This full text paper was peer reviewed at the direction of IEEE Communications Society subject matter experts for publication in the IEEE Globecom 2011 proceedings.

Joint Rate and Power Control for Elastic and Inelastic Traffic in Multihop Wireless Networks

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***Abstract*—The current optimal joint rate and power control algorithms for wireless networks are mainly for elastic traffic which has strictly concave utility functions. In the multiclass service networks such that elastic and inelastic, the inelastic traffic is usually associated with the sigmoidal utilities which are nonconcave functions. Therefore, the corresponding Network Utility Maximization (NUM) problem is nonconvex in both objective and constraints. The current approaches cannot be applied and the problem is difficult to solve for the global optimal solution even with the centralized method. This paper proposes the joint rate and power control algorithm which can be distributively implemented. We approximate the nonconvex NUM to the convex problem which is easily solved by dual decomposition method. After a series of approximations, the algorithm converges the local optimal solution.**

I. Introduction

In the multihop wireless networks, the current works for optimal rate and power allocation are mainly for elastic traffic with the assumption of strictly concave utilities, [1]–[3]. The network utility maximization (NUM) problem is nonconvex because of the nonconvexity of the link capacity constraints. The author in [1] transforms the NUM problem into convex form by the high SIR assumption and solves it using dual decomposition method. In [3], the constraints in NUM problem is approximated to a convex form. After a series of approximations, the solution converges to the KKT point of the original problem without the high SIR assumption.

The concave utilities are merely suitable for elastic traffic such as the traffic of non-real-time applications. The inelastic traffic of real-time applications is usually modeled by sigmoidal functions which is convex in the lower half and concave in the upper half (see Figure 1), [4]. Hence, the joint rate and power NUM problem is nonconvex in both objective and constraints and it cannot be solved by the methods in [1], [3].

On the other hand, the current works that address the NUM for rate allocation of inelastic traffic directly solve the problem by using standard dual decomposition method, [5]–[7]. Certainly, the standard dual-based algorithm does not always converge to the global optimum because of the nonconvexity of the objective. In [6], [7], the authors find the conditions for which the algorithm converges to the global optimal solution. It turns out that the link capacity must greater than the critical values. Using the Sum-of-Square method, the nonconvex problem is relaxed and solved by semidefinite programming, [8]. However, this method requires a centralized computation.

Designing a distributed rate allocation for the multiclass service networks is a hard problem. Combining rate control and power control makes the problem be even harder, the NUM is nonconvex in both objective and constraints. We propose a distributed algorithm that converges to the suboptimal solution for any link capacity. Instead of solving NUM problem directly, we replace it by another equivalent problem (Problem 2) with has same optimal/suboptimal allocation, then we approximate this new problem to a convex problem (Problem 5) which is efficiently solved by dual decomposition method. After many approximations, the result converges to the KKT point of the original problem which is the local optimal solution. The successive approximation approach is introduced in [9] and usually applied to geometric programming in power control problems, [2], [3], [10]. [10] has a clearly introduction about this method. In this paper, we make the approximations for both utilities and constraints to overcome the nonconvexity.

The structure of the paper is as follows. Section I of our paper introduces the motivation and related works. Section II analysis and proposes the joint rate and power control algorithm. And finally, the simulation and conclusions are given in section III and IV, respectively.

II. Analysis

A. Network model

In the multihop wireless networks with the sets of sources and links are $\mathcal{N}$ and $\mathcal{L}$, and N and L are their cardinalities respectively, the NUM problem for joint rate and power control is stated as

$$\text{Problem 1}: \quad \text{Max.} \sum_{s\in\mathcal{N}} U_s(x_s)$$
$$\text{s.t.} \sum_{s:l\in L(s)} x_s \le c_l(\mathbf{P}),\ \forall l \in \mathcal{L}$$
$$\mathbf{P}^{\min} \le \mathbf{P} \le \mathbf{P}^{\max},$$

where x_s is the rate of source s; P_l is the transmitting power of link l; and $c_l(\mathbf{P})$ is the capacity of link l, a function of

This research was supported by Basic Science Research Program through the National Research Foundation of Korea(NRF) funded by MEST (No. 2010-0027645).

