인지 무선 네트워크를 위한 유틸리티 기반 신호전력제어 단일 최적화 방안
(Utility-Based Power Control for Cognitive Radio Networks: Monotonic Optimization Approach)

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요 약 기본 네트워크와 공존하는 인지 무선 네트워크(CRN)의 특성 때문에, 파워 제어는 매우 중요하다. 본 논문에서는 제2사용자(secondary user)의 전체 유틸리티를 최대화하기 위해 인지 무선 네트워크의 신호전력제어 기법을 제안한다, 기본 사용자(Primary user, PU)는 제2사용자의 간섭으로부터 보호되어야 한다. 본 논문은 기본 사용자를 보호하기 위해 간섭을 임계치 이하로 유지해주는 간섭제한 임계치 방식을 사용한다. 제2사용자의 유틸리티 함수는 아무 형태의 증가 함수이다. 본 논문에서는 전체적인 최적값을 얻기 위해 중앙 집중방식을 사용하여 신호전력제어 문제를 단일 최적화 방안으로 공식화 하였다.

키워드 : 인지 무선 네트워크, 신호전력제어, 글로벌 최적화

Abstract Because of the properties of cognitive radio networks (CRNs), which coexist with primary network, power control problem is very important. In this paper, we propose a power control algorithm for cognitive radio networks (CRNs) that maximizes total utility of secondary users (SUs). Primary users (PUs) need to be protected from interference of secondary users. We use interference temperature constraints that keep interference under a threshold to protect primary users (PUs). Utility functions of SUs can be any increasing functions. We formulate our power control problem as a monotonic optimization that can be solved in centralization to achieve global optimum.

Key words : Cognitive radio network, power control, global optimization

1. Introduction

There are two approaches for spectrum access in CRNs: spectrum overlay and spectrum underlay. In spectrum overlay approach, SUs (unlicensed users) sense the available channels that are not in used by PUs (licensed user) so that they can transmit in these channels. In this scenario, it is not necessary to impose some restrictions on transmission power of SUs, since they will not be harmful to PUs. On the other hand, SUs in spectrum underlay approach can transmit simultaneously on the same channels with PUs. In this approach, interference from the SUs to PUs can be harmful to active PUs. So, power control is very important in this approach.

Recently, there are a lot of papers focusing on power control problem for cognitive radio network. Some of them base on optimization approach[4-6]. In [1], Le considers joint rate and power control

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problem as an optimization problem and transforms it into convex optimization problem[2]. uses the concept of interference temperature limit to protect primary users from secondary users and solve their optimization problem using sub-gradient method.[3] solves their power control problem by using branch and bound method to obtain sub-optimal solution. Other papers [4], [5] base on game theory where they model secondary users as players competing for transmission.

In this work, we use spectrum underlay approach for SUs to access the channel. We propose a power control problem that maximizes total utility of the SUs. Because of the non-concave property of $log(1+SINR)$, some previous works considered high SINR regime where the SINR (signal-to-interference-and-noise-ratio) of each link is much greater than 0 dB. In this case, the $log(1+SINR)$ becomes $log(SINR)$ and the optimization problem can be transformed into convex optimization problem in the form of geometric programming (GP) [6]; hence it can be efficiently solved for global optimality. In this paper, however we consider general *SINR* regime and the utility functions can be any increasing functions without any assumptions. We use the concept of interference temperature limit, introduced by Federal Communications Commission (FCC), as a constraint in order to keep interference from the SUs to PUs within a tolerable range such that PUs can decode their received signal.

Our power control optimization problem is formulated as Monotonic Optimization (MO) problem which is introduced in [7] and first applied into network optimization by Qian et al in [8]. By applied the *Polyblock outer approximation algorithm* in [7], we can achieve the global optimum for our proposal.

Typically, we use bold-face letters $\mathbf{x}, \mathbf{y}, \mathbf{z}, \ldots$ and $\mathbf{x}^{(k)}, \mathbf{y}^{(k)}, \mathbf{z}^{(k)}, \ldots$ to denote vectors and $x_i, y_i, z_i, \ldots, x_i^{(k)}, y_i^{(k)}, z_i^{(k)}, \ldots$ to denote their ith dimension respectively. $\mathbf{x} \le \mathbf{y}$ means $x_i \le y_i$ for every $i = 1, 2, \ldots, n$. R_+^n is the set of n-dimensional column vectors, which each element is in R_+ (nonnegative real numbers).

