# Throughput Maximization for The Secondary User over Multi-channel Cognitive Radio Networks

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***Abstract*—This paper studies average waiting time analysis of opportunistic access in multi-channel cognitive radio networks with a single secondary user and multiple primary users by applying the M/G/1 preemptive priority queueing scheme. By employing convex optimization tool, the secondary user finds the optimal way to distribute the packets to all channels in the system. For sensitive delay network, we proposed an algorithm to distribute the secondary user's packets to the only group of channels which satisfy the delay constraint. Simulation are used to validate the results, and simulation results demonstrate a high degree of accuracy for the derived expressions. Results indicate that the performance of the secondary user depends on the data traffic characteristics of the primary users and also the delay constraint of the secondary user.**

## I. INTRODUCTION

Radio spectrum is one of the most scarce and valuable resources for wireless communications. Some surveys performing actual measurements have shown that most of the allocated spectrum is largely under-utilized [1]. Similar views on the under-utilization of allocated spectrum were reported by the Spectrum-Policy Task Force appointed by Federal Communications Commissions (FCC) [2]. Cognitive radio has been proposed as a way to improve spectrum efficiency by exploiting the unused spectrum in dynamically changing environments [3]. In a cognitive radio network, there are two types of users, namely, primary users and secondary users or cognitive users. In the cognitive radio concept, the transmission channel is licensed to the primary users while the secondary user opportunistically accesses the channel resources when it is not used by any primary user. Accordingly, a secondary user is required to detect exactly a vacant channel for transmission and vacate it when a primary user wishes to use the channel. This transmission strategy is called opportunistic spectrum access [4], [5].

In recent research, queueing theory has been employed in performance analysis of cognitive radio networks [5], [7], [6] where some results have been obtained, such as packet waiting time inside the queue and delay. In view of the above, studies on average waiting time of packets in cognitive radio networks have been given large attention. Specifically, the average waiting time of packets for secondary user is studied in [5], [7], under various preemptive priority queueing models such as M/D/1 (Poisson arrival process, deterministic distribution of service time, single server) and M/G/1 (Poisson arrival process, general distribution of service time, single server). However, these works do not examine the system in a multi-channel model. In [8], the authors analyzed the secondary call traffic with frequency agility features of cognitive radio in a multi-channel system. The performance measures were derived in terms of waiting time, blocking probability and forced termination probability.

In this paper, we derive the average waiting time of packets with different priorities for primary user and secondary user by employing a preemptive priority M/G/1 model. Furthermore, based on the waiting time analysis, we find an optimal way to allocate how many packets should enter which channel in multi-channel system by employing convex optimization. And we also proposed an adaptive algorithm for delay sensitive networks. To the best of our knowledge, this is the first paper analyzing this kind of problem.

The remainder of this paper is organized as follows. The system model is introduced in Section II. In Section III, the performance analysis of the average waiting time of packets for different scenarios in terms of multiple primary users and single secondary user in a multi-channel system. Then, numerical results are reported in Section IV. Finally, we draw conclusions in Section V.

## II. SYSTEM MODEL

We consider a time slotted cognitive wireless network where primary users has a license to use the channels. This network has $N$ primary users and one secondary user. The frequency band is equally divided into $N$ channels each of which is exclusively occupied by one licensee named primary user. According to the access technology of the secondary users, we assume spectrum sharing between the secondary user and the primary users is categorized as spectrum overlay. It means the secondary user can only use the licensed spectrum when primary users are not transmitting. When the primary users wish to transmit, it is given a priority over the secondary user. This is implemented by having the secondary user perform spectrum sensing with perfect sensing assumed. If there is no signal of the primary user then the secondary user is allowed to occupy the channel. If the primary users join the channel again, then the secondary user must leave the channel within certain period of time. This channel occupation behavior can

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be described by a preemptive priority queueing model [9]. And with the preemptive channel occupation approach, it is ensured that the primary users have a designated usage of their licensed channel. The network is assumed to operate in ideal channel conditions (e.g. no noise and error-free). Poisson process is used for packet arrivals, so that the inter-arrival times are exponentially distributed. The primary user $i-th$ arrival rate is $\lambda_i^P$ and secondary user arrival rate is $\lambda_S$. Infinite buffers are assumed. Fig. 1 shows the preemptive priority queueing model for single secondary user and $N$ primary users in a multi-channel system. It looks like a system which have $N$ servers with different transmission capability. Each primary user $i$ operates in channel $i$ but the secondary user can operates in all $N$ channel.