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978-1-4244-9268-8/11/$26.00 ©2011 IEEE

power vector, [1].

In the wireless systems without perfect orthogonal channels such as CDMA, a receiver node is interfered by transmitter nodes of all other links. Thus, the link capacity is calculated by

$$c_l(\mathbf{P}) = W\log(1+K\mathrm{SIR_l}), \forall \mathrm{l} \in \mathcal{L}, \quad (1)$$

where $\mathrm{SIR_l}(\mathbf{P}) = \frac{\mathrm{P_l G_{ll}}}{\sum_{\mathrm{k}\neq \mathrm{l}} \mathrm{P_k G_{lk}} + \mathrm{n_l}}$, G_{lk} is the channel gain from transmitter of link k to link l, W is the baseband bandwidth, and K is a constant depending on the modulation, coding scheme, and bit-error rate (BER). For the sake of brevity, we assume that $W = 1$ and K is absorbed in the channel gain G_{ll}. Hence, we can rewrite the capacity formula (1) as follows

$$\begin{aligned} c_l(\mathbf{P}) &= \log(1+\mathrm{SIR_l}) \\ &= \log(\sum_{k\in\mathcal{L}} G_{lk}P_k + n_l) - \log(\sum_{k\neq l} G_{lk}P_k + n_l), \forall l \in \mathcal{L}. \end{aligned} \quad (2)$$

We consider two groups of utilities in this paper (see Figure 1). The concave utilities for elastic flows

$$U(x) = \begin{cases} \log(x+1), & \text{if } \alpha = 1, \\ \frac{(x+1)^{(1-\alpha)}-1}{1-\alpha}, & \text{if } \alpha \in (0,1)\cup(1,\infty) \end{cases} \quad (3)$$

and the sigmoidal utilities for inelastic flows

$$U(x) = \frac{1}{1+\mathrm{e}^{-\mathrm{a(x-b)}}}, \forall a > 0, b > 0. \quad (4)$$

It can be seen that Problem 1 is a nonconvex problem because of the nonconcavity of (2) and (4). In the next section, we will present the method to transform and approximate them to the concave functions. (We use the italic characters to denote the variables and the bold characters to denote the vectors in the paper. For example, x_s is the rate of source s and **x** is the rate vector with elements are rates of all sources.)

B. Convex approximation method

Instead of solving Problem 1 directly, we replace Problem 1 by the following problem

$$\begin{aligned} \text{Problem 2 :} \quad & \text{Max. } \log\Big(\sum_{s\in\mathcal{N}} U_s(x_s)\Big) \\ & \text{s.t. } \sum_{s:l\in L(s)} x_s \leq c_l(\mathbf{P}) \\ & \mathbf{P}^{\min} \leq \mathbf{P} \leq \mathbf{P}^{\max}. \end{aligned}$$

Result 1: Problem 1 and Problem 2 share the same optimal/suboptimal rate and power allocation.

Proof: The proof is quite straightforward because the logarithm is a monotonically increasing function. ■

For the use of successive approximations method which requires the nonconvex problem must have the convex-form objective, [9], we transform Problem 2 to the new equivalent problem in epigraph form, [11, p. 4.2.4],

$$\begin{aligned} \text{Problem 3 :} \quad & \text{Max. } t \\ & \text{s.t. } t \leq \log\Big(\sum_{s\in\mathcal{N}} U_s(x_s)\Big) \\ & \sum_{s:l\in L(s)} x_s \leq c_l(\mathbf{P}), \forall l \in \mathcal{L} \\ & \mathbf{P}^{\min} \leq \mathbf{P} \leq \mathbf{P}^{\max}. \end{aligned}$$

