Joint Rate Adaption, Power Control, and Spectrum Allocation for OFDMA-Based Multi-hop CRNs

Mui Van Nguyen and Choong Seon Hong
Dept. of Computer Engineering,
Kyung Hee University, 449-701, Republic of Korea
Email: {nvmui, cshong}@khu.ac.kr

Il Kwon Cho
Department of Network Planning,
National Inform. Soc. Agency, 100-775, Republic of Korea
Email: ikcho@nia.or.kr

***Abstract*—The overall performance of multi-hop cognitive radio networks (CRNs) can be improved significantly by deploying the channel diversity of orthogonal licensed channels in spectrum underlay fashion. However, interference due to the sharing of common radio channels among links and congestion due to the contention among those flows that share the same link become an obstacle to meet this challenge. How to control congestion efficiently and allocate power as well as channel optimally in order to obtain a high end-to-end throughput motivates a framework of cross-layer optimization design for OFDMA-based multi-hop CRNs. By taking into account the problem of joint rate adaption, power control, and spectrum allocation (JRPS), we propose a new cross-layer optimization framework. The formulation is shown to be a mix-integer non-linear programming (MINLP) problem, which is NP-hard in general. To solve the problem, we develop a partially distributed solution, which has been proved to converge to the global optimum within reasonable times.**

I. INTRODUCTION

Cognitive Radio (CR) has been realized to be a new communication paradigm for more efficient utilization of radio spectrum. In fact, CRNs are based on the principles of spectrum sensing and dynamic spectrum access. However, recently as proposed by many researchers, secondary transmission may be done with the primary users (PUs) over the same spectrum band simultaneously on the condition that the harmful interference introduced by the secondary users (SUs) to the PU receiver is below an acceptable threshold, known as spectrum underlay [12].

In multi-hop wireless networks (MHWNs), congestion control mechanisms regulates source rates x_s according to the available capacity of wireless links l to guarantee that the offered load on any link does not exceed its capacity. The problem of joint congestion control and power control (JCPC) has been well-studied in [4], [13] via the underlying network utility maximization (NUM) under *physical interference* model:

$$\text{(JCPC problem)}: \quad \max_{\mathbf{x},\mathbf{P}\geq 0} \sum_{s} U_s(x_s)$$
$$s.t. \quad \sum_{s\in\mathcal{S}_l} x_s \leq C_l(\mathbf{P}), \qquad \forall l$$

where $\mathcal{S}_l$ is the set of sources traverse link l, and the capacity of wireless links is a *global* function of the transmit power vector $\mathbf{P}$ of interfering links. However, in multi-hop CRNs, the capacity of CR links strongly depends on not only the mutual interference among them but also spectrum that they are allowed to use. Hence, the end-to-end network utility of multi-hop CRNs calls for newly efficient spectrum and power allocation strategies coupled with congestion control mechanisms.

In this work, we investigate a spectrum and power allocation policy that the diversity of orthogonal licensed channels is taken into account for CR links along with a congestion control mechanism under *protocol interference* model, where a group of CR links is active on the same band if they make no interference with each other. In this regard, the mutual interference among CR links can be eliminated and thereby the capacity of CR link l is a function of its *local* power vector $\mathbf{P}_l$ and spectrum allocation vector $\mathbf{a}_l$. The key difference from JCPC problem is that the global dependence of optimization variables $(\mathbf{x}, \mathbf{P})$ in JCPC problem leads a non-convex optimization problem which calls for a globally optimal solution while our approach formulates the JRPS problem as a MINLP problem which can solved for the global optimum with a lower complexity. Some recent works [2], [9], [10], [17] model the interference relationship among wireless links using protocol interference model and propose *back-pressure-based* cross-layer scheduling algorithms for MHWNs with the same assumption of fixed transmit power of wireless links. It is shown that back-pressure based algorithm tends to explore all paths in a network (including loop paths and dead-end paths) that can not guide sources to their desired destination.

