Power Control for Cognitive Radio Networks: Monotonic Optimization Approach

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Abstract

In this paper, we propose the power control problem for cognitive radio networks (CRNs) that maximizes the total utility of the secondary users (SUs). We use the interference temperature constraints to protect the primary users (PUs). The utility functions of SUs can be any increasing functions. We formulate the power control problem as monotonic optimization that can be solved in centralization to achieve the global optimum.

Key words: Cognitive radio, power control.

1. Introduction

Nowadays, the rapid development of wireless services and applications makes the using of frequency is more significant. Therefore, the cognitive radio is proposed in order to improve the radio spectrum utilization and is interested by a lot of researchers. There are two approaches for dynamic spectrum access in CRNs: spectrum overlay and spectrum underlay. In spectrum overlay, the secondary users (unlicensed users) sense the available channels that are not in used by primary users (licensed user) and transmit in these channels. On the other hand, SUs in spectrum underlay approach can transmit simultaneously on the same channels with PUs.

In this work, we use spectrum underlay approach for the SUs to access the channel. We propose the power control problem that maximizes the total utility of the SUs. Because of the non-concave property of *log(1+SINR)*, some previous works considered the high SINR regime where the SINR of each link is much greater than 0 dB. The *log(1+SINR)* becomes *log(SINR)* and optimization problem can be transformed into convex optimization problem in the form of geometric programming (GP) [1]; hence it can be efficiently solved for global optimality. However, we consider the general SINR regime and the utility functions can be any increasing function without any assumptions. And in order to keep the interference to the PUs under the threshold, we use interference temperature limit, introduced by Federal Communications Commission (FCC), as the constraint to protect the PUs from SUs. The power control optimization problem is formulated as Monotonic Optimization (MO) problem which is introduced in [2] and first applied into network optimization by Qian et al in [3]. By applied the *Polyblock outer approximation algorithm* in [2], we can achieve the global optimum for our proposal.

2. System Model and Problem Formulation

1. System model

We consider a cognitive radio network consists of L links denoted by set $\mathcal{L}$ = {1,2,...,L}. The SUs communicate in ad hoc mode and coexist with primary network which has M PUs. The signal to interference noise ratio (SINR) of each secondary link l can be expressed as:

$$\gamma_l(\mathbf{p}) = \frac{G_{ll}p_l}{\sum_{i \neq l} G_{il}p_i + \eta_l} \quad (1)$$

where the p_l is the transmission power from the transmitter of link l and G_{il} is the channel gain from transmitter of link i to the receiver of link l. η_l denotes the additive noise at the receiver of link l. From Shannon capacity formula, the corresponding data rate on link l can be expressed as:

$$R_l(\mathbf{p}) = \log(1+\gamma_l(\mathbf{p})) \quad (2)$$

In order to protect the PUs from the SUs, FCC introduced the concept of interference temperature limit. The maximum interference tolerance for PUs can be calculated as

$$Q^{max} = k\mathbf{T}^{max}$$

where k is Boltzman's Constant and T^{max} is the interference temperature limit. We assume that SUs can be aware of the total interference to PUs and interference threshold.

2. Problem Formulation

In this paper, our objective is to maximize the total utility of all SUs:

$$\sum_{l=1}^{L} U_l(R_l(\mathbf{p}))$$

where $U_l(.)$ can be any increasing function. The limitation of the power at each SU can be expressed as the constraints on the optimization problem:

$$0 \le p_l \le p_l^{max} \qquad 1 \le l \le L$$

Our power control problem can be expressed as:

$$\text{(P1): max} \quad \sum_{l=1}^{L} U_l(\log(1+\gamma_l(\mathbf{p})))$$

$$\text{st.} \quad 0 \le p_l \le p_l^{max} \qquad 1 \le l \le L$$

$$\sum_{l=1}^{L} h_{lm} p_l \le Q_m^{\max} \qquad 1 \le m \le M \quad (3)$$

where h_{lm} is the channel gain from secondary link l to primary user m and is known by secondary network. The last constraints guarantee the interference from secondary network to primary network is below the interference tolerance.

