to access a set of $\mathcal{M} = \{1, 2, ..., M\}$ orthogonal primary channels under the interference-limited communication model. A set of logical links $\mathcal{L} = \{1, 2, 3, ..., L\}$ shared by a set of flow sources $\mathcal{S} = \{1, 2, 3, ..., S\}$ simultaneously access the licensed frequency band. Note that we use the special index of $i = 0$ to denote those relevant to primary link. Suppose that flow $s \in \mathcal{S}$ traverses multiple hops to get its destination through the ordered set of links $L(s) \subseteq \mathcal{L}$ called routes. We also adapt the code division multiple access (CDMA) technique to physical layer model where the PUs and the SUs can simultaneously transmit in a common frequency band. We assume that each source with an allocated data rate $x_s^{min} \leqslant x_s \leqslant x_s^{max}$, source s obtains a benefit $U(x_s) : \mathbb{R}_+ \to \mathbb{R}$ which is continuously differentiable, nondecreasing and strictly concave.

[1]In this paper, the term "user" and "node" are interchangeably used.

A. Interference Model and Link Capacity Constraint

Let η_0 denote the thermal noise power under the baseband bandwidth W at receiver of link l. The average signal-to-interference ratio $\overline{\gamma}_l^m(\mathbf{P}^m)$ at link l on band m:

$$\overline{\gamma}_l^m(\mathbf{P}^m) = \frac{G_{ll}^m P_l^m}{\eta_0 + \sum_{k \neq l} G_{lk}^m P_k^m + G_{l0}^m P_0^m} \tag{1}$$

where $\mathbf{P}^m = [P_1^m, P_2^m, ..., P_L^m]$ is a vector of secondary link powers and P_0^m is the transmit power of PU-Tx on the band m. The average capacity of link l modeled on the Shannon capacity and ignore the fading-margin can be approximated as a nonlinear function of transmit power vector $\mathbf{P} = (\mathbf{P}^m, m \in \mathcal{M})$.

$$C_l(\mathbf{P}) \simeq W \sum_m^M \log\left(K \overline{\gamma}_l^m(\mathbf{P}^m)\right), \tag{2}$$

Here constant $K = -\phi_1 / \log(\phi_2 BER)$, where ϕ_1 and ϕ_2 are constants depending on the modulation method, coding scheme and bit-error rate (BER) [5] and $K\overline{\gamma}_l^m(\mathbf{P}^m)$ is assumed to be much greater than 1.

For each link l in multihop networks, the ingress rate should not exceed its link capacity:

$$\sum_{s \in S(l)} x_s \leq W \sum_m^M \log\left(K \overline{\gamma}_l^m(\mathbf{P}^m)\right), \forall l \tag{3}$$

where $S(l) = \{s : l \in L(s)\}$ is the set of flow source uses link l. Without loss of generality, we assume that K and W is unit, henceforth.

B. Primary User Protection

Since the harmful interference caused by all SUs transmitting on band m can make the m^{th} PU-Rx's reception unsuccessful. To guarantee its quality of service (QoS), we propose the PU's outage probability stays below a certain target, denoted by ζ_{th}^m. This constraints can be written as

$$\Pr[\gamma_0^m(\mathbf{P}^m) \leq \gamma_{th}^m] \leq \zeta_{th}^m \tag{4}$$

where γ_{th}^m is the SIR threshold at the m^{th} PU-Rx. On the other words, the outage probability at PU-Rx for a given secondary transmit power vector $\mathbf{P}^m$ is [6]:

$$\Pr[\gamma_0^m(\mathbf{P}^m) \leq \gamma_{th}^m] = 1 - (1 - \zeta_0^m) \prod_{l=1}^{L} \left(1 + \frac{G_{0l}^m P_l^m \gamma_{th}^m}{P_0^m G_{00}^m}\right)^{-1} \tag{5}$$

where $\zeta_0^m = 1 - \exp(\frac{-\eta_0 \gamma_{th}^m}{P_0^m G_{00}^m})$ is the outage probability of PR-Rx in the absence of SUs. Substitute (5) into (4), rewrite the resulting inequality as an lower bound on a posynomial function in $\mathbf{P}^m$, then take logarithm on both sides, we have

$$\sum_{l=1}^{L} \log(1+\rho_l^m P_l^m) \leq \log \mu^m. \quad (6)$$

where $\mu^m = (1-\zeta_0^m)/(1-\zeta_{th}^m)$ and $\rho_l^m = \frac{G_{0l}^m \gamma_{th}^m}{G_{00}^m P_0^m}$. We assume that the primary requirements including the transmit power P_0^m, ζ_0^m, and ζ_{th}^m must be declared a priori to all secondary nodes.