For all vectors $\boldsymbol{\gamma} = [\gamma_1, .., \gamma_N]$, $\boldsymbol{\theta}^l = [\theta_1^l, .., \theta_{L+1}^l]$, $\forall l = 1, .., L$, such that $\boldsymbol{\gamma} \succ 0$, $\mathbf{1}^T\boldsymbol{\gamma} = 1$, $\boldsymbol{\theta}^l \succ 0$, and $\mathbf{1}^T\boldsymbol{\theta}^l = 1$, let

$$\tilde{U}_s^{\gamma_s}(x_s) \triangleq \gamma_s \log\left(\frac{U_s(x_s)}{\gamma_s}\right), \text{ and} \quad (5)$$

$$\begin{aligned} \hat{c}_l^{\boldsymbol{\theta}^l}(\mathbf{P}) \triangleq & \sum_{l\in\mathcal{L}} \theta_k^l \log\left(\frac{G_{lk}P_k}{\theta_k^l}\right) + \theta_{L+1}^l \log\left(\frac{n_l}{\theta_{L+1}^l}\right) \\ & - \log\Big(\sum_{k\neq l} G_{lk}P_k + n_l\Big), \end{aligned} \quad (6)$$

we have the following inequalities.

Result 2:

$$\log\Big(\sum_{s\in\mathcal{N}} U_s(x_s)\Big) \geq \sum_{s\in\mathcal{N}} \tilde{U}_s^{\gamma_s}(x_s), \text{ and} \quad (7)$$

$$c_l(\mathbf{P}) \geq \hat{c}_l^{\boldsymbol{\theta}^l}(\mathbf{P}), \quad l = 1, .., L+1. \quad (8)$$

Proof: From the arithmetic-geometric mean inequality, we have

$$\sum_{s\in\mathcal{N}} U_s(x_s) \geq \prod_{s\in\mathcal{N}} \left(\frac{U_s(x_s)}{\gamma_s}\right)^{\gamma_s}. \quad (9)$$

Taking the logarithm of both sides of the inequality, we obtain (7). The equality holds if and only if

$$\gamma_k = \frac{U_k(x_k)}{\sum_{s\in\mathcal{N}} U_s(x_s)}, \forall k = 1, .., N. \quad (10)$$

Similarly, we also have the following inequality from applying the arithmetic-geometric mean inequality

$$\sum_{k\in\mathcal{L}} G_{lk}P_k + n_l \geq \prod_{k\in\mathcal{L}} \left(\frac{G_{lk}P_k}{\theta_k^l}\right)^{\theta_k^l} \left(\frac{n_l}{\theta_{L+1}^l}\right), \forall l = 1, .., L. \quad (11)$$

Taking the logarithm of both sides of above inequality,

$$\begin{aligned} \log\Big(\sum_{k\in\mathcal{L}} G_{lk}P_k + n_l\Big) \geq & \sum_{k\in\mathcal{L}} \theta_k^l \log\left(\frac{G_{lk}P_k}{\theta_k^l}\right) \\ & + \theta_{L+1}^l \log\left(\frac{n_l}{\theta_{L+1}^l}\right), \forall l = 1, .., L. \end{aligned}$$

With the use of link capacity formula (2), we get (8). The equality holds if and only if

$$\begin{aligned} \theta_k^l &= \frac{G_{lk}P_k}{\sum_{k\in\mathcal{L}} G_{lk}P_k + n_l} = \frac{G_{kk}\mathrm{SIR_k}}{G_{lk} + G_{kk}\mathrm{SIR_k}}, \forall k = 1, .., L, \\ \theta_{L+1}^l &= \frac{n_l}{\sum_{k\in\mathcal{L}} G_{lk}P_k + n_l}. \end{aligned} \quad (12)$$

