

# Maximize Throughput for The Secondary User over Multi-channel Cognitive Radio Networks

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## Abstract

This paper studies average waiting time analysis of opportunistic access in multi-channel cognitive radio networks with a single secondary user and multiple primary users by applying the M/D/1 preemptive priority queuing scheme. By employing convex optimization tool, the secondary user finds the optimal way to distribute the packets to all channels in the system. Simulations results indicate that the performance of the secondary user depends on the data traffic characteristics of the primary users and also the delay constraint of the secondary user.

## 1. Introduction

Radio spectrum is one of the most scarce and valuable resources for wireless communications. Cognitive radio (CR) has been proposed as a way to improve spectrum efficiency by exploiting the unused spectrum in dynamically changing environments [1]. Among existing analysis tools, queuing theory been employed in performance analysis of cognitive radio networks [2], [3] where some results have been obtained, such as packet waiting time in queue and delay. Specifically, the average waiting time of packets for SU is studied in [2], [5], under various preemptive priority queuing models such as M/D/1 (Poisson arrival process, deterministic distribution of service time, single server) and M/G/1 (Poisson arrival process, general distribution of service time, single server). Based on the average waiting time analysis, we find an optimal way to allocate how many packets should enter which channel with multi-channel system in the context of time division multiple access (TDMA) by employing point-wise maximum of convex functions. To the best of our knowledge, this is the first paper analyzing this problem.

## 2. System model

We consider a time slotted cognitive wireless network where a primary users is the owner of the network. We adopted similar model as used in [2] where the authors considered a cognitive network with  $N$  primary and 1 secondary links. When the primary wishes to transmit, it is given a priority over the secondary user. This is implemented by having the secondary user perform spectrum sensing at the beginning of the slot.

If there is no signal at the beginning of the slot, the remainder of the slot can be utilized for secondary transmission. Perfect time synchronization and perfect sensing are assumed. All packets are assumed to be one slot in duration. The network is assumed to operate in ideal channel conditions (e.g. no noise and error-free). Fig. 2 shows example of realization of packet arrivals and departures. Poisson process is used for packet arrivals, so that the inter-arrival times are exponentially distributed. The primary user  $i$  arrival rate is  $\lambda_{p,i}$  and secondary user arrival rate is  $\lambda_s$ . Note that transmission of packets can only start at the beginning of the slot, so that even if a packet arrives at the middle of the slot, it has to wait half of the slot duration, even if the channel is free. Infinite buffers are assumed. Fig. 1 shows the queuing model for single SU and  $N$  PUs in a multi-channel system. Each PU  $i$  operates in channel  $i$  but SU can operate in all  $N$  channels.

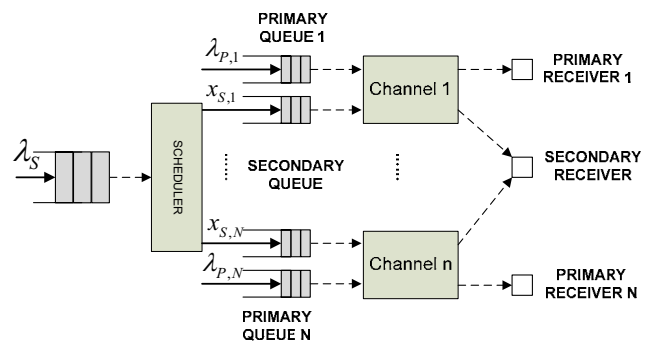


Fig. 1. A multi-channel cognitive wireless network.

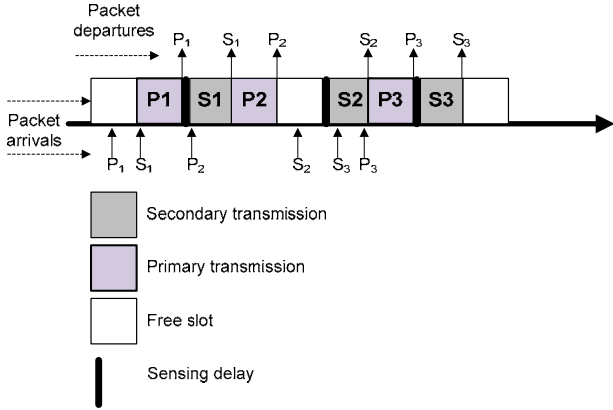


Fig. 2. Priority service discipline.