2. Monotonic Optimization

In this section, we want to remind about monotonic optimization (MO) based on [7].

Definition 1 (Normal set): A set $G \subset R_+^n$ is called normal if for any two elements $x, x' \in R_+^n$ such that $x \le x'$, if $x \in G$ then $x' \in G$ too.

Proposition 1: The intersection and the union of a family of normal sets are normal sets.

Definition 2 (MO): An optimization problem is called MO if it can be represented by the following formulation:

$$\begin{aligned} &max && f(x) \\ &st. && x \in G \end{aligned} \quad (1)$$

where set $G \subset [0, b] \subset R_+^n$ is a nonempty normal closed set and the objective function $f(x)$ is an increasing function over $[0, b]$.

Definition 3 (Upper boundary): An element $y \in R_+^n$ is called an upper boundary point of bounded normal set G if $y \in G$ while

$$K_{\mathbf{y}} = \{\mathbf{y}' \in \mathbf{R}_+^n \mid \mathbf{y}' > \mathbf{y}\} \in \{\mathbf{R}_+^n \setminus G\}.$$

The set of upper boundary points of G is the upper boundary of G and is denoted by $\partial^+ G$.

Proposition 2: The maximization of $f(x)$ over G of the monotonic optimization problem (1), if it exists, is attained on $\partial^+ G$.

Definition 4 (Polyblock): Given any finite set $T \subset R_+^n$ with elements v_i, the union of all the boxes $[0, v_i]$ is called a *polyblock* with vertex set T (A box is defined as $[0, v_i] = \{x \mid 0 \le x \le v_i\}$).

Definition 5 (Proper): An element $v \in T$ is called proper if there does not exist $v' \in T$ such that $v' \neq v$ and $v' \ge v$. Set T is a proper set if all of its elements are proper.

Proposition 3: If G in (1) is a *polyblock*, then the optimal solution of (1) is attained at one proper of this *polyblock*.

Definition 6 (Projection): Let $G \subset [0, b]$ be a nonempty normal set. For every point $z \in R_+^n$, the half line from 0 through z meets $\partial^+ G$ at a unique point, $\pi_G(z)$, which is defined by:

$$\pi_G(\mathbf{z}) = \lambda \mathbf{z}, \lambda = max\{\alpha \ge 0 \mid \alpha \mathbf{z} \in G\}.$$

Detailed illustrations about these propositions, definitions and their proofs are omitted due to space limitation. Interested readers can refer to [7] for more details.

3. System Model and Problem Formulation

3.1 System Model

We consider a cognitive radio network consisting of L links denoted by set $\mathcal{L} = \{1, 2, ..., L\}$. SUs communicate in ad hoc mode and coexist with primary network which has M PUs. The signal to interference and noise ratio (SINR) of each secondary link l can be expressed as:

$$\gamma_l(\boldsymbol{p}) = \frac{G_{ll} p_l}{\sum_{i \neq l} G_{il} p_i + \eta_l} \tag{2}$$

where p_l is the transmission power from the transmitter of link l, $\boldsymbol{p} = (p_1, p_2, ..., p_n)^T$ and G_{il} is the channel gain from the transmitter of link i to the receiver of link l. η_l denotes additive noise at the receiver of link l. From Shannon capacity formula, the corresponding data rate on link l can be expressed as:

$$R_l(\boldsymbol{p}) = \log_2(1 + \gamma_l(\boldsymbol{p})) \tag{3}$$

In order to protect the PUs from the SUs, we use the concept of interference temperature limit. The maximum interference tolerance for PUs can be calculated as:

$$Q^{max} = kT^{max}$$

where k is Boltzman's constant and T^{max} is the interference temperature limit. We assume that SUs can be aware of the total interference to PUs and interference threshold.