Particularly,

$$\theta_l^l = \frac{\mathrm{SIR_l}}{\mathrm{SIR_l}+1}, \;\; \forall l = 1,..,L. \tag{13}$$

■

Inspired by Result 2, we approximate Problem 3 to the new problem with the convex approximation constraints as follows

$$\begin{aligned}\text{Problem 4}: \quad & \text{Max. } t \\ & \text{s.t. } t \le \sum_{s\in\mathcal{N}} \tilde{U}_s^{\gamma_s}(x_s) \\ & \sum_{s:l\in L(s)} x_s \le \tilde{c}_l^{\boldsymbol{\theta}^l}(\mathbf{P}), \;\forall l \in \mathcal{L} \\ & \mathbf{P}^{\min} \preceq \mathbf{P} \preceq \mathbf{P}^{\max}.\end{aligned}$$

Denoting $\tilde{P}_l \triangleq \log(P_l), \forall l \in \mathcal{L}$ and transforming Problem 4 back to the canonical form, the new approximate problem is given by

$$\begin{aligned}\text{Problem 5}: \quad & \text{Max. } \sum_{s\in N} \tilde{U}_s^{\gamma_s}(x_s) \\ & \text{s.t. } \sum_{s:l\in L(s)} x_s \le \tilde{c}_l^{\boldsymbol{\theta}^l}(\tilde{\mathbf{P}}), \;\forall l \in \mathcal{L} \\ & \tilde{\mathbf{P}}^{\min} \preceq \tilde{\mathbf{P}} \preceq \tilde{\mathbf{P}}^{\max},\end{aligned}$$

where $\tilde{P}_l^{\min} \triangleq \log(P_l^{\min})$, $\tilde{P}_l^{\max} \triangleq \log(P_l^{\max})$, and

$$\begin{aligned}\tilde{c}_l^{\boldsymbol{\theta}^l}(\tilde{\mathbf{P}}) \triangleq & \sum_l \theta_k^l \tilde{P}_k + \sum_l \theta_k^l \log\left(\frac{G_{lk}}{\theta_k^l}\right) + \theta_{L+1}^l \log\left(\frac{n_l}{\theta_{L+1}^l}\right) \\ & - \log\left(\sum_{k\ne l} G_{lk} \mathrm{e}^{\tilde{P}_k} + \mathrm{n_l}\right), \;\; \forall l \in \mathcal{L}.\end{aligned} \tag{14}$$

Result 3: The functions $\tilde{U}_s^{\gamma_s}(x_s), \forall s \in \mathcal{N}$ and $\tilde{c}_l^{\boldsymbol{\theta}^l}(\tilde{\mathbf{P}}), \forall l \in \mathcal{L}$ are concave.

Proof: $\tilde{c}_l^{\boldsymbol{\theta}^l}(\tilde{\mathbf{P}})$ is concave because it has the form of subtracting a log-sum-exp function from an affine function (log-sum-exp function is convex [11, p. 3.1.5]). $\tilde{U}_s^{\gamma_s}(x_s)$ is concave function because sigmoidal functions are log-concave and logarithm of concave functions are also concave. ■

From Result 3, Problem 5 is a convex optimization problem. Hence, it is efficiently solved by Lagrange dual decomposition method. Note that Problem 4 is the approximation of Problem 3, thus Problem 5 is the approximation of Problem 2, not Problem 1.