### 3. Problem formulation

We assume that packets arrive at SU according to a Poisson process with rate  $\lambda_s$ . Each packet is independently routed with probability one of  $N$  channel. In [6], the authors showed that arrival processes at each channel are Poisson with rate  $x_{s,i} = p_i \lambda_s$ . Therefore, we can model the multichannel system like  $N$  parallel single channel systems. In [2], the average waiting time of a packet analysis with a single channel system was presented. It consists of three parts: time until the beginning/start of the next slot, time spent in a queue waiting time for the service to begin, and the average service time (transmission time). The packets of SU and PU both are served according to a first come first served discipline (FCFS), but a packet of SU queues may start its transmission at the beginning of a time slot only if there are no packets of PU (i.e. empty primary user queues) in the each channel. Given the fact that packets arrive according to Poisson process and that the system time is slotted with a fixed unit time slot, it is straightforward to estimate the average time spent by a newly arrived packet waiting in a queue for the start of the next slot: on average, a packet has to spent  $T_D = 1/2$  slot waiting for the next slot to start. By applying M/D/1 priority queuing scheme the average delay per packet for the SU in channel  $i$  is given by

$$W_{S,i} = \bar{X}_i + \frac{T_D}{\left(1 - \frac{x_{p,i}}{\mu_i}\right) \left(1 - \frac{x_{p,i}}{\mu_i} - \frac{x_{s,i}}{\mu_i}\right)} \quad (1)$$

where is the average service time of a packet in channel  $i$  and  $\mu_i$  is the average service time of a packet in channel  $i$ . We have already assumed that the service time is one slot and that a newly arrived

packet has to wait for  $1/2$  slot before the beginning of slot. We can substitute the corresponding values in and to obtain

$$W_{S,i} = \frac{1}{\mu_i} + \frac{\mu_i^2}{2(\mu_i - \lambda_{p,i})(\mu_i - \lambda_{p,i} - x_{s,i})} \quad (2)$$

and the average time spent by primary packet in each channel  $i$  as

$$W_i^P = \frac{1}{\mu_i} + \frac{\mu_i}{2(\mu_i - \lambda_{p,i})} \quad (3)$$

#### A. Scenario 1: maximum the total arrival rate

In this section we want to maximize the total packets can be transmitted in  $N$  channel by maximize the sum arrival rate in each channel. With these assumptions in the above section, the optimization can be formulated as

$$P1: \max_x \sum_{i=1}^N x_{S,i} \quad (4)$$

$$s.t. \sum_{i=1}^N x_{S,i} \leq \lambda_s, \quad (5)$$

$$\max_i \{W_{S,i}\} \leq W_{\max}, \quad (6)$$

$$0 \leq x_{S,i} < \mu_i - \lambda_{p,i}, \forall i \in \{1, 2, \dots, N\}, \quad (7)$$

where  $\mathbf{x} = \{x_{s,1}, x_{s,2}, \dots, x_{s,N}\}$  is a vector of arrival rate allocation over  $N$  channel for the SU. We will prove that P1 is a convex optimization problem. The constraint (7) is represented queue stability. Loynes theorem in [8] states that if arrival and departure rates of a queuing system are stationary and the average arrival rate  $\lambda$  is less than the average departure rate  $\mu$ ,  $\lambda < \mu$ , then the queue is stable [7]. In the constraints (6)  $W_{\max}$  is the given delay threshold for each packet under some delay sensitive application. The constraints (6) can be formulated as

$$f(x) \leq W_{\max}, \quad (8)$$

Where  $f(x) = \max_i \{f_i(x)\}$

$$\text{and } f_i(x) = W_{S,i} = \frac{1}{\mu_i} + \frac{\mu_i^2}{2(\mu_i - \lambda_{p,i})(\mu_i - \lambda_{p,i} - x_{s,i})}$$

It can be seen that if  $\mu_i, \lambda_{p,i}$  are given then  $f_i(x)$  is the convex function hence their point-wise maximum function  $f(x)$  is also convex [4]. Therefore with given  $\mu_i, \lambda_{p,i}, \lambda_s$  and  $W_{\max}$ , P1 is a convex optimization problem and can be solved by the algorithms in [4].

### B. Scenario 2: minimum the maximum waiting time

In the delay sensitive application network, the first requirement is minimization the delay or the waiting time of a packet in the system. The optimization problem can be formulated as below

$$P2: \min_x \max_i \{W_{S,i}\} \quad (9)$$

$$s.t. \sum_{i=1}^N x_{S,i} \leq \lambda_S, \quad (10)$$

$$0 \leq x_{S,i} < \mu_i - \lambda_{P,i}, \forall i \in \{1, 2, \dots, N\}, \quad (11)$$

