Queuing Analysis for Overlay/Underlay Spectrum Access in Cognitive Radio Networks

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Abstract
In this paper, we present theoretical queuing analysis for hybrid overlay/underlay Cognitive Radio (CR) system by applying M/M/1 queuing model where the rate of arrival and the service capacity are subject to Poisson alterations. Numerical results are used to prove a high degree of accuracy for the derived expressions. The result can be used as a benchmark to evaluate the performance of a hybrid overlay/underlay CR system.

1. Introduction

Recently overlay/underlay framework [1] in Cognitive Radio has been studied and it demonstrated the benefits such as spectrum efficiency and channel capacity maximization. Arising from these works, we supposed the secondary user can operate under overlay mode when the primary user is absent and operate under underlay mode when the primary user is present. It leads to improving the spectrum utilization for secondary transmissions. According to our best knowledge, analyzing sojourn time and queue length of this type of system has not been performed before.

2. System model

We consider a CR system with a single Primary User (PU) band. PUs have a license to use the band. And when the primary users wish to transmit, it is given a priority over Secondary User (SU). This is implemented by having the secondary user performs spectrum sensing with perfect sensing assumed. If there is no signal of the primary users, the secondary user will operate under overlay framework. Otherwise if the band is occupied by the primary users, the secondary user will operate under underlay framework. The situation shown in Fig.1 can be interpreted to be an example of the hybrid overlay/underlay spectrum access framework. We assume that PU sojourn time (i.e., the amount of time that PU band is in state ON) is random and exponentially distributed with mean $1/\eta$. And the amount of time that elapses between the end of a sojourn and the starting of the next sojourn (i.e., the amount of time that PU band is in state OFF) is random and exponential with parameter ξ. The PU band can be considered as a server which oscillates between two feasible states ON/OFF which can be modeled by using Markov ON/OFF channel model [2].

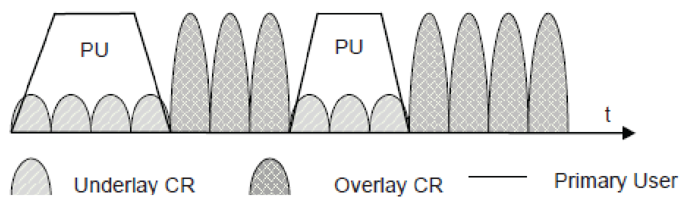

Fig.1 Hybrid overlay/underlay spectrum access.

Consequently, the secondary user operates also as a server that oscillates between two modes: underlay corresponding to state ON and overlay corresponding to state OFF. We denote underlay mode by 1 and overlay mode by 0. For both case, we assume that SU customers are served with the service times which are exponentially distributed with rate μ_0 in overlay mode and μ_1 in underlay mode respectively. We assume that $\mu_0 > \mu_1$. Because in spectrum overlay mode, the secondary user can achieve the maximum throughput while in the underlay mode the secondary user cannot achieve the maximum throughput due to constraints on transmission power. Poisson process is used for packet arrivals so that the inter-arrival times are exponentially distributed with parameter λ. And we assume that the probability that two and more customers arrive at the same time is zero. A first-in-first-out (FIFO) queue is used for SU packets.

3. Expected Sojourn Time Analysis

In this study, we use the M/M/1 queuing model with heterogeneous service rate [3] to analyze the expected sojourn time of SU customers. Based on that, the strategies of customers are analyzed in the next sections.

Fig. 2 shows the Markov process corresponding to the system evolution. The steady state probability of the secondary user working under overlay mode is $P_0 = \eta/(\eta + \xi)$ and underlay mode is $P_1 = \xi/(\eta + \xi)$. The

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Markov process is positive recurrent if the average arrival rate is less than average service rate. The average service rate is defined by

$$\hat{\mu} = \mu_0 P_0 + \mu_1 P_1. \quad (1)$$

Then the steady-state conditions of the system is

$$\lambda - \hat{\mu} < 0. \quad (2)$$

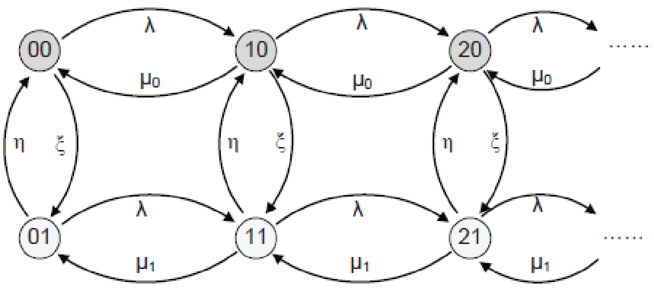

Fig.2 Transition-rate diagram

Under the above condition, the set of balance equations is given by

$$(\lambda + \eta)P_{01} = \xi P_{00} + \mu_1 P_{11}, \quad (3)$$

$$(\lambda+\eta+\mu_1)P_{n1} = \lambda P_{n-1,1}+\xi P_{n,0}+\mu_1 P_{n+1,1},\ n > 0, \quad (4)$$