C. Solution to the approximation problem

We apply the Lagrange dual decomposition method to solve the approximate problem, the dual function is given by

$$\begin{aligned}D(\boldsymbol{\lambda}) =& \max_{\mathbf{x}\succeq 0, \tilde{\mathbf{P}}^{\min}\preceq\tilde{\mathbf{P}}\preceq\tilde{\mathbf{P}}^{\max}} \left(L(\mathbf{x},\boldsymbol{\lambda})\right) \\ =& \max_{\mathbf{x}\succeq 0, \tilde{\mathbf{P}}^{\min}\preceq\tilde{\mathbf{P}}\preceq\tilde{\mathbf{P}}^{\max}} \Bigg(\sum_{s\in\mathcal{N}} \tilde{U}_s^{\gamma_s}(x_s) \\ & - \sum_{l\in\mathcal{L}} \lambda_l \Big(\sum_{s:l\in L(s)} x_s - \tilde{c}_l^{\boldsymbol{\theta}^l}(\tilde{\mathbf{P}}) \Big) \Bigg) \\ =& \max_{\mathbf{x}\succeq 0} \left(\sum_{s\in\mathcal{N}} \left(\tilde{U}_s^{\gamma_s}(x_s) - (\sum_{l\in L(s)} \lambda_l) x_s \right) \right) \\ & + \max_{\tilde{\mathbf{P}}^{\min}\preceq\tilde{\mathbf{P}}\preceq\tilde{\mathbf{P}}^{\max}} \left(\sum_{l\in\mathcal{L}} \lambda_l \tilde{c}_l^{\boldsymbol{\theta}^l}(\tilde{\mathbf{P}}) \right) \\ =& \max_{\mathbf{x}\succeq 0} L_x(\mathbf{x},\boldsymbol{\lambda}) + \max_{\tilde{\mathbf{P}}^{\min}\preceq\tilde{\mathbf{P}}\preceq\tilde{\mathbf{P}}^{\max}} L_P(\tilde{\mathbf{P}},\boldsymbol{\lambda}),\end{aligned} \tag{15}$$

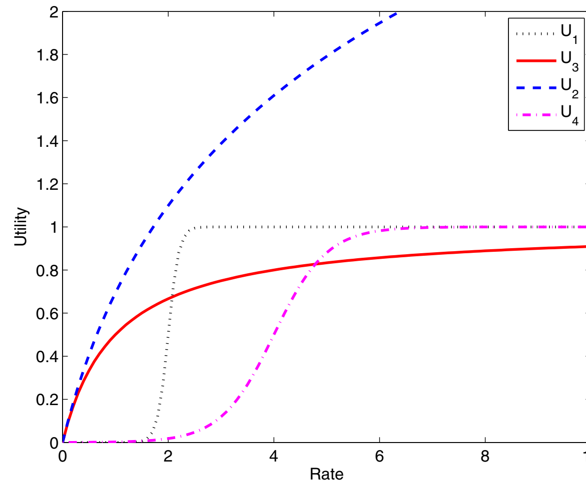

Fig. 1. Utility functions: $U_1(x) = \frac{1}{1+\mathrm{e}^{-10(x-2)}}$, $U_2(x) = \log(x+1)$, $U_3(x) = \frac{x}{x+1}$, and $U_4(x) = \frac{1}{1+\mathrm{e}^{-2(x-4)}}$.

where $L_x(\mathbf{x},\boldsymbol{\lambda}) \triangleq \sum_{s\in\mathcal{N}} \left(\tilde{U}_s^{\gamma_s}(x_s) - (\sum_{l\in L(s)} \lambda_l) x_s\right)$ and $L_P(\tilde{\mathbf{P}},\boldsymbol{\lambda}) \triangleq \sum_{l\in\mathcal{L}} \lambda_l \tilde{c}_l^{\boldsymbol{\theta}^l}(\tilde{\mathbf{P}})$.

The dual problem is

$$\min_{\boldsymbol{\lambda}\succeq 0} D(\boldsymbol{\lambda}). \tag{16}$$

Solving the first subproblem, we get the *rate update* as follows

$$x_s(t+1) = \tilde{U}_s^{\gamma_s' -1}(q_s), \forall s \in \mathcal{N}, \tag{17}$$

where $q_s = \sum_{l\in L(s)} \lambda_l(t)$ is the path price of source s.