More specifically, some other works [11], [16] has studied the JRPS problem for multi-channel multi-radio MHWNs with an assumption of static spectrum. In this work, we investigate JRPS problem in spectrum underlay manner, which has not been addressed yet. Since the harmful interference emitted by the SUs can make the PU's reception unsuccessful, some existing studies [6], [14], [15] proposed the different solutions to optimally allocate power for the SUs in spectrum underlay fashion. In [6], Hasan *at al.* proposed the suboptimal and optimal algorithms to allocate power under a fixed power budget with a risk return model which considers the reliability and availability of licensed spectrum bands. The authors in [15] introduced the transmit power constraints and interference power constraints for the SUs. Son *at al.* in [14] introduced a new interference power outage constraint to protect the PUs along with a transmit power constraint for the SUs' power budget. The optimal and suboptimal algorithms to maximize the capacity of the SUs are derived in [14]. However, most above works focus on the CRNs with infrastructure where the secondary transmission only occurs in the single hop between the SUs and CR base station. Hence, the mutual interference among the SUs has not taken into consideration yet.

II. SYSTEM MODEL AND PROBLEM FORMULATION

We consider an OFDMA-based multi-hop CRN with a set of secondary nodes[1] $\mathcal{N} = \{1, ..., N\}$, a set of directed links $\mathcal{L} = \{1, ..., L\}$, and a set of orthogonal frequency bands $\mathcal{M} = \{1, ..., M\}$. Each band with bandwidth W is correspondingly licensed to one pair of PUs. We assume that each secondary node is equipped with multiple reconfigurable transceivers for data communication and one for signalling. Each CR link can switch to different licensed bands simultaneously during transmission. We further assume that the network is shared by a set of sources $\mathcal{S} = \{1, ..., S\}$ indexed by s. Each source $s \in \mathcal{S}$ traverses multiple hops to get its destination through the set of links, $\mathcal{L}_s \subseteq \mathcal{L}$, which is called a route. Without loss of generality, the routes of all source flows in this work are predefined. We also assume that each source has an infinite amount of data to send such that no delay constraint is considered.

A. Channel Assignment and Capacity Constraint

In a OFDMA-based multi-hop CRN, links are separated by the different bands in order to avoid co-channel interference. We define the band allocation variable $a_l^m \in \{0, 1\}$, where 1 indicates that band m is assigned to link l and 0 otherwise. Then, we have

$$\sum_{m=1}^{M} a_l^m = 1, \, \forall l \tag{1}$$

which states that each link l operates on at most one band at one time. Let η_l denote the thermal noise power under the baseband bandwidth W at the receiver of link l on band m. According to Shannon, the instantaneous capacity at the link l is given by:

$$C_l(\mathbf{P}_l, \mathbf{a}_l) = W \sum_{m=1}^{M} a_l^m \log\left(1 + \gamma_l^m P_l^m\right) \tag{2}$$

where $\mathbf{P}_l = [P_l^1, ..., P_l^M]$ and $\mathbf{a}_l = [a_l^1, ..., a_l^M]$ are power allocation vector and band allocation indicator vector of the lth link, respectively. We use the special index 0 to denote the primary link (e.g., P_0^m is the transmit power of PU-Tx on the band m). $\gamma_l^m = \frac{G_{ll}^m}{\eta_l + G_{l0}^m P_0^m}$ is channel gain-to-interference ratio (CIR) of link l on band m. Generally, G_{lk}^m represent the channel gain between the kth link's transmitter and the lth link's receiver on channel m. In this paper, we assume that $G_{lk}^m = d_{lk}^{-n}$, which only depends on the physical link distance d_{lk} with the path loss exponent n. For ease of exposition, without loss of generality, we assume that W is unit henceforth.

We assume that two CR links are interfering if they share a common node on the same band at the same time [7]. We define $\mathcal{I}_{-l}^m$ as the set of other links that interfere with the given link l on band m. We further assume that the interference relationship is symmetric, i.e., if link l interferes with link k, then link k also interferes with link l. The constraints

$$\sum_{l \in \mathcal{I}_l^m} a_l^m = 1, \quad \forall m \tag{3}$$

allow only one link $l \in \mathcal{I}_l^m \doteq \mathcal{I}_{-l}^m \cup l$ to transmit data on band m at any time instant. To avoid overwhelming link capacity, the offered load on each link does not exceed its capacity $C_l(\mathbf{P}_l, \mathbf{a}_l)$:

$$\sum_{s \in \mathcal{S}_l} x_s \leq \sum_{m=1}^{M} a_l^m \log\left(1 + \gamma_l^m P_l^m\right), \forall l \tag{4}$$

where $\mathcal{S}_l = \{s : l \in \mathcal{L}_s\}$ is the set of sources using link l.