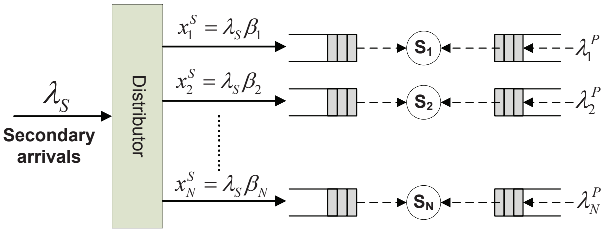

Fig. 1. A multi-channel cognitive wireless network.

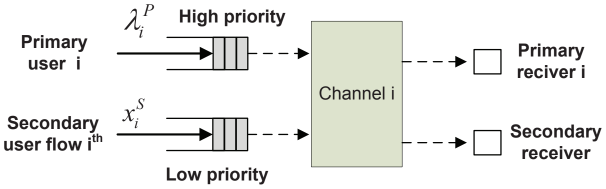

Fig. 2. Priority service discipline.

## III. PROBLEM FORMULATION

First, let us show the nomenclature in Table I. Fig. 2 shows the priority queuing system used for modeling the cognitive radio. We assume that packets arrive at the secondary user according to a Poisson process with rate $\lambda_S$. Each packet is routed randomly and independently to $N$ channels, the ratio of distribution to a channel $i$ is $\beta_i$ where $\sum_{i=1}^{N}\beta_i \le 1$. In [9], the authors showed that the arrival processes at each channel is Poisson with rate $x_i^S=\beta_i\lambda_S$. Therefore, we can model the multi-channel system like $N$ parallel single channel systems. Since each channel has identical features on traffic management for both designated primary traffic and the distributed secondary traffic, the mathematical modeling could be done for a single channel system like in Fig.2. The packets of the secondary user and primary user both are served according to a first come first served discipline (FCFS). The primary users are assumed to be able to preempt the transmission of the secondary users. When the transmission of the secondary user is preempted by the primary user, the rest of the secondary transmission would be taken up into the priority queue. Note that this discipline is called preemptive resume [9]. Under these schemes, lower priority transmission would always wait in the queue for the transmission of higher priority class.

TABLE I
MODEL PARAMETERS

| Symbol | Explanation |
|---|---|
| S | Secondary user |
| P | Primary user |
| $\lambda$ | Arrival rate |
| $\mu$ | Service rate |
| $E[X_i]=\frac{1}{\mu_i}$ | Average service time |
| $E\left[X_i^2\right]=\overline{X_i^2}$ | Second moment of the average service time |
| $N_{Q,i}$ | Average number of packets in queue |
| $W_{Q,i}$ | Average queueing time |
| $\rho_i=\frac{\lambda_i}{\mu_i}$ | Utilization factor |
| $T_i$ | Total time spent in the system |
| $R_i$ | The mean residual time |

We model the traffic conditions of the system by adopting an M/G/1 priority queueing model. The waiting time of a packet in the system consists of three parts: the mean residual service time, time spent in a queue waiting for the service to begin, and the average service time( transmission time). Starting from the well known analysis result of the M/G/1, known as Pollaczek-Khintchine and following similar approach as in [9]. The sojourn time of a secondary user packet depends not only on the packets found upon arrival (contain the average number of packets in $i-th$ queue of the secondary user denoted by $N_{Q,i}^S$ and the average number of packets in queue of the primary user $i$ denoted by $N_{Q,i}^P$ ), but also on subsequent arrivals at the primary user queue. We first derive the expression for the waiting time $W_{Q,i}^P$ for the primary user which has the highest-priority class as