Solving the second subproblem

$$\frac{\partial L_P(\tilde{\mathbf{P}},\boldsymbol{\lambda})}{\partial \tilde{P}_l} = \lambda_l \theta_l^l - \sum_{k\ne l} \frac{\lambda_k G_{kl} \mathrm{e}^{\tilde{P}_l}}{\sum_{j\ne k} G_{kj} \mathrm{e}^{\tilde{P}_j} + n_k} = 0, \forall l \in \mathcal{L} \tag{18}$$

and transforming (18) back to $\mathbf{P}$ space, we obtain the *power update*

$$P_l(t+1) = \left[\frac{\lambda_l(t)\theta_l^l}{\sum_{k\ne l} G_{kl} m_k(t)}\right]_{P_l^{\min}}^{P_l^{\max}}, \forall l \in \mathcal{L} \tag{19}$$

where $[a]_c^b = \min(\max(a,c),b)$ and

$$m_k(t) = \frac{\lambda_k(t)\mathrm{SIR_k}(t)}{P_k(t) G_{kk}}. \tag{20}$$

We use the gradient method to solve the dual problem (16), the *price update* is given by

$$\lambda_l(t+1) = \Big[\lambda_l(t) + \kappa\Big(\sum_{s:l\in L(s)} x_s(t) - \tilde{c}_l^{\boldsymbol{\theta}^l}(\tilde{\mathbf{P}})\Big)\Big]^+, \forall l \in \mathcal{L}, \tag{21}$$

where κ is a small enough step-size for the convergence of the algorithm, $[a]^+ = \max(a, 0)$.

D. Successive approximation algorithm for joint rate and power control

Algorithm 1: Initialize $\mathbf{x} = 0$, $\mathbf{P} = 0$, $\boldsymbol{\gamma} = 0$, and $\boldsymbol{\theta} = 0$. In the τ-th iteration,

1) Updating $\boldsymbol{\gamma}(\tau)$ and $\boldsymbol{\theta}(\tau)$ with (10) and (13).
2) (Inner iterations) Updating the rate, power, and congestion prices according to (17), (19), and (21) respectively to solve Problem 5 until convergence to the stationary point $\mathbf{x}^o(\tau), \mathbf{P}^o(\tau)$.
3) Increasing τ and go back to step 1.

Theorem 1: Algorithm 1 converges to the stationary point satisfying the Karush-Kuhn-Tucker conditions of Problem 2.

Proof: Denoting $f_u(\mathbf{x}) \triangleq \frac{t}{\log(\sum_s U_s(x_s))}$, $\tilde{f}_u(\mathbf{x}) \triangleq \frac{t}{\sum_s \tilde{U}_s^{\gamma_s}}(x_s)$, $f_c(\mathbf{x},\mathbf{P}) \triangleq \frac{\sum_{s:l\in L(s)} x_s}{c_l(\mathbf{P})}$, and $\tilde{f}_c(\mathbf{x},\mathbf{P}) \triangleq \frac{\sum_{s:l\in P(s)} x_s}{\hat{c}_l(\mathbf{P})}$. According to [9], [10], we need to prove the following three conditions for the convergence to the KKT point of the algorithm:

1) $f_u(\mathbf{x}) \le \tilde{f}_u(\mathbf{x})$ and $f_c(\mathbf{x},\mathbf{P}) \le \tilde{f}_c(\mathbf{x},\mathbf{P})$
2) $f_u(\mathbf{x}^o) = \tilde{f}_u(\mathbf{x}^o)$ and $f_c(\mathbf{x}^o,\mathbf{P}^o) = \tilde{f}_c(\mathbf{x}^o,\mathbf{P}^o)$
3) $\nabla f(\mathbf{x})|_{\mathbf{x}=\mathbf{x}^o} = \nabla \tilde{f}(\mathbf{x})|_{\mathbf{x}=\mathbf{x}^o}$ and $\nabla f_c(\mathbf{x},\mathbf{P})|_{\mathbf{x}=\mathbf{x}^o;\mathbf{P}=\mathbf{P}^o} = \nabla \tilde{f}_c(\mathbf{x},\mathbf{P})|_{\mathbf{x}=\mathbf{x}^o;\mathbf{P}=\mathbf{P}^o}$, where $(\mathbf{x}^o,\mathbf{P}^o)$ is the optimal solution of the previous iteration.