B. PU Protection

Harmful interference introduced by SUs to the PUs occurs via two different forms: out-of-band emission (OBE) and in-band emission (IBE). OBE interference are due to SU's power leakages in the sidelobes of an OFDM signal while IBE interference is because the the SU's and PU's transmission concurrently occur on the same band. In this work, we assume that the PU system also employs an OFDMA scheme, where each band is licensed to each pair of PUs [3]. Hence, we can ignore the OBE interference. The IBE interference power introduced in a specific band, which is occupied by a pair of PUs on band m, as a result of the lth link's transmission with transmit power P_l^m, can be expressed as

$$J_l^m(d^m, P_l^m) = P_l^m \beta^m \tag{5}$$

where $\beta^m = T \int_{-W_m/2}^{+W_m/2} \phi_l^m(f) df$ denotes the interference factor, which depends on the OFDM symbol duration $T = 1/W$ and the power density spectrum (PSD) $\phi_l^m(f)$ of the mth subcarrier (e.g., $\phi_l^m(f) = \left(\frac{\sin \pi f T}{\pi f T}\right)^2$ [1]).

We assume that channel state information (CSI) between SU transmitters and PU receivers is known to SU transmitters, thereby SU system is allowed to transmit provided that the total interference power level caused by them at the PU receiver m is kept below a tolerable threshold μ^m:

$$\sum_{l} a_l^m G_{0l}^m J_l^m \leq \mu^m, \forall m \tag{6}$$

C. Problem Formulation

Our joint rate adaption, power control and spectrum allocation problem with the PU protection for multi-hop CRNs is

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[1] In this paper, the term "secondary user" and "secondary node" are interchangeably used.

formulated via NUM problem as following.

$$(\mathbf{P1}) \qquad \max_{\mathbf{x}\in\mathcal{X},\mathbf{P}\in\mathcal{P},\mathbf{a}} \sum_{s\in\mathcal{S}} U_s(x_s) \tag{7}$$

subject to

$$\sum_{s\in\mathcal{S}_l} x_s \le \sum_{m=1}^{M} a_l^m \log(1+\gamma_l^m P_l^m), \quad \forall l, \tag{8}$$

$$\sum_{l=1}^{L} a_l^m G_{0l}^m \beta^m P_l^m \le \mu^m, \quad \forall m, \tag{9}$$

$$\sum_{l\in\mathcal{I}_l^m} a_l^m = 1, \quad \forall m, \tag{10}$$

$$\sum_{m=1}^{M} a_l^m = 1, \quad \forall l, \tag{11}$$

$$a_l^m \in \{0,1\}, \forall m, l, \tag{12}$$

where $\mathcal{X} = \{x_s; s \in \mathcal{S} | x_s^{min} \le x_s \le x_s^{max}\}$ and $\mathcal{P} = \{P_l^m; l \in \mathcal{L}, m \in \mathcal{M} | P_l^{m,min} \le P_l^m \le P_l^{m,max}\}$ indicate quality of service (QoS) constraints for each source and power restriction for each CR node per link, respectively. The utility function $U_s(x_s)$ are assumed to be twice continuously differentiable, non-decreasing and strictly concave in its domain. It is straightforward that $\mathbf{P1}$ belongs to the class of MINLP problem. The optimal solution is known to be $\mathcal{NP}$-Hard in general.