$$W_{Q,i}^P = R_i^P + \frac{1}{\mu_i^P}N_{Q,i}^P \tag{1}$$

where $R_i^P = \frac{\lambda_i^P\overline{X_{P,i}^2}}{2}$. By substituting $N_{Q,i}^P=\lambda_i^P W_{Q,i}^P$ from Little's Theorem, equation (1) becomes

$$W_{Q,i}^P = R_i^P + \rho_i^P W_{Q,i}^P \tag{2}$$

which can be rewritten as

$$W_{Q,i}^P = \frac{R_i^P}{1-\rho_i^P} \tag{3}$$

and finally we can express the average delay per packet for the primary user $i$ as

$$T_i^P = \frac{1}{\mu_i^P} + \frac{R_i^P}{1-\rho_i^P} \tag{4}$$

For the secondary user, the average waiting time in the system consists of three terms:

1) The average service time $\frac{1}{\mu_i^S}$.
2) The average waiting time required, upon arrival of the secondary user packet, to service the primary user packet

and also the secondary user packets already in the system (i.e., the average unfinished work). It can be seen that this time is equal to the average waiting time in the corresponding, ordinary M/G/1 system (without priorities), that is calculated in [9] as $\frac{R_i^S}{1-\rho_i^P-\rho_i^S}$ where $R_i^S = \frac{\lambda_i^P \overline{X_{P,i}^2}+x_i^S \overline{X_{S,i}^2}}{2}$

3) The average waiting time for the primary user packets which arrive while the secondary user packets are being processed in the system. This term is $\frac{1}{\mu_i^P}\left(\lambda_i^P T_i^S\right) = \rho_i^P T_i^S$

By summing the three components of the delay above, we can establish the equation for the average packet delay from the secondary user in queue $i$ as

$$T_i^S = \frac{1}{\mu_i^S} + \frac{R_i^S}{1-\rho_i^P-\rho_i^S} + \rho_i^P T_i^S \tag{5}$$

and the final result is

$$T_i^S = \frac{1}{\mu_i^S(1-\rho_i^P)} + \frac{R_i^S}{(1-\rho_i^P-\rho_i^S)(1-\rho_i^P)} \tag{6}$$

### *A. Scenario 1: Maximum number of total transmitted packets under average delay bound*

In this section we want to maximize the total packets which can be transmitted in $N$ channels by maximizing the sum $\sum\limits_{i=1}^{N} x_i^S$. With the assumptions in the above section, the optimization can be formulated as

$$P1 : \max_x \sum_{i=1}^{N} x_i^S \tag{7}$$

$$s.t. \quad \sum_{i=1}^{N} x_i^S \le \lambda_S, \tag{8}$$

$$\frac{\sum_{i=1}^{N} T_i^S}{N} \le T_{\max}, \tag{9}$$

$$0 \le x_i^S, \forall i \in \{1,2,...,N\}, \tag{10}$$

$$\rho_i^P + \rho_i^S < 1, \forall i \in \{1,2,...,N\}, \tag{11}$$

where $\mathbf{x} = \left\{x_1^S, x_2^S, ..., x_N^S\right\}$ is a vector of arrival rate allocation over $N$ channels for the secondary user. We will prove that **P1** is a convex optimization problem. The constraints in (13) represents queue stability. The constraints in (11) keep the average delay in $N$ channels under a predefined delay threshold $T_{\max}$. The constraints in (11) can be formulated as

$$f(x) \le T_{\max} \tag{12}$$

where $f_i(x) = T_i^S = \frac{1}{\mu_i^S(1-\rho_i^P)} + \frac{\mu_i^S \mu_i^P (\lambda_i^P \overline{X_{P,i}^2}+x_i^S \overline{X_{S,i}^2})}{2(\mu_i^S \mu_i^P - x_i^S \mu_i^P - \lambda_i^P \mu_i^S)(1-\rho_i^P)}$ and $f(x) = \sum_{i=1}^{N} f_i(x) \Big/ N$. It can be seen that if $\mu_i$,$\lambda_i^P$ and $\mu_i^S$are given then $f_i(x)$ is the convex function hence the nonnegative weighted sums function $f(x) = \sum_{i=1}^{N} \frac{1}{N} f_i(x)$ is also convex [10]. Therefore with given $\lambda_S$ ,$\bar{X}_{S,i}^2$, $T_{\max}$, $\bar{X}_{P,i}^2$, $\mu_i^S$, and $\mu_i^P$ then **P1** is a convex optimization problem and can be solved by the interior-point algorithms in [10].