Condition 1) and 2) are clearly satisfied by (7) and (8). It is straightforward to verify condition 3) by taking partial derivatives. ■

The convergence point of Algorithm 1 is also the local optimal solution according to [9]. Note that the convergence point of Algorithm 1 is the KKT point of Problem 2, not Problem 1. We can verify that if the triplet $(\mathbf{x}^*, \mathbf{P}^*, \mathbf{q}^*)$ is a KKT point of Problem 2 then $(\mathbf{x}^*, \mathbf{P}^*, (\sum_{s\in\mathcal{N}} U_s(x_s^*))\mathbf{q}^*)$ is a KKT point of Problem 1. Moreover, $(\mathbf{x}^*, \mathbf{P}^*)$ is the local optimum of Problem 1 due to the monotonically increasing property of logarithm function.

Remark 1: Distributed implementation
At inner iterations of Algorithm 1, each link l calculates the value m_l locally according to (20) and pass this information to all other links in the network for the power update in (19). At outer iterations, we need to update $\boldsymbol{\gamma}$ and $\boldsymbol{\theta}$. $\theta_l^l, \forall l \in \mathcal{L}$ is updated locally in (13). Each source s calculates its utility and passes this information to all other sources in the network for $\boldsymbol{\gamma}$ updates in (10).

Remark 2: Log-concave utility
The conditions on utility functions which the above analysis can be applied are that the utilities must be positive, increasing, and continuously differentiable functions. Moreover, $\gamma \log(\frac{U(x)}{\gamma})$ must be concave. In the other words, $U(x)$ must be log-concave functions or $UU'' \le U'^2$, for example,

- all positive, increasing, and continuously differentiable concave functions,
- polynomials having real roots such as x, x^2, $x(x+1)$,...
- sigmoidal-like functions $\frac{x^a}{k+x^a}$, where $a > 1, k > 0$.

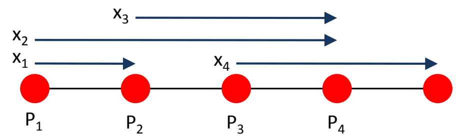

Fig. 2. Network topology

III. Simulation

We consider the network topology as Figure 2. The parameters of the simulation are same as in [3]: $W = 1$ MHz; $K = -1.5/\log(5\text{BER})$ with $\text{BER} = 10^{-3}$ for MQAM modulation. The channel gains are calculated by $h(d) = h_o(d/15)^{-4}$, where h_o is a reference channel gain at a distance 15 m. The maximum and minimum power are $\mathbf{P}^{\max} = 100$ mW and $\mathbf{P}^{\min} = 5$ mW. The utility functions of flows 1, 2, 3, and 4 are $U_1(x_1) = \frac{1}{1+\mathrm{e}^{-10(x_1-2)}}$, $U_2(x_2) = \log(x_2+1)$, $U_3(x_3) = \frac{x_3}{x_3+1}$, and $U_4(x_4) = \frac{1}{1+\mathrm{e}^{-2(x_4-4)}}$, respectively. The unit of rate for calculating utility is Mbps.

For each type of considering utility functions, we calculate the rate updates from (17) as follows:

1) $x_1(t+1) = 2 - \frac{1}{10}\log(\frac{q_1}{10\gamma_1 - q_1})$,
2) $x_2(t+1) = \frac{\gamma_2/q_2}{W(\gamma_2/q_2)} - 1$, where $W(.)$ is the Lambert W-function, the inverse function of $f(W) = W\mathrm{e}^W$,
3) $x_3(t+1) = \sqrt{\frac{1}{4} + \frac{\gamma_3}{q_3}} - \frac{1}{2}$, and
4) $x_4(t+1) = 4 - \frac{1}{2}\log(\frac{q_4}{2\gamma_4 - q_4})$.