III. CROSS-LAYER DESIGN VIA DUAL DECOMPOSITION

A. Lagrange Dual Problem

By augmenting the objective function with a weighted sum of the constraints (8) and (9), we obtain the partial Lagrangian of $\mathbf{P1}$:

$$L(\mathbf{x},\mathbf{P},\mathbf{a},\boldsymbol{\lambda},\boldsymbol{\nu}) = \sum_{s\in\mathcal{S}} U_s(x_s) - \\ -\sum_{l} \lambda_l \left(\sum_{s\in\mathcal{S}_l} x_s - \sum_{m} a_l^m \log(1+\gamma_l^m P_l^m) \right) \\ -\sum_{m} \nu_m \left(\sum_{l} a_l^m G_{0l}^m \beta^m P_l^m - \mu^m \right) \tag{13}$$

where $\boldsymbol{\lambda} = [\lambda_1, \lambda_2, ..., \lambda_L]$ and $\boldsymbol{\nu} = [\nu_1, \nu_2, ..., \nu_M]$ are the Lagrangian nonnegative multipliers which are interpreted as congestion prices and intereference prices, respectively. The former reflects the degree of congestion on CR links while the latter is on the interference status of licensed bands.

We refer to the problem $\mathbf{P1}$ as the *primal* problem, then it's *dual* problem can be described as

$$(\mathbf{D}) \qquad \min_{\boldsymbol{\lambda}\succeq 0, \boldsymbol{\nu}\succeq 0} g(\boldsymbol{\lambda},\boldsymbol{\nu}) \tag{14}$$

where

$$g(\boldsymbol{\lambda},\boldsymbol{\nu}) = \max_{\mathbf{x}\in\mathcal{X},\mathbf{P}\in\mathcal{P},\mathbf{a}} L(\mathbf{x},\mathbf{P},\mathbf{a},\boldsymbol{\lambda},\boldsymbol{\nu}) \\ \text{subject to} \quad (10),(11),(12). \tag{15}$$

B. Dual Algorithm

By dual decomposition, the maximization problem in (15) can be decomposed into two subproblems as in (16) and (17).

1) Subproblem 1 (Rate Adaption): As can clearly be observed from (16), L_x is strictly concave and separable in $\mathbf{x}$. Hence, given the multipliers $\boldsymbol{\lambda}$, the optimal rate can be found by Karush-Kuhn-Tucker (KKT) condition [5]:

$$x_s^*(\boldsymbol{\lambda}) = \left[U_s^{'-1} \left(\sum_{l\in\mathcal{L}_s} \lambda_l \right) \right]^{\mathcal{X}} \tag{18}$$

where $U_s^{'}(.)$ is the first derivative of utility and $[x]^{\mathcal{X}}$ is the projection of x onto the set $\mathcal{X}$. As a result, source s adjusts its rates according to the total congestion price along its path.

2) Subproblem 2 (Power Control and Spectrum Allocation): The subproblem (17) is combinatorial with respect to $\mathbf{a}$ while convex with respect to $\mathbf{P}$. It's main objective is how to allocate M bands to L different secondary links and corresponding transmit power per each band to maximize the sum of weights $\Gamma_l^m(P_l^m, \lambda_l, \nu_m) \forall m, l$ subject to the contention constraints (10), (11), and (12). Since the primal variables $\mathbf{P}$ and $\mathbf{a}$ appear in the sum of products, the subproblem (18) can be further decomposed into two subproblems:

Subproblem 2.1: Power Control

Given a pair of dual variables (λ_l, ν_m), optimize the transmit power P_l^m such that the weight $\Gamma_l^m(P_l^m, \lambda_l, \nu_m)$ in (17) is maximized. The optimal power can be obtained by Karush-Kuhn-Tucker (KKT) condition [5].

$$P_l^{m,*} = \left[\left(\frac{\lambda_l}{\nu_m} \frac{1}{G_{0l}^m \beta^m} - \frac{1}{\gamma_l^m} \right) \right]^{\mathcal{P}}. \tag{19}$$