C. Problem Formulation

Our resource allocation problem with primary protection is formulated via NUM problem as following

$$(\mathbf{P1}) \quad \max_{\mathbf{x}\in\mathcal{X},\mathbf{P}\in\mathcal{P}} \sum_{s\in\mathcal{S}} U_s(x_s) \quad (7)$$

subject to (3), (6).

where $\mathcal{X} = \{x_s; s\in\mathcal{S} | x_s^{min} \leq x_s \leq x_s^{max}\}$,

$\mathcal{P} = \{P_l^m; l\in\mathcal{L}, m\in\mathcal{M} | P_l^{min} \leq P_l^m \leq P_l^{max}\}$.

III. Interference-Limited Resource Allocation Algorithm (ILRA)

The constraint (3) is non-convex on $(\mathbf{x},\mathbf{P})$ space. Without loss of optimality, we can perform a variable transformation, $\hat{\mathbf{P}} = \log \mathbf{P}$. Let $\hat{\mathcal{P}} = \{\hat{P}_l^m; l\in\mathcal{L}, m\in\mathcal{M} | \log P_l^{min} \leq \hat{P}_l^m \leq \log P_l^{max}\}$. The resulting optimization problem can be written as

$$(\mathbf{P2}) \quad \max_{\mathbf{x}\in\mathcal{X},\hat{\mathbf{P}}\in\hat{\mathcal{P}}} \sum_{s\in\mathcal{S}} U_s(x_s) \quad (8)$$

subject to

$$\sum_{s\in S(l)} x_s \leq \sum_{m\in\mathcal{M}} \log(\overline{\gamma}_l^m(e^{\hat{\mathbf{P}}^m})), \quad \forall l \quad (9)$$

$$\sum_{l=1}^{L} \log(1+\rho_l^m e^{\hat{P}_l^m}) \leq \log\mu^m, \quad \forall m \quad (10)$$

Since the objective of $\mathbf{P2}$ is assumed to be a concave function. On the other hand, the log-sum-exponential function is a convex function the sets of constraints in (9) and (10) are convex on new variable space $(\mathbf{x},\hat{\mathbf{P}})$. As a result, $\mathbf{P2}$ is a convex optimization problem [7].

By augmenting the objective function with a weighted sum of the constraint functions, we obtain the Lagrangian function of $\mathbf{P2}$:

$$\begin{aligned} L(\mathbf{x},\hat{\mathbf{P}},\boldsymbol{\lambda},\boldsymbol{\nu}) &= \sum_{s\in\mathcal{S}} U_s(x_s) \quad (11)\\ &- \sum_{l\in\mathcal{L}} \lambda_l \left(\sum_{s\in S(l)} x_s - \sum_{m\in\mathcal{M}} \log(\overline{\gamma}_l^m(e^{\hat{\mathbf{P}}^m})) \right)\\ &- \sum_{m\in\mathcal{M}} \nu_m \left(\sum_{l=1}^{L} \log(1+\rho_l^m e^{\hat{P}_l^m}) - \log\mu^m \right) \end{aligned}$$

where $\boldsymbol{\lambda} = [\lambda_1,\lambda_2,...,\lambda_L]$ and $\boldsymbol{\nu} = [\nu_1,\nu_2,...,\nu_M]$ are the Lagrangian nonnegative multipliers which are interpreted as link prices, and primary outage prices. The dual problem of $\mathbf{P2}$ can be described as a min-max optimization problem:

$$\min_{\boldsymbol{\lambda},\boldsymbol{\nu}} \max_{\mathbf{x},\hat{\mathbf{P}}} L(\mathbf{x},\hat{\mathbf{P}},\boldsymbol{\lambda},\boldsymbol{\nu}) \quad (12)$$

Since $\mathbf{P2}$ is a convex optimization problem, there exists a feasible point satisfying the Slater constraint qualification in its domain [7]. From the Strong Duality Theorem [8], there is no duality gap. Hence, the optimal solution of maximization problem in (12) can then be obtained by solving the minimization problem in (12) via the ILRA algorithm.

IRLA is implemented in a distributed manner, in which source s gets the total congestion price $\lambda_s = \sum_{l\in L(s)} \lambda_l$ accumulated through a feedback message from its destination in the path of s and decides its rate by (13). The receiver of link k locally measures $\overline{\gamma}_k^m(\mathbf{P}^m)$ on each band and broadcasts its control message containing $m_k^{m,(t)}$. The transmitter of link k receives $m_j^{m,(t)}$ and $P_j^{m,(t)}$ from the other link, updates power on each band as (14) through congestion price (15) and primary outage price (16).