3. Monotonic Optimization and Power Control Problem

1. Monotonic Optimization

In this subsection, we want to remind about the monotonic optimization (MO) based on [2].

Definition 1 (Normal set): A set G $\subset R_+^n$ is called normal if for any two points x, x' $\in R_+^n$ such that x' $\le$ x, if x $\in$ G then x' $\in$ G too.

Proposition 1: The intersection and the union of a family of normal sets are normal sets.

Definition 2 (MO): An optimization problem is called MO if it can be represented by the following formulation:

$$\begin{array}{ll} max & f(\mathrm{x}) \\ st. & \mathrm{x} \in G \end{array} \quad (4)$$

where set G $\subset$ [0,b] $\subset R_+^n$ is a nonempty normal closed set and the objective function f(x) is an increasing function on [0,b].

Definition 2 (Upper boundary): A point y $\in R_+^n$ is called an upper boundary point of bounded normal set G if y $\in$ G while $K_{\mathbf{y}} = \{\mathbf{y}' \in R_+^n \mid \mathbf{y}' > \mathbf{y}\} \in \{R_+^n \setminus G\}$. The set of upper boundary points of G is the upper boundary of G and is denoted by ∂^+G.

Proposition 2: The maximization of f(x) over G, if it exists, is attained on ∂^+G.

Definition 3 (Polyblock): Given any finite set T $\subset R_+^n$ with elements v_i, the union of all the boxes [0,v_i] is called polyblock with vertex set T (Box [0,v_i]={x|0≤x≤ v_i}).

Definition 4 (Proper): An element v $\in$ T is proper if there does not exist v' $\in$ T, such that v' ≠ v and v' ≥ v. Set T is a proper set if all of its elements is proper.

Proposition 3: If G in (4) is a polyblock, then the optimal solution is attained at one proper vertex of this polyblock.

Definition 5 (Projection): Let G $\subset$[0,b] be a nonempty normal set, for every point $\mathbf{z} \in R_+^n \setminus \{\mathbf{0}\}$, the half line from 0 through z meets ∂^+G at a unique point π_G(z), which is defined by:

$$\pi_G(\mathbf{z}) = \lambda\mathbf{z}, \lambda = max\{\alpha \ge 0 \mid \alpha\mathbf{z} \in G\}.$$

The detailed illustrations about these propositions, definitions and their proofs are omitted due to space limitation. Interested readers can refer to [2] for more details.

2. Power Control Problem

The power control problem (P1) can be rewritten as:

$$\text{(P2):} \quad \text{max} \quad \Gamma(\mathbf{z}) = \sum_{l=1}^{L} U_l(\log(z_l))$$

$$\text{st.} \quad \mathbf{z} \in \Pi,$$

where the feasible set Π is defined by:

$$\Pi = \{\mathbf{z} \mid 1 \le z_l \le 1+\gamma_l, \forall l \in \mathcal{L}, \mathbf{p} \in \mathcal{P}\},$$

with

$$\mathcal{P} = \{\mathbf{p} \mid 0 \le p_l \le p_l^{max}, \sum_{l=1}^{L} h_{mp} p_l \le Q_m^{max}, \forall 1 \le m \le M\}.$$

The feasible set Π is the union of infinite number of boxes, each box corresponds to a feasible $\mathbf{p} \in \mathcal{P}$. Therefore by proposition 1, Π is normal set. Together with $\Gamma(\mathbf{z})$ being an increasing function in z, problem (P2) is a MO problem. Hence we can solve this problem using the *Polyblock Outer Approximation Algorithm* in [2] with slight modifications. The details are shown in Algorithm 1:

Algorithm 1:

Initialization: Select ϵ≥0 (tolerance). Let $\bar{\mathbf{x}}^0$ be feasible solution available and current best value (CBV) equals to $f(\bar{\mathbf{x}}^0)$. Let $T_1 = \{\mathbf{b}\}$, where

$$b_l = 1 + \frac{G_{ll} p_l^{\max}}{\eta_l}$$

It is obvious that box $[\mathbf{0}, \mathbf{b}]$ containing Π. Set k=1.