Subproblem 2.2: Spectrum Allocation

In (17), given the optimized weights $\Gamma_l^m(P_l^m, \lambda_l, \nu_m)$, the spectrum allocation problem

$$\max_{\mathbf{a}} \sum_{l=1}^{L} \sum_{m=1}^{M} a_l^m \Gamma_l^m \qquad \text{s.t.} \quad (10),(11),(12). \tag{20}$$

is equivalent to a *maximum weighted matching problem* on the multi-band weighted conflict graph $G_c = (\mathcal{V}, \mathcal{E})$. In graph G_c, each vertex corresponds to a pair of link-band (l, m) associated with its weight Γ_l^m, thereby we have $|\mathcal{V}| = M \times L$. For constraints (10), the edge between two vertices in G_c corresponds to the link-band pairs interfere with each other. For constraints (11), there will be the additional edges between two vertices with the same link but different bands. Figure 1 shows an example of a simplified multi-hop CRN with two orthogonal bands and its weighted contention graph. In this example, we assume the model of two-hop interference [7] is used at link layer.

In Fig.1, for example, vertex $(a, 1, 3)$ corresponds to a pair of link a and band 1 associated with its weight 3. There is an edge between two vertices $(a, 1, 3)$ and $(b, 1, 4)$ because they interfere with each other. Additionally, there exists an edge between two vertices $(a, 1, 3)$ and $(a, 2, 1)$ because they come from the same link 1 but different channels.

$$\max_{\mathbf{x}\in\mathcal{X}} \left\{ L_x(\mathbf{x},\boldsymbol{\lambda}) \triangleq \sum_{s} U_s(x_s) - \sum_{l\in\mathcal{L}} \lambda_l \sum_{s\in\mathcal{S}_l} x_s \right\}. \tag{16}$$

$$\max_{\mathbf{P}\in\mathcal{P},\mathbf{a}} \left\{ L_{P,a}(\mathbf{P},\mathbf{a},\boldsymbol{\lambda},\boldsymbol{\nu}) \triangleq \sum_{l=1}^{L} \sum_{m=1}^{M} a_l^m \underbrace{\left[\lambda_l \log(1+\gamma_l^m P_l^m) - \nu_m G_{0l}^m \beta^m P_l^m\right]}_{\Gamma_l^m(P_l^m,\lambda_l,\nu_m)} \right\} \tag{17}$$

$$\text{subject to} \quad (11), (12), (13).$$

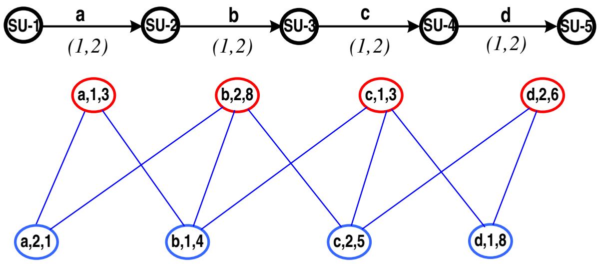

Fig. 1: Example of a multi-hop CRN with 5 SUs, 4 links, and 2 bands and the corresponding weighted conflict graph.

Then, the global optimal solution $\mathbf{a}^*$ can be found through the maximum independent set (MIS) of G_c (i.e., an independent set with maximum total weight). This MIS can achieved by the exhaustive search or augmenting path algorithms [8] in a centralized manner.

C. Primal Solution

Since $L(\mathbf{x},\mathbf{P},\mathbf{a},\boldsymbol{\lambda},\boldsymbol{\nu})$ is affine in $(\boldsymbol{\lambda},\boldsymbol{\nu})$, the optimal multipliers (λ_l^*,ν_m^*) to minimize $g(\boldsymbol{\lambda},\boldsymbol{\nu})$ can obtained using the projected gradient-descent method [5] as (21) and (22).

IV. OPTIMAL JOINT RATE ADAPTION, POWER CONTROL AND SPECTRUM ALLOCATION (JRPS-OP) ALGORITHM

The above dual algorithm motivates an optimal joint rate adaption, power control and band assignment design where each source s locally adjusts it's rates according to the aggregate congestion price $\sum_{l\in\mathcal{L}_s} \lambda_l^{(t)}$, and link l individually controls it's power per band and updates it's congestion prices and interference prices. The link layer nodes notify the central node their weights and local connectivity information such that the central node constructs a multi-band weighted conflict graph, computes MIS, and then sends feedback to them.