Similarly, P2 can be proved to be a convex problem. But minimizing the maximum delay of the secondary user's packets will decrease throughput, which reduces significantly the amount of packet transmissions. In order to tradeoff between delay constraints and high utilization factor, we add a tradeoff component in to  $f_i(x)$  such that

$$f_i(x) = W_{S,i} = \frac{1}{\mu_i} + \frac{\mu_i^2}{2(\mu_i - \lambda_{P,i})(\mu_i - \lambda_{P,i} - x_{S,i})} - \alpha \log\left(\frac{x_{S,i}}{\mu_i}\right)$$

Then P2 become

$$P3: \min_x \max_i \{f_i(x)\} \quad (12)$$

$$s.t. \sum_{i=1}^N x_{S,i} \leq \lambda_S, \quad (13)$$

$$0 \leq x_{S,i} < \mu_i - \lambda_{P,i}, \forall i \in \{1, 2, \dots, N\}, \quad (14)$$

where  $\alpha$  is called a tradeoff coefficients. We can consider P2 is just one of special case of P3 when  $\alpha=0$ .

### 4. Numeric analysis

In the simulation, we consider a system with 1 secondary user and 5 primary users with 5 sharing channels. The initial parameters are given in Table I. The simulation is performed by cvx package [7].

	Ch1	Ch2	Ch3	Ch4	Ch5
$\mu_i$	0.5	0.7	0.6	0.8	1.0
$\lambda_{P,i}$	0.1	0.4	0.3	0.2	0.4
$\mu_i - \lambda_{P,i}$	0.4	0.3	0.3	0.6	0.6
$\lambda_{P,i} / \mu_i$	0.20	0.57	0.50	0.25	0.40
$W_{P,i}$	2.625	2.595	2.667	1.917	1.833

Table I. Initial Parameters

Figure 3 shows that the total number of transmitted packets increases with delay threshold. But the increasing depresses when the total arrival packets on 5 channels is reasonably large that means the

channels are busier. And the variety in quality of channels makes the difference of the distribution of packet over 5 channels which can be observed in Fig.3. Fig.4 presents the performance and proportion among 5 channels with different value  $\alpha$ . As can be

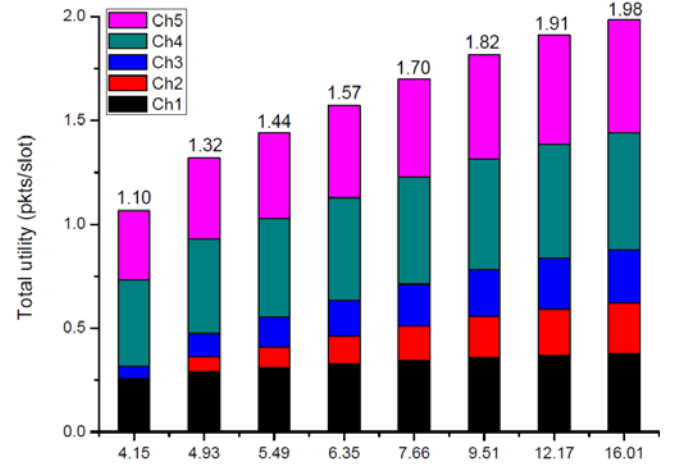


Fig. 3. The distribution of packet over the channels in scenario A

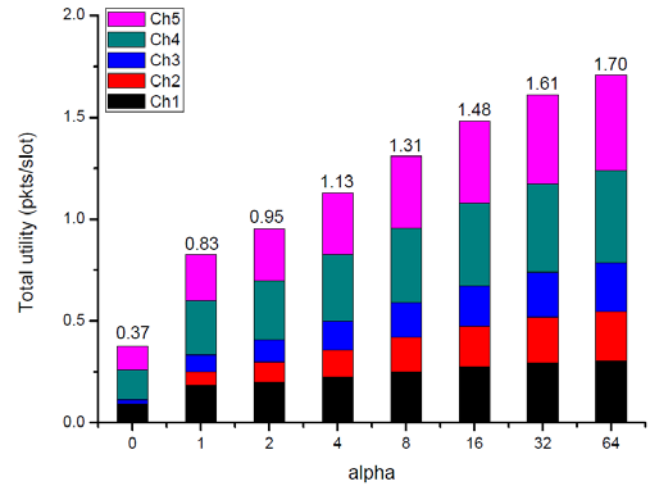


Fig. 4. The distribution of packet over the channels in scenario B seen, when  $\alpha$  increase the throughput will be rise up but the packets must tolerate more delay.

### 5. Conclusion

In this paper, based on the analysis of average waiting time of the primary users' and the secondary user's packets, the distribution of the secondary user's packets over multi-channel has been derived. Our numerical results show that the distribution of packet of the secondary user depends on the data traffic characteristics of the primary users. Furthermore, the performance of the secondary user is connected with the delay constraint.

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