In the implementation, we can consider the algorithm gets the stationary point if $\frac{|\sum_{s\in\mathcal{N}} U_s(x_s(t+1)) - \sum_{s\in\mathcal{N}} U_s(x_s(t))|}{|\sum_{s\in\mathcal{N}} U_s(x_s(t))|} < \epsilon$ for inner iterations and $\frac{|\sum_{s\in\mathcal{N}} U_s(x_s^o(\tau+1)) - \sum_{s\in\mathcal{N}} U_s(x_s^o(\tau))|}{|\sum_{s\in\mathcal{N}} U_s(x_s^o(\tau))|} < \epsilon$ for outer iterations. The error bound in our simulation is $\epsilon = 10^{-4}$. With the above setup parameters, Algorithm 1 converges after 16 outer iterations. Figure 3 and 4 show the convergence of the rate and power allocation for each user. The local optimal values are $\mathbf{x}^* = [0\ 1.0891\ 2.0590\ 3.8954]$ Mbps and $\mathbf{P}^* = [0.0081\ 0.0113\ 0.0252\ 0.0050]$ W along with $\boldsymbol{\gamma}^* = [0.0000\ 0.3966\ 0.3623\ 0.2411]$ and $\boldsymbol{\theta}_l^{l*} = [0.2348\ 0.6049\ 0.9460\ 0.7080]$.

IV. Conclusions

We have solved and proposed the joint rate and power allocation algorithm for the wireless system supporting multiclass services. The nonconvex problem is approximated to a convex problem, and after a series of approximations, our algorithm converges to the local optimal solution. The simulation has shown the convergence of the algorithm.

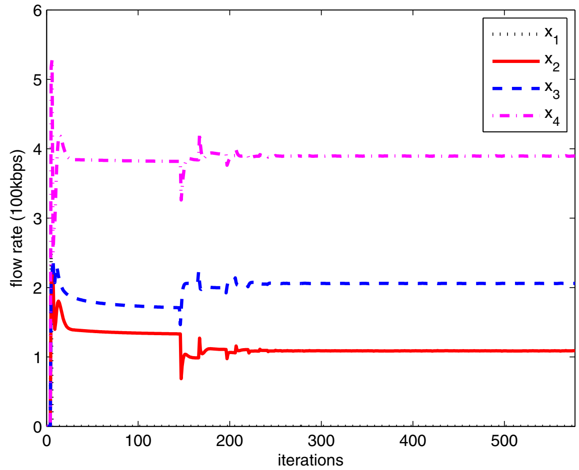

Fig. 3. Convergence of rate

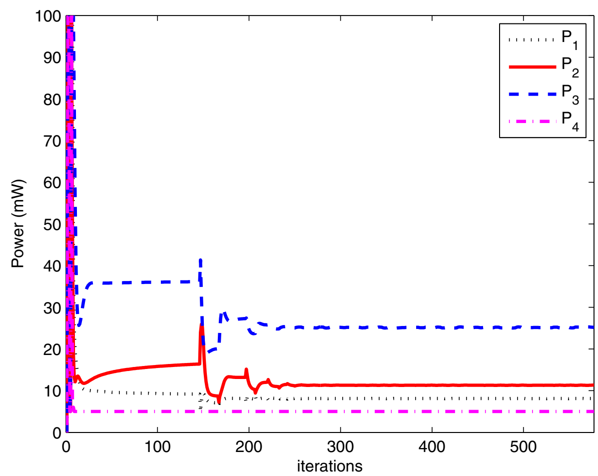

Fig. 4. Convergence of power

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