

# Power control with QoS and Interference Temperature Constraints in Cognitive Radio Networks

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## Abstract

This paper derive an optimal distributed power control strategy under both the probability of dropping a packet due to buffer overflow constraints at the secondary users and the interference constraints to the primary users.

## 1. Introduction

Cognitive radio(CR) is the key enabling technology that enables the next generation communication networks to utilize the spectrum more efficiently in an opportunistic fashion without interfering with the primary users. The main contributions of this paper is to minimize the total transmit power of all secondary user transmitter (SU-Tx) nodes under both the probability of dropping a packet due to buffer overflow constraints of each SU-Tx and the interference constraint at primary receiver (PU-Rx). Our interest is to design a distributed power control (DPC) algorithm in the SU networks without any central control node.

## 2. System model

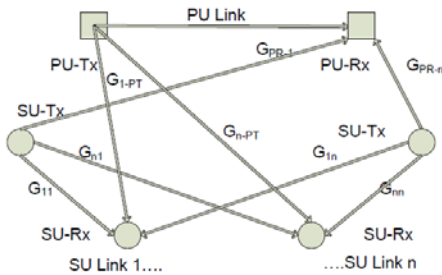


Fig.1. SU networks with M SU links and 1 PU link.

We consider a CR network with  $M$  SU user links and one primary user link just as the illustration in Fig. 1. There is no information exchange between primary users and SU users. The SU nodes access the spectrum in the underlay model, so long as the interference generated by SU nodes to primary users should not exceed the maximum level that primary users can tolerate. Denote  $I$  as the maximum interference tolerance at PR, which characterizes the "worst case" of the RF environment [1]. So the main

constraint of power control in CR networks is to make the interference caused by SUs to the PUs below  $I$ , i.e.,

$$\sum_{i=1}^M G_{PR-i} P_i \leq I$$

where  $G_{PR-i}$  is the channel gain between the SU-TX of link  $i$  and PU-RX, and  $P_i$  represents the transmit power of SU-TX of link  $i$ . The signal-to-interference-noise-ratio (SINR) at the SU-Rx of link  $i$

$$\text{is } SINR_i = \frac{G_i P_i}{\sum_{j \neq i}^M G_{ij} P_j + G_{PT-i} P_{PT} + \sigma^2}$$

where  $G$  denotes the channel gain of the SU,PU link and  $\sigma^2$  is the background noise at the SU-Rx of link  $i$  which is assumed to be AWGN. The transmit rate  $R_i$  of SU-Tx is

$$\text{defined in [3] as } R_i = \frac{1}{T} \log_2 (1 + K \bullet SINR_i)$$

The probability  $P_{BO}$  of dropping a packet due to buffer overflow at a node is also important in several applications. Setting an upper bound  $P_{BO,i,max}$  on the buffer overflow probability also gives a polynomial lower bound constraint in  $P$ :  $(T\lambda_i / \Gamma \log_2(1 + K \bullet SINR_i))^{B+1} \leq P_{BO,i,max}$ , or equivalently,  $ISR_i(P) \leq K / (2^{\Psi} - 1)$  where  $\Psi = (T\lambda_i) / (\Gamma (P_{BO,i,max})^{1/B+1})$ ,  $B$  is the buffer size and  $ISR_i$  is the inverse of the  $SINR_i$  [4]. For simplicity, we define  $\pi_i \triangleq K / (2^{\Psi} - 1)$ , which is a constant when all the parameters of link  $i$  are fixed.

## 3. Problem formulation and Distributed Power Control Algorithm

Our optimization objective is to minimize the total SU-Tx transmit power of all links under both  $P_{BO,i,max}$  constraints of each SU-Tx and the interference

constraint at PU-Rx. The optimization problem can be written as:

$$\begin{aligned} & \min \sum_{i=1}^M \exp(p_i) \quad s.t. \\ & \exp(p_i) \leq P_{\max}, \forall i, \sum_{i=1}^M G_{PR-i} \exp(p_i) \leq I \\ & \log(\exp(-p_i)(\exp(z_i) + N_i)) \leq \log(\Pi_i G_{ii}), \forall i \\ & \exp(z_i) = \sum_{j \neq i}^M G_{ij} \exp(p_j), \forall i \end{aligned} \quad (1)$$

Where  $p_i = \log P_i$ ,  $N_i \triangleq G_{i-PT} P_{PT} + \sigma^2$ ,  $P_{\max}$  is the maximum transmit power of SU-Tx. We assume that every user  $i$  has the capability to estimate the interference  $Z_i \triangleq \sum_{j \neq i}^M G_{ij} P_j$  from other secondary users and keep a copy locally and introduce a new auxiliary variable  $z_i = \log Z_i$ .