In JRPS-OP algorithm, we assume that the control messages between CR links and the central node are exchanged through a common control channel (CCC). All CR links and sources set the same initial values at the first iteration. The algorithm will stop whenever the convergence reached. For the sake of convenience, we use the same step-size k_t for all updates without loss of generality.

Algorithm 1: JRPS-OP Algorithm

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Sources and links initialize $\mathbf{x}^{(0)}, \mathbf{P}^{(0)}, \boldsymbol{\lambda}^{(0)}, \boldsymbol{\nu}^{(0)}$. At time $t$:
Source Algorithm: For each source $s \in \mathcal{S}$
    1) Receive the total price that accumulates the intermediate links' link price $\lambda_l^{(t)}$ along it's path through a feedback message from it's destination.
    2) Update rate $x_s^{(t+1)}$ using (18) with $\lambda_l^{(t)}$.
Link Algorithm: For each link $l \in \mathcal{L}$
    1) Receive spectrum allocation information $\mathbf{a}_l^{(t)}$ and interference price $\nu^{m,(t)}$ from the central node and transmit data on allocated band.
    2) Get ingress rates $\sum_{s\in S_l} x_s^{(t)}$ from input queue. Update congestion price $\lambda_l^{(t+1)}$ using (22).
    3) Update power $P_l^{m,(t+1)}$ using (19).
    4) Compute weights $\Gamma_l^m, \forall m \in \mathcal{M}$. Send it's weights, connectivity information, and powers to the central node.
Central node Algorithm:
    1) Receive the links' weights, connectivity information, and powers.
    2) Construct conflict graph $G_c$. Perform exhaust search algorithm to find MIS.
    3) Update interference prices $\nu^{m,(t+1)}, \forall m \in \mathcal{M}$ using (21).
    4) Send spectrum allocation information $\mathbf{a}$ and interference prices $\nu^{m,(t+1)}$ to links.
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Theorem 1. *For any initial source rate $\mathbf{x}^{(0)} \in \mathcal{X}$, link power $\mathbf{P}^{(0)} \in \mathcal{P}$, spectrum allocation $\boldsymbol{a} \in \{0,1\}$, and shadow prices $(\boldsymbol{\lambda}^{(0)},\boldsymbol{\nu}^{(0)}) \geq 0$, the sequence of primal-dual variables generated by **JRPS-OP** converges to the global optimum of the original problem $\mathbf{P1}$ provided that the stepsize satisfies*

$$\kappa_t \geq 0, \quad \sum_{t=0}^{\infty}(\kappa_t) = \infty, \quad \sum_{t=0}^{\infty}(\kappa_t)^2 < \infty. \tag{23}$$

Proof: For a given vector $\mathbf{a}$, the problem $\mathbf{P1}$ is convex. Hence, with any initial values, source rates (18), link powers (19) and dual variables $(\boldsymbol{\lambda},\boldsymbol{\nu})$ in (21) and (22) by JRPS-OP converge to the unique optimum with a sufficiently small step size κ_t satisfying (23) [5]. Since the global solutions of spectrum allocation $\mathbf{a}^{(\mathbf{t})}$ (obtained via MIS at each iteration t) keep improving their schedules according to the weights resulting from $(\mathbf{P}^{(t)},\boldsymbol{\lambda}^{(t)},\boldsymbol{\nu}^{(t)})$, they eventually reach the convergence condition. Hence, the resultant unique optimum is globally optimal. ■

$$\nu_m^{(t+1)} = \left[\nu_m^{(t)} + \kappa_t \left(\sum_{l\in\mathcal{L}} a_l^{m,(t)} G_{0l}^m \beta^m P_l^{m,(t)} - \mu^m \right)\right]^{\mathbf{R}_+} \tag{21}$$

$$\lambda_l^{(t+1)} = \left[\lambda_l^{(t)} + \kappa_t \left(\sum_{s\in\mathcal{S}_l} x_s^{(t)} - \sum_{m=1}^{M} a_l^{m,(t)} \log(1+\gamma_l^m P_l^{m,(t)}) \right)\right]^{\mathbf{R}_+} \tag{22}$$