3.2 Problem Formulation

In this paper, we formulate our power control problem as an optimization problem. Our objective is to maximize total utility of all active links in CRN. The objective can be written as follow

$$\sum_{l=1}^{L} U_l(R_l(\boldsymbol{p}))$$

where utility function $U_l(.)$ of link l can be any monotonic increasing functions. The meaning of the utility function is the satisfaction of secondary links. The higher data rate on the link, the more satisfaction. The limitation of the power at each SU can be expressed as the constraints on the optimization problem:

$$0 \le p_l \le p_l^{max} \qquad 1 \le l \le L.$$

Together with the interference temperature limit constraints, our power control problem can be expressed as follow

$$\text{(P1):} \quad max \quad \sum_{l=1}^{L} U_l(\log(1 + \gamma_l(\boldsymbol{p})))$$

$$S.t \quad 0 \le p_l \le p_l^{max} \qquad 1 \le l \le L$$

$$\sum_{l=1}^{L} h_{lm} p_l \le Q_m^{\max} \qquad 1 \le m \le M.$$

In (P1), h_{lm} is the channel gain from the transmitter of secondary link l to PU m and is assumed to be known by secondary network. Because of the non-concavity of $log(1+SINR)$, (P1) is not convex optimization problem. In order to make it convexity, [6] considered high $SINR$ regime and transformed it into geometric programming.

4. Power allocation algorithm

The power control problem (P1) can be rewritten as

$$\text{(P2):} \quad \max \quad \Gamma(\boldsymbol{z}) = \sum_{l=1}^{L} U_l(\log(z_l))$$

$$\text{s.t} \quad \boldsymbol{z} \in \Pi$$

where feasible set Π is defined by:

$$\Pi = \{\boldsymbol{z} \mid 1 \le z_l \le 1 + \gamma_l, \forall l \in \mathcal{L}, \boldsymbol{p} \in \mathcal{P}\},$$

with

$$P = \{\boldsymbol{p} \mid 0 \le p_l \le p_l^{max}, \sum_{l=1}^{L} h_{mp} p_l \le Q_m^{max}, \forall 1 \le m \le M\}.$$

It is obviously that the feasible set Π is the union of infinite numbers of boxes, each box corresponds to a feasible $\boldsymbol{p} \in \mathcal{P}$. Therefore, by proposition 1, Π is normal set. Together with $\Gamma(\boldsymbol{z})$ being an increasing function in z, problem (P2) is a MO problem. Hence we can solve this problem by using the *Polyblock Outer Approximation Algorithm* proposed in [7] with slight modifications. The details are shown in Algorithm 1 as below:

The current best value (CBV), which is updated every iteration in algorithm 1, is the best optimal value the algorithm can attain until this iteration. This CVB is obtained at $\bar{\boldsymbol{x}}^{(k)}$. The projection

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Algorithm 1:

Initialization: Select $\varepsilon > 0$ (error tolerance). Let $\overset{-(0)}{\boldsymbol{x}}$ be feasible solution available and set current best value (CBV) equals to $f(\overset{-(0)}{\boldsymbol{x}})$. Let $T_1 = \{\boldsymbol{b}\}$, where

$$b_l = 1 + \frac{G_{ll} p_l^{\max}}{\eta_l}$$

It is obvious that box $[\mathbf{0}, \boldsymbol{b}]$ contains Π. Set k=1.

Step 1: Select $z^{(k)} \in argmax\{f(z) \mid z \in T_k, z \geq \mathbf{1}\}$. Compute $\boldsymbol{x}^{(k)} = \pi_G(z^{(k)})$. Determine CBV $= \max\{f(\overset{-(k-1)}{\boldsymbol{x}}), f(\boldsymbol{x}^{(k)})\}$ and current feasible solution $\overset{-(k)}{\boldsymbol{x}}$ corresponding to that value, that means $\overset{-(k)}{\boldsymbol{x}} = \arg\max\{f(\overset{-(k-1)}{\boldsymbol{x}}), f(\boldsymbol{x}^{(k)})\}$.

Step 2: Set T_{k+1} is attained from $(T_k \setminus \{z^{(k)}\}) \cup \{z^{(k)} - (z_i^{(k)} - x_i^{(k)})\mathbf{e}^i, i = 1, ..., l\}$ after removing improper elements (where $\boldsymbol{e}^i$ is ith unit vector of $\boldsymbol{R}_+^l$).