Algorithm 1: ILRA

Congestion Control: The source rate updates

$$x_s^{(t+1)}(\lambda_s) = \left[U_s^{'-1}\left(\lambda_s^{(t)}\right) \right]^{\mathcal{X}_s} \quad (13)$$

where $U_s^{'-1}(.)$ is the inverse of the first derivative of utility.
Power Control: The link power updates

$$\begin{aligned} P_l^{m,(t+1)} = \Bigg[P_l^{m,(t)} + \kappa_t \Bigg(&\frac{\lambda_l^{(t)}}{P_l^{m,(t)}} - \sum_{k\neq l} m_k^{m,(t)} G_{kl}\\ &- \nu_m^{(t)} \frac{\rho_l^m}{1+\rho_l^m P_l^{m,(t)}} \Bigg) \Bigg]^{\mathcal{P}_l} \quad (14) \end{aligned}$$

where $m_k^{m,(t)} = \frac{\lambda_k^{(t)} \overline{\gamma}_k^{m,(t)}}{G_{kk} P_k^{m,(t)}}$.
Link Congestion Price update:

$$\lambda_l^{(t+1)} = \left[\lambda_l^{(t)} + \kappa_t \left(\sum_{s\in S(l)} x_s^{(t)} - C_l(\mathbf{P}^{(t)}) \right) \right]^{\mathbf{R}_+} \quad (15)$$

Primary Outage Price update:

$$\nu_m^{(t+1)} = \left[\nu_m^{(t)} + \kappa_t \left(\sum_{l=1}^{L} \log(1+\rho_l^m P_l^{m,(t)}) - \log\mu^m \right) \right]^{\mathbf{R}_+} \quad (16)$$

where $[x]^{\mathcal{A}}$ is the projection of x onto the feasible set $\mathcal{A}$ and κ_t is the positive scalar diminishing step-size.

Proposition 1. *The rate updates* (13) *and power updates* (14) *solve the maximization problem in* (12) *for a pair of fixed primal variables* $(\boldsymbol{\lambda},\boldsymbol{\nu})$.

Proof:

$$x_s = \arg\max_{x_s} \sum_s U_s(x_s) - \lambda_s x_s$$

can be found by the Karush-Kuhn-Tucker (KKT) theorem by taking the first-order derivative of the objective with respect to x_s.

$$P_l^m = \arg\max_{P_l^m} \sum_{l,m} \lambda_l \log(\overline{\gamma}_l^m(e^{\hat{\mathbf{P}}^m})) - \nu_m \log(1+\rho_l^m e^{\hat{P}_l^m}),$$

which appears in (14) adopting the projected gradient-descent method [8] with a step size $\kappa_t \geq 0$. ■

Coming back to the dual problem (12), its objective is affine for all $\boldsymbol{\lambda}$ and $\boldsymbol{\nu}$. Therefore, we can also apply the projected gradient-descent method [8] to solve the dual problem (12) via link congestion price updates (15) and PU outage price updates (16).

Proposition 2. *For any initial source rate* $\mathbf{x}^{(0)} \in \mathcal{X}$*, link power* $\mathbf{P}^{(0)} \in \mathcal{P}$ *and shadow prices* $(\boldsymbol{\lambda}^{(0)},\boldsymbol{\nu}^{(0)}) \geq 0$*, the sequence of primal-dual variables generated by* ***ILRA*** *converges to the global optimum of the original problem* $\mathbf{P1}$ *provided that the stepsize satisfies*

$$\kappa_t \geq 0, \quad \lim_{t\to\infty} \kappa_t = 0, \quad \sum_{t=0}^{\infty}(\kappa_t) = \infty, \quad \sum_{t=0}^{\infty}(\kappa_t)^2 < \infty \quad (17)$$

Proof: From **Proposition 1**, we conclude that **ILRA** solves the problem **P2**. For any initial values of the primal and dual variables and the step-size satisfying (17), **ILRA** always converges to a unique point[8]. Since **P2** is convex optimization problem, any locally optimal point achieved from **ILRA** is also the global optimum [7]. ■

TABLE I
THE OPTIMAL LINK POWER COMPARISON

$LinkPowers(mW)$	P_1	P_2	P_3	P_4
Band 1.	3.8	14.3	153.2	191.4
Band 2.	352	98.7	25	1.5

IV. PERFORMANCE EVALUATION

A. Simulation Settings

In this section, we present the illustrative numerical results for the proposed algorithm. We consider a multihop CRN system illustrated in Fig.1. Each secondary link with a maximum transmit power of 26dBm can access both the licensed bands, baseband bandwidth of each is $125KHz$. The minimum data rate for each elastic flow is $100bps$ and the target BER is 10^{-3} with $K = -1.5/\log(5BER)$. The outage thresholds for the licensed band 1 and 2 are 9% and 20%, respectively. The SIR thresholds for PU-Rx 1 and 2 are 0.97dB and 1.31dB at transmit power 20dBm and 23dBm, respectively. The power spectrum density of white noise is assumed to be -80dBm/Hz at PU and SU receivers. We choose $U_s(x_s) = log(x_s)$ as source's utility function for all secondary nodes.