$$(\lambda + \xi)P_{00} = \mu_0 P_{10} + \eta P_{01}, \quad (5)$$

$$(\lambda+\xi+\mu_0)P_{n0} = \lambda P_{n-1,0}+\eta P_{n,1}+\mu_0 P_{n+1,0},\ n > 0. \quad (6)$$

By employing vertical cuts in Fig. 2, we obtain

$$\lambda P_{n0} + \lambda P_{n1} = \mu_0 P_{n+1,0} + \mu_1 P_{n+1,1},\ n \geq 0. \quad (7)$$

By summing above equation over n we get

$$\lambda = \mu_0(P_0 - P_{00}) + \mu_1(P_1 - P_{01}), \quad (8)$$

Or

$$\mu_0 P_{00} + \mu_1 P_{01} = \hat{\mu} - \lambda. \quad (9)$$

where $P_i = \sum_{k=0}^{\infty} P_{n,i}, i = 0,1.$ By following similar approach as in [3], we define the partial generating function of the system as

$$G_i(z) = \sum_{n=0}^{\infty} z^n P_{n,i}, i = 0,1, |z| \leq 1. \quad (10)$$

Multiplying each equations (4),(5) and (6),(7) by z^n and sum over n we obtain

$$(\lambda+\eta+\mu_1)G_1(z) = \lambda z G_1(z) + \xi G_0(z)$$
$$+\frac{\mu_1}{z}[G_1(z) - P_{01}] + \mu_1 P_{01}, \quad (11)$$

$$(\lambda+\xi+\mu_0)G_0(z) = \lambda z G_0(z) + \eta G_0(z)$$
$$+\frac{\mu_0}{z}[G_0(z) - P_{00}] + \mu_0 P_{00}. \quad (12)$$

In order to calculate P_{01} and P_{00}, a polynomial of the third degree, g(z) is defined as follows

$$g(z) = \lambda^2 z^3 - (\lambda^2 + \mu_0\lambda + \lambda\mu_1 + \xi\lambda + \eta\lambda)z^2$$
$$+(\mu_0\mu_1 + \mu_0\lambda + \lambda\mu_1 + \eta\mu_0 + \xi\mu_1)z - \mu_0\mu_1. \quad (13)$$

(13), (14) and (15) are used to derive the bellow formula

$$g(z)G_0(z) = P_{01}\lambda\eta\mu_1 z +$$
$$P_{00}\mu_0[\eta z + \lambda z(1-z) - \mu_1(1-z)]. \quad (14)$$

Theorem 1: For positive μ_0 and μ_1 and finite η and ξ, the polynomial g(z) possesses a unique root $z0$ in the open interval (0,1). The proof was presented in [3]. By substituting $z = z_0$ in (14) we can eliminate the left hand side and obtain

$$0 = P_{01}\lambda\eta\mu_1 z_0 + P_{00}\mu_0[\eta z_0 + \lambda z_0(1-z_0) - \mu_1(1-z_0)].$$

(15)

Using (9), we eliminate P_{01} in the above equation and get

$$P_{00} = \frac{\eta(\hat{\mu}-\lambda)z_0}{\mu_0(1-z_0)(\mu_1 - z_0\lambda)}, \quad (16)$$

and, similarly we obtain

$$P_{01} = \frac{\xi(\hat{\mu}-\lambda)z_0}{\mu_1(1-z_0)(\mu_0 - z_0\lambda)}. \quad (17)$$

Combining (11), (12) and (13), we get

$$G_0(z) = [\eta(\hat{\mu}-\lambda)z + \mu_0 P_{00}(1-z)(\lambda z - \mu_1)] / g(z), \quad (18)$$

$$G_1(z) = [\xi(\hat{\mu}-\lambda)z + \mu_1 P_{01}(1-z)(\lambda z - \mu_0)] / g(z). \quad (19)$$