Step 3: If $(1+\epsilon)\Gamma(\overset{-(k)}{\boldsymbol{x}}) \geq \Gamma(z^{(k)})$, terminate. Otherwise, $k \leftarrow k+1$ and return to Step 1.

$\pi_G(z^{(k)}) = \lambda_k z^{(k)}$ in Step 1 can be obtained by solving the max-min problem:

$$\begin{aligned}\lambda_k &= \max\{\lambda \mid \lambda z^{(k)} \in \Pi\} \\ &= \max\{\lambda \mid \lambda \leq \min_{1 \leq l \leq L} \frac{1+\gamma_l(\boldsymbol{p})}{z_l^{(k)}}, \boldsymbol{p} \in \mathcal{P}\} \\ &= \max_{\mathbf{p} \in \mathcal{P}} \min_{1 \leq l \leq L} \frac{f_l(\boldsymbol{p})}{z_l^{(k)} g_l(\boldsymbol{p})}\end{aligned}$$

where $f_l(\mathbf{p}) = \sum_{i \in \mathcal{L}} G_{il} p_i + \eta_l$ and

$g_l(\mathbf{p}) = \sum_{i \in \mathcal{L}, i \neq l} G_{il} p_i + \eta_l$.

This is a generalized linear fractional programming problem that can be solved using the Dinkelbach-type algorithm as the same in [8]. This algorithm solves a series of Linear Programming (LP) until convergence.

In algorithm 1, after each iteration, we always have $\Pi \subset O_{k+1} \subset O_k$, where O_k is a polyblock with its vertex set T_k. By this way, we can construct a series of polyblocks containing Π that approximate the normal set Π with an increasing level of accuracy.

To make it easy to understand the algorithm, the geometric illustration in case of two dimensions is shown in figure 1. At the beginning, the vertex set has two vertices z and v. Without loss of generality, supposes that the objective value at z is greater than at v, so z is selected at the step 1. According to step 2, z^1 and z^2 are attained by $\{z - (z_i - (\pi_G(z))_i)\mathbf{e}^i, i = 1, 2\}$, respectively. However, z^2 is improper so we remove it and the proper set after this is $\{z^1, v\}$. This vertex set makes a new polyblock that approximate the feasible region more accurate.

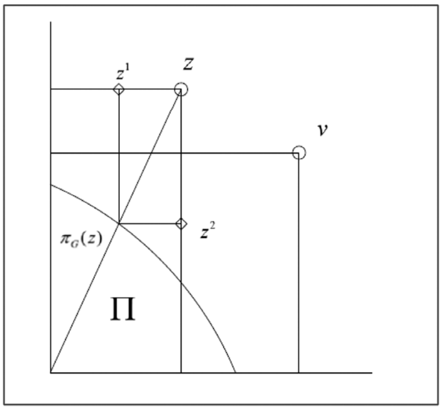

Fig. 1 Geometric illustration of shrinking the outer polyblock

Theorem 1: If algorithm 1 is infinite, each of the generated sequences $\{z^{(k)}\}, \{\boldsymbol{x}^{(k)}\}$ contains a subsequence converging to an optimal solution. Therefore (P2) converges to global optimal.

Proof: The proof of Theorem 1 is the same as the proof of theorem 1 in [7] and is summarized as follows. If algorithm 1 is infinite, it generates at least one infinite subsequence $z^{(l_1)}, z^{(l_2)}, ..., z^{(l_k)}, ...$ such that $z^{(l_{k+1})} = z^{(l_k)} - (z_{i_k}^{(l_k)} - x_{i_k}^{(l_k)})\mathbf{e}^{i_k}$ for a certain i_k. It's obviously that $z^{(l_1)} \geq z^{(l_2)} \geq ... \geq z^{(l_k)} \geq ... \geq 0$.