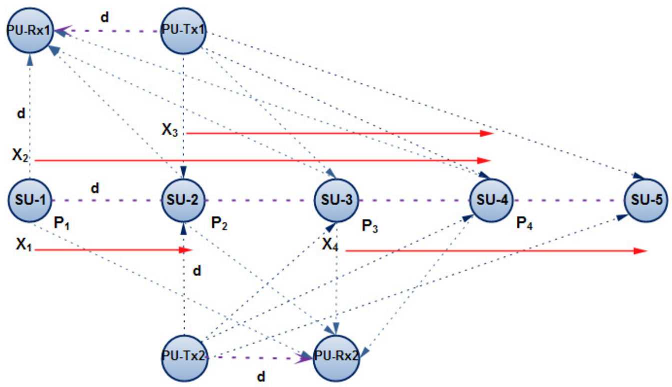


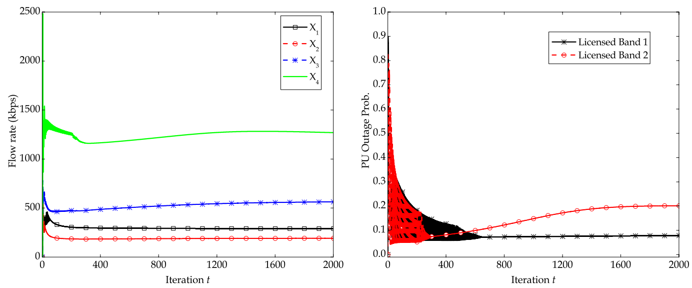
Fig. 1: Physical and logical topologies for simulation

B. Numerical Results

The criterion used to evaluate the convergence speed is $\max \|\mathbf{P}^{*(t)} - \mathbf{P}^{*(t-1)}\| \leq \varepsilon$, where $\varepsilon = 10^{-5}$ is the error tolerance. Fig.3 shows the aggregate utility of our proposed algorithm converges to the optimum within reasonable times. This optimal value compared to those in [9] on single licensed band is indistinguishable. The aggregate utility will be much higher as the number of licensed band allowed to access increases. In Table 1, we compare the optimal powers allocated to each link on both bands. It is clear that the transmit power of secondary nodes depends on not only the mutual interference levels among them but also their physical distance to PU-Rx. Also, the optimal rate in Fig.2a shows that the greedy sources try to get a larger rate to obtain a higher utility. However, sources must follow the diminishing marginal return due to the concave property of utility function. They must adjust their rate via both congestion price and primary outage price which reflect the cost to pay. Fig.2b shows that the outage probabilities caused by secondary nodes at band 1 and 2 converges to the outage thresholds allowed by PUs.

V. CONCLUSION

In this paper, we proposed a cross-layer design for resource allocation and congestion control considering the co-existence of the licensed and unlicensed users in multichannel multihop wireless networks. The resulting optimization problem is convex with nonlinear constraints. By applying the dual theory and the layering technique, we can decompose the original problem into the two function subproblem. The optimal solution is obtained through a distributed algorithm. Fair resource allocation also is achieved by using a utility-based scheme of logarithm function.



(a) (b)

Fig. 2: Convergence of Rates (a) and Outage probability (b).

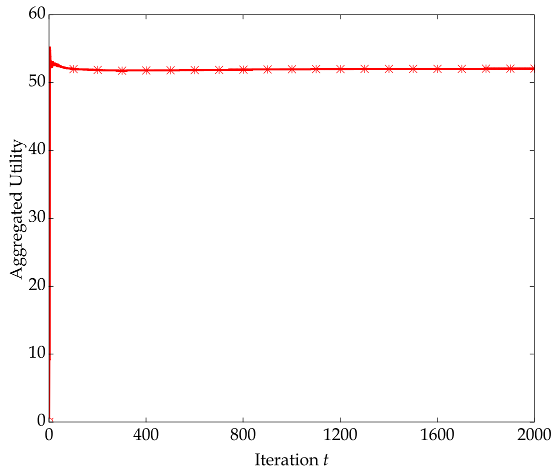


Fig. 3: Aggregated Utility of Algorithm

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