Step 1: Select $\mathbf{z}^k \in argmax\{f(\mathbf{z}) \mid \mathbf{z} \in T_k, \mathbf{z} \ge \mathbf{1}\}$. Compute $\mathbf{x}^k = \pi_G(\mathbf{z}^k)$. Determine CBV = $\max\{f(\bar{\mathbf{x}}^{k-1}), f(\mathbf{x}^k)\}$ and current feasible solution $\bar{\mathbf{x}}^k$ corresponding to that value.

Step 2: The set T_{k+1} is attained from $(T_k \setminus \{\mathbf{z}^k\}) \cup \{\mathbf{z}^k - (z_i^k - x_i^k)\mathbf{e_i}, i = 1,...,l\}$ after removing the improper elements.

Step 3: If $(1+\epsilon)\Gamma(\bar{\mathbf{x}}^k) \ge \Gamma(\mathbf{z}^k)$, termination. Otherwise, k= k+1 and return to Step 1.

The projection $\pi_G(\mathbf{z}^k) = \lambda_k z^k$ in Step 1 can be obtained by solving the max-min problem:

$$\lambda_k = \max\{\lambda \mid \lambda \mathbf{z}^k \in \Pi\}$$

$$= \max\{\lambda \mid \lambda \le \min_{1 \le l \le L} \frac{1+\gamma_l(\mathbf{p})}{z_l^k}, \mathbf{p} \in \mathcal{P}\}$$

$$= \max_{\mathbf{p} \in \mathcal{P}} \min_{1 \le l \le L} \frac{f_l(\mathbf{p})}{z_l^k g_l(\mathbf{p})} \quad (5)$$

where $f_l(\mathbf{p}) = \sum_{i \in \mathcal{L}} G_{il} p_i + \eta_l$ and $g_l(\mathbf{p}) = \sum_{i \in \mathcal{L}, i \ne l} G_{il} p_i + \eta_l$. This is the generalized linear fractional programming problem that can be solved using the Dinkelbach-type algorithm as in [3].

In algorithm 1, after each iteration, we always have $\Pi \subset O_{k+1} \subset O_k$, where O_k is the polyblock with its vertex set T_k. By this way, we can construct a series of polyblocks containing Π that approximate the normal set Π with an increasing level of accuracy. The geometric illustration of that step in case of two dimensions is shown in fig. 1. $\mathbf{z}^1$ and $\mathbf{z}^2$ are attained by $\{\mathbf{z} - (z_i - (\pi_G(\mathbf{z}))_i)\mathbf{e_i}, i = 1,2\}$, respectively. However, $\mathbf{z}^2$ is improper so we remove it and the proper set after this is $\{\mathbf{z}^1, \mathbf{v}\}$.

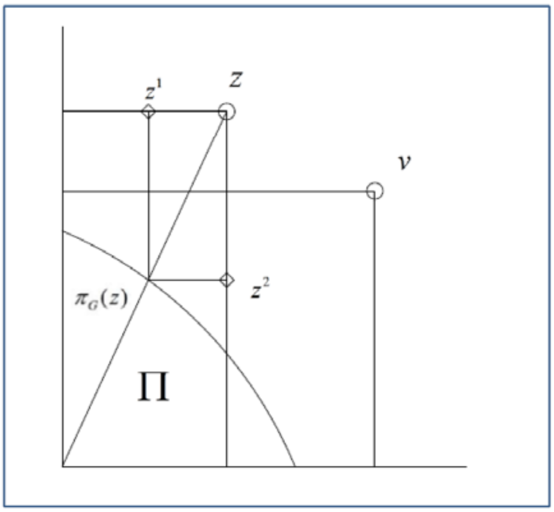

Fig.1. Geometric illustration of shrinking the outer polyblock

Theorem 1: If algorithm 1 is infinite, each of the generated sequences $\{z^k\}, \{x^k\}$ contains a subsequence converging to an optimal solution. Therefore (P2) converges to global optimal.