So, there exists z^* such that $z^* = \lim_{k\to+\infty} z^{(l_k)}$. This implies $z^{(l_k)} - z^{(l_{k+1})} \to 0$, and hence $z_{i_k}^{(l_k)} - x_{i_k}^{(l_k)} \to 0$. On the other hand $z_{i_k}^{(l_k)} - x_{i_k}^{(l_k)} = (1-\lambda_{l_k})z_{i_k}^{(l_k)}$ and $z_{i_k}^{(l_k)} \geq 1$ then $\lambda_{l_k} \to 1$. That means $z^{(l_k)} - x^{(l_k)} \to 0$. Consequently, $z^* = \lim_{k\to+\infty} z^{(l_k)} = \lim_{k\to+\infty} x^{(l_k)}$ belongs to Π and $f(z^*) \geq f(x), \forall x \in \Pi$, i.e., z^* is global optimal solution.

5. Numerical results

We consider the cognitive radio network with 3 links (i.e., L=3) and one PU (i.e., M=1). Assume that $p_l^{max} = 0.5mW$ and $\eta_l = 0.5\mu W$ for all links. We consider a realization of the channel gains, represented by matrix G:

$$G = \begin{pmatrix} 0.075 & 0.015 & 0.020 \\ 0.015 & 0.045 & 0.002 \\ 0.020 & 0.002 & 0.085 \end{pmatrix}$$

In the first simulation, we choose the utility function $U_l(x) = x$ in order to maximize the total throughput of the secondary network. The aggregated utility gets closer to its upper bound until the gap between them is acceptable (tolerance ϵ). The aggregated utility when ϵ is chosen equals to 0.1 is show in figure 2. The optimal power solutions in this case are shown in table 1 (in mW). In figure 2, value at Zn is defined the utility at Zn. We can see

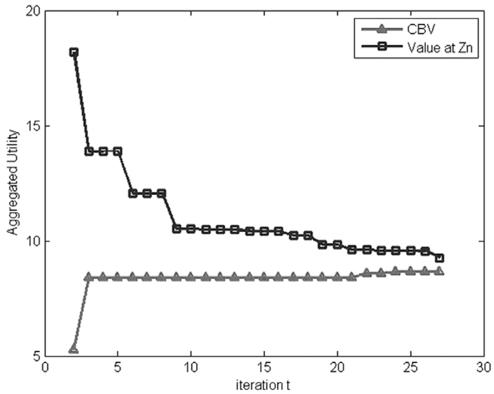

Fig. 2 Utility function $U_l(x) = x$ and error tolerance $\epsilon = 0.1$

Table 1 Power optimal solution

P1	P2	P3
0.0075	0.3524	0.5

Table 2 Number of iterations

ϵ	0.5	0.1	0.05	0.01
Number of iterations	6	27	34	78

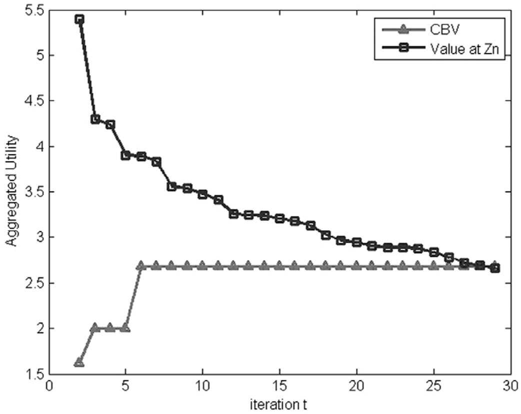

Fig. 2 Utility function $U_l(x) = \ln(x)$ and error tolerance ϵ=0.001 or 0.0001

that CBV gets closer to value at Zn until the gap between them is less than ϵ, that means it converges to optimal value. There is a tradeoff between optimal solution and convergent time. It depends on the tolerable value of ϵ. Table 2 shows the number of iterations for each value of ϵ.

In the second simulation, we choose the utility function $U_l(x) = \ln(x)$ in order to get the proportional fairness among secondary users. Figure 3 shows that after just 29 iterations, the aggregated utility is very close to its upper bound. Actually, the results are the same when we select ϵ=0.001 or 0.0001. That means our proposed algorithm can quickly obtain the global optimal solution.

6. Conclusion

In this paper, we formulate the power control problem as Monotonic Optimization problem. The algorithm 1 is guaranteed to converge to global optimal solution despite of non-convexity of the problem. But the convergent time is large; therefore, our proposal only provides benchmark for performance evaluation of the other power control heuristics in this area.

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정보과학회논문지 : 정보통신

제 39 권 제 2 호 참조