V. PERFORMANCE EVALUATION

A. Simulation Settings

In this section, we evaluate the performance of the proposed algorithms JRPS-OP with JRPS-SOP. We consider a simplified multi-hop CRN with 5 secondary nodes, 4 flows, and 3 pairs of PUs with 3 corresponding licensed bands as shown in Fig.2. Each secondary link with a transmit power range from $1.76dBm$ to $27dBm$ is allocated one band and a power level at each period. Bandwidth of each band is $125KHz$. We assume that the distance $d = 1m$. The minimum data rate for each flow is assumed to be $100bps$. For PUs, we predefine the interference power thresholds for the licensed band 1, 2, and 3 are $1mW$, $72mW$, and $2.5mW$. The power spectrum density of white noise is assumed to be -80dBm/Hz at both PU and SU receivers. We choose $U_s(x_s) = log(x_s)$ as source's utility function for all secondary nodes.

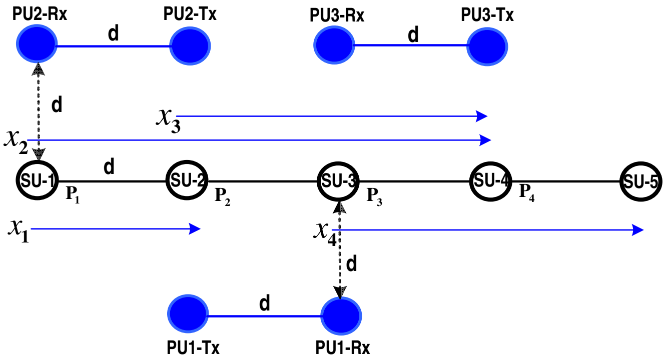


Fig. 2: Physical and logical topologies for simulation

TABLE I
THE OPTIMAL LINK POWERS AND ALLOCATED BANDS

P (mW)	JRPS-OP				JRPS-SOP			
	$l_{1,2}^3$	$l_{2,3}^2$	$l_{3,4}^1$	$l_{4,5}^2$	$l_{1,2}^3$	$l_{2,3}^2$	$l_{3,4}^1$	$l_{4,5}^3$
B_1	$--$	$--$	160	$--$	$--$	$--$	73	$--$
B_2	$--$	395	$--$	1.5	$--$	397	$--$	$--$
B_3	500	$--$	$--$	$--$	500	$--$	$--$	1.5

B. Numerical Results

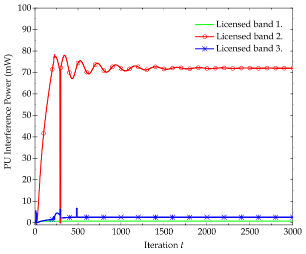
First, we investigate the optimality of algorithm JRPS-OP by randomly assigning the licensed bands to all links provided that satisfying contention constraints (10), (11), and (12) and fix them during power and rate update period. For ease of presentation, we call this method JRPS-SOP. Fig. 3 shows the interference powers caused by SU system converge to the target thresholds allowed by PU system while Fig. 4 illustrates the balance of the link capacity C_l and it's ingress rate (i.e., $\sum_{s\in\mathcal{S}_l} x_s$) at the optimum for both JRPS-OP and JRPS-SOP. More importantly, Fig. 4 and 5 show that JRPS-OP significantly outperforms JRPS-SOP, demonstrating that not only power and rate control but also spectrum allocation are jointly designed to obtain the maximum network utility under contention-limited spectrum sharing system. Hence, the simulation results prove that the solution of JRPS-OP is globally optimal for the JRPS problem **P1**.