Proof: The proof of Theorem 1 is the same as the proof of theorem 1 in [2] and is summarized as follows. If algorithm 1 is infinite, it generates at least one infinite subsequence $\mathbf{z}^{l_1}, \mathbf{z}^{l_2}, ..., \mathbf{z}^{l_k}, ...$ such that $\mathbf{z}^{l_{k+1}} = \mathbf{z}^{l_k} - (z_{ik}^{l_k} - x_{ik}^{l_k})\mathbf{e}_{ik}$. It's obviously that $\mathbf{z}^{l_1} \geq \mathbf{z}^{l_2} \geq ... \geq \mathbf{z}^{l_k} \geq ... \geq 0$. So, there exists $\mathbf{z}^*$ such that $\mathbf{z}^* = \lim_{k\to+\infty} \mathbf{z}^{l_k}$. This implies $\mathbf{z}^{l_k} - \mathbf{z}^{l_{k+1}} \to 0$, and hence $z_{ik}^{l_k} - x_{ik}^{l_k} \to 0$. On the other hand, $z_{ik}^{l_k} - x_{ik}^{l_k} = (1-\lambda_{l_k})z_{ik}^{l_k}$ and $z_{ik}^{l_k} \geq 1$ then $\lambda_{l_k} \to 1$, that means $\mathbf{z}^{l_k} - \mathbf{x}^{l_k} \to 0$. Consequently, $\mathbf{z}^* = \lim_{k\to+\infty} \mathbf{z}^{l_k} = \lim_{k\to+\infty} \mathbf{x}^{l_k}$ belongs to Π and $f(\mathbf{z}^*) \geq f(\mathbf{x}), \forall \mathbf{x} \in \Pi$, i.e., $\mathbf{z}^*$ is global optimal solution□

Because of the tradeoff between convergence time and the optimality, we can select ϵ>0 and algorithm will converge to ϵ-optimal solution.

4. Numerical results

We consider the cognitive radio network with 3 links (i.e., *L*=3) and one PU (i.e., *M*=1). Assume that $p_l^{max} = 0.5mW$ and $\eta_l = 0.5\mu W$ for all links. We consider a realization of the channel gains, represented by matrix *G*:

$$G = \begin{pmatrix} 0.075 & 0.015 & 0.020 \\ 0.015 & 0.045 & 0.002 \\ 0.020 & 0.002 & 0.085 \end{pmatrix}$$

The utility function is selected $U_l(x) = x$ in order to maximize the total throughput of the secondary network. Fig. 2 shows that the total utility of secondary network converges to the sub-optimal solution with the error tolerance ϵ chosen equals to 0.05.

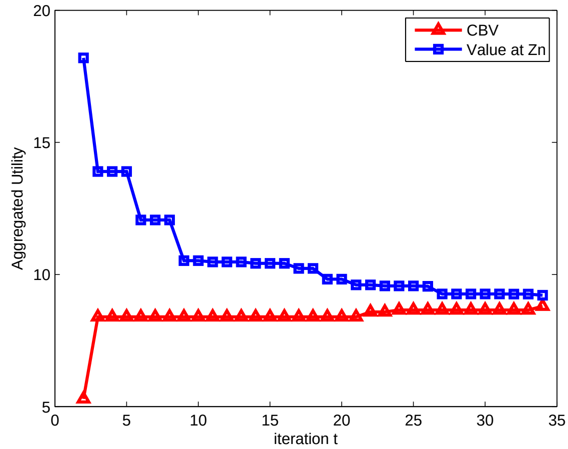

Fig.2. Total throughput when error tolerance ϵ=0.05.

5. Conclusion

In this paper, we formulate the power control problem as Monotonic Optimization problem. The algorithm 1 is guaranteed to converge to global optimal solution despite of non-convexity of the problem. Therefore, our proposal provides benchmark for performance evaluation of the other power control heuristics in this area.

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