Proposition 1: The power control problem (1) is a convex optimization problem in  $\{p_i\}$  [1].

The power control problem (1) could be solved by solving its dual problem. In this section, we will construct the sub gradient iterative algorithm to solve the dual problem. We can apply the decomposition method of Lagrange relaxation of the coupling constraint which is proposed in [1],[4] and construct the sub gradient algorithm to solve the dual problem. The dual function is

$$\begin{aligned} & D(\{\mu_i\}, \nu, \{\xi_i\}, \{\gamma_i\}) \\ & = \sum_{i=1}^M \min_{p_i, z_i} L_i(\{p_i\}, \{z_i\}, \{\mu_i\}, \nu, \{\xi_i\}, \{\gamma_i\}) \\ & - \sum_{i=1}^M \mu_i P_{\max} - \nu I - \sum_{i=1}^M \xi_i \log(\Pi_i G_{ii}) \end{aligned} \quad (2)$$

Where

$$\begin{aligned} & L_i(\{p_i\}, \{z_i\}, \{\mu_i\}, \nu, \{\xi_i\}, \{\gamma_i\}) \\ & = \exp(p_i) + \mu_i \exp(p_i) + \nu G_{PR-i} \exp(p_i) \\ & + \xi_i \log(\exp(-p_i)(\exp(z_i) + N_i)) \\ & + (\sum_{j \neq i}^M \gamma_j G_{ji}) \exp(p_i) - \gamma_i \exp(z_i) \end{aligned} \quad (3)$$

According to KKT condition [5], the optimal transmit power of each SU-Tx can be obtained through the

following equation:  $\frac{\partial L_i(\{p_i\}, \{z_i\}, \{\mu_i\}, \nu, \{\xi_i\}, \{\gamma_i\})}{\partial p_i} = 0$

and the solution is  $P_i^* = \frac{\xi_i}{1 + \mu_i + \nu G_{PR-i} + \sum_{j \neq i}^M \gamma_j G_{ji}}$

;  $\frac{\partial L_i(\{p_i\}, \{z_i\}, \{\mu_i\}, \nu, \{\xi_i\}, \{\gamma_i\})}{\partial z_i} = 0$  and the solution is

$\exp(Z_i) = \xi_i / \gamma_i - N_i$ . The dual problem can be solved using sub gradient method [2].

#### 4. Numerical Results and Conclusions

The simulation parameters are  $\lambda_i = 2000$  pk/s,  $\Gamma = 30$ bits.  $T=1/10$ s,  $B=100$ ,  $P_{BO,i,max} = 0.05$ . The distances between three SU-Tx nodes and PU-Rx are 1000m, 1300m and 1800m respectively.  $P_{\max}=1$ W and  $N_i=10^{-10}$ W. Channel gains are defined using a simple path loss model,  $G_{ij} = Ld^{-4}_{ij}$  and  $G_{PR-i} = Ld^{-3}_{PR-i}$ , where  $L$  is a constant.

In conclusions, based on convex optimization theory, we have designed a distributed algorithm to solve the dual problem since the duality gap of a convex problem is zero. The algorithm proposed in this paper could be carried out at each SU nodes with limited message exchange and have fast convergence performances.

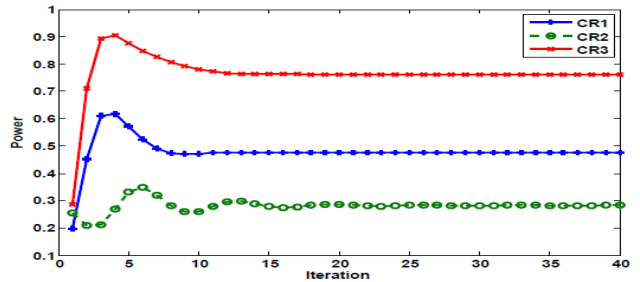


Fig.2. The power convergence of each user in DPC

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