Next, we examine the energy and spectrum efficiency of JRPS-OP. Table 1 shows the optimal powers and bands for each link for both JRPS-OP and JRPS-SOP. Power per band on each link is optimally controlled to meet exactly rate requirements which fairly maximize source's utility. For example, link between SU-4 and SU-5 only transmits at the minimum power while link between SU-1 and SU-2 transmits at the maximum power. This is because the source 1's rate demand and spectrum opportunity on band 3 is the highest and while source 4 suffers both the most congested and lowest spectrum opportunity on band 2 (shared with the more congested link between SU-2 and SU-3 for JRPS-OP) and band 3 (shared with the more congested link between SU-1 and SU-2 for JRPS-SOP). Moreover, in JRPS-OP, weight of links on all bands are always calculated and depends on congestion prices and interference prices. Hence, links will properly switch to the best band without energy wastage. Fig. 5b shows that the SU system can exploit more spectrum to get the higher network utility if the PUs' interference power threshold increases.

Finally, we consider the convergence speed of algorithms. Fig. 5a and Fig. 6 shows the trajectory the network utility and source rates for both JRPS-OP and JRPS-SOP. Clearly, they converge to the optimal points within reasonable times, and the algorithms's convergence speed are almost indistinguishable. However, there is a very small oscillation of SUs' flow rates in JRPS-OP and in practice, JRPS-OP's convergence speed will be slower due to additional message passing for spectrum allocation.

VI. CONCLUSION

In this paper, we proposed a cross-layer framework for the problem of joint rate adaption, power control and spectrum allocation under a contention-limited spectrum underlay system for OFDMA-based multi-hop CRNs. Our major objective is to deploy the diversity of multiple orthogonal licensed bands for CR links and efficiently solve the problem of congestion and interference to maximize the overall utility. The proposed



(a) JRPS-OP (b) JRPS-SOP

Fig. 3: Convergence of interference powers at PUs.

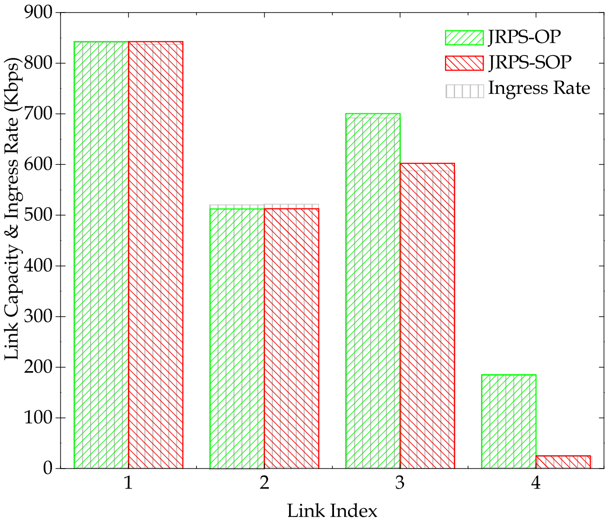


Fig. 4: Comparison of link capacity of algorithms

algorithm JRPS-OP is proved to converge to the global optimal solution.

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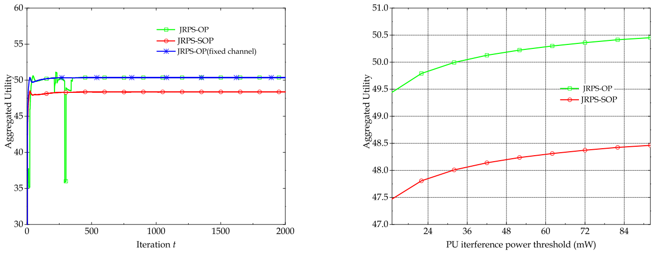
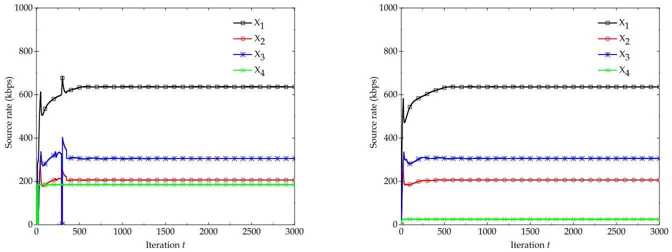


(a) (b)

Fig. 5: Trajectory of Aggregated Utility (a) and Utility versus varying PU-Rx 2's interference power threshold while fixed on others (b).



(a) JRPS-OP (b) JRPS-SOP

Fig. 6: Convergence of Source Rates.

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