Define $E[L_i] = \sum_{n=0}^{\infty} nP_{n,i}$ as the contribution of state i to the mean queue size. Using (10) we have

$$E[L_i] = (d/dz)G_i(z)|_{z=1} \quad (20)$$

Then, using (13), (18), (19) and (20) to obtain the (unconditional) expected queue size $[L]$ as

$$E[L] = (d/dz)G_0(z)|_{z=1} + (d/dz)G_1(z)|_{z=1}$$
$$= \frac{\lambda}{\hat{\mu}-\lambda} + \frac{\mu_1(\mu_0-\lambda)P_{01} + \mu_0(\mu_1-\lambda)P_{00} - (\mu_0-\lambda)(\mu_1-\lambda)}{(\eta+\xi)(\hat{\mu}-\lambda)}.$$

(21)

Applying Little's formula to $[L]$, we get the expected sojourn time of an arbitrary SU customer in the system

$$\bar{T}(\lambda) = \frac{E[L\ ]}{\lambda}. \quad (22)$$

4. Special case: Overlay system

A special case of a model is that the service station is incapacitated from time to time and resumes its

operation after a random time. This system can be modeled by using queuing model with heterogeneous arrivals and service but can be considered as a special case μ_1 = 0 which was described in [3]. In particular, this special case interprets the overlay mode of cognitive radio network which is also referred to as opportunistic spectrum access (OSA).
A slotted OSA cognitive network was modeled by M/D/1 priority queuing scheme [4] and M/G/1 queuing model [5]. In contrast, we are concerned with modeling and analysis of an OSA cognitive network by M/M/1 queuing model.
By substituting μ_1 = 0 in (9), we obtain

$$P_{00} = P_0 - \frac{\lambda}{\mu_0}. \quad (22)$$

Substituting this value in (21) we have the expected queue size [L] as

$$E[L\ ] = \frac{\lambda + P_1 \dfrac{\lambda\mu_0}{\eta+\xi}}{\mu_0 P_0 - \lambda}. \quad (23)$$

For steady-state condition, the relation $\mu_0 P_0 > \lambda$ must hold. Then we get the expected sojourn time of an arbitrary packet in the system as

$$\bar{T} = \frac{E[L\ ]}{\lambda} = \frac{1 + P_1 \dfrac{\mu_0}{\eta+\xi}}{\mu_0 P_0 - \lambda}. \quad (24)$$

4. Numerical analysis

The numerical analysis is performed by “Maple”. Fig.3 presents the expected sojourn time for secondary user under hybrid overlay/underlay framework. The parameters are set as ξ = 3; μ_0 = 7; μ_1 = 3. We vary value of η with 3, 5 and 7. As can be seen, when η increases, the sojourn time of the secondary user decreases. The reason is the transition rate from state OFF to state ON is faster. Consequently, the secondary user operates under overlay more time, then, the expected sojourn time will decrease.
Fig.4 shows the expected sojourn time under overlay mode vs. overlay/underlay mode. The parameters are given as η = 7; ξ = 3; μ_0 = 7. We can see that the delay in only overlay is much higher than in hybrid overlay/underlay.

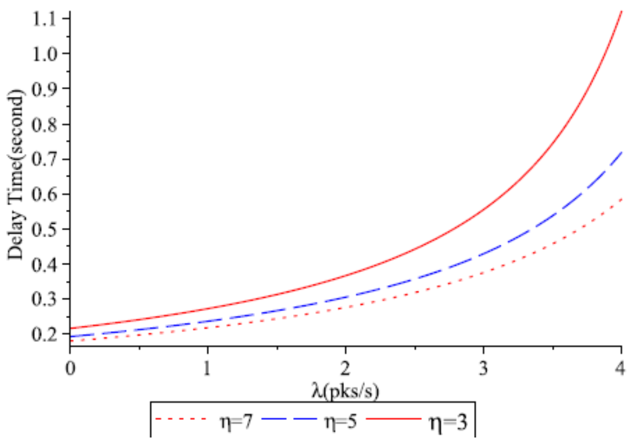

Fig. 3. Expected sojourn time under hybrid overlay/underlay CR.

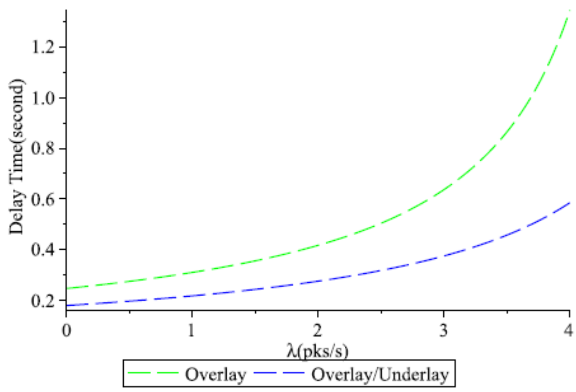

Fig. 4. Expected sojourn time under Overlay vs Hybrid overlay/underlay CR.

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