### *B. Scenario 2: Adaptive algorithm for delay sensitive networks*

In the delay sensitive application networks, the first requirement is minimization of the delay or the waiting time of a packet in the system. It means that we should tighten the delay bound $T_{\max}$. However, small $T_{\max}$ makes the optimization **P1** infeasible. The optimization **P1** is infeasible with $N$ channels because some poor channels(i.e,.channels which have long average waiting time of the packets) increase the average delay. But the optimization **P1** may be feasible with less than $N$ channel if we remove some poor channels. Therefore, we need to discard these channels from our selected channels for transmission with step by step elimination of the worst channel(i.e.,the channel which has maximum secondary packet delay $T_i^S$). Our proposed algorithm is as follow:

1) initial value $i = N$;
2) solve the optimization problem $P_i$;
3) if $P_i$ is infeasible then
    - if $i > 1$ then eliminate the worse channel;go to step 4
    - else return "the problem is infeasible" and terminate;

    else return the sum of transmitted packet $S_i = \sum\limits_{i=1}^{N} x_i^S$ ;
4) if $i > 1$ then $i = i - 1$ ; go to step 2;
5) compare between $S_1$, $S_2$,.., $S_N$ and choose the maximum $S_j$ then return the optimal $S_j$ and the optimal distribution of $P_j$

We consider the "worse channel" is the channel that has the maximum average waiting time of the secondary user's packets among the channels.

## IV. NUMERICAL ANALYSIS

In the simulation, we consider a system with 1 secondary user and 5 primary users with 5 sharing channels. The initial parameters are given in Table II. The simulation is performed by cvx package [11].

TABLE II
INITIAL PARAMETERS

| | Ch 1 | Ch 2 | Ch 3 | Ch 4 | Ch 5 |
|---|---|---|---|---|---|
| $\mu_i^P = \mu_i^S$ | 0.5 | 0.7 | 0.6 | 0.8 | 1.0 |
| $\lambda_i^P$ | 0.1 | 0.2 | 0.1 | 0.2 | 0.4 |
| $\overline{X_{P,i}^2} = \overline{X_{S,i}^2}$ [12] | 4.400 | 2.244 | 3.055 | 1.718 | 1.100 |
| $\rho_i^P = \lambda_i^P \big/ \mu_i$ | 0.20 | 0.57 | 0.50 | 0.25 | 0.40 |
| $T_i^P$ | 2.275 | 1.743 | 1.850 | 1.480 | 1.367 |

At first, we want to examine the effect of average delay $T_{\max}$ on the sum of transmit packet as well as the distribution of the packet over 5 channel. The result is shown in Table III and Fig.3. As can be observed, the average delay will increase with the total number of transmitted packets. Because of the constraints (13), the maximum sum $\sum\limits_{i=1}^{N} x_i^S$ can be reached is 2.6 packets/slot. At the beginning, the sum $\sum\limits_{i=1}^{N} x_i^S$

TABLE III

THE OPTIMAL RESULT

| $x_1^S$ | $x_2^S$ | $x_3^S$ | $x_4^S$ | $x_5^S$ | $T_{\max}$ |
|---|---|---|---|---|---|
| 0 | 0 | 0.005 | 0.079 | 0.017 | 2.4 |
| 0.025 | 0.103 | 0.132 | 0.212 | 0.167 | 2.8 |
| 0.06 | 0.14 | 0.167 | 0.249 | 0.207 | 3 |
| 0.169 | 0.256 | 0.274 | 0.361 | 0.333 | 4 |
| 0.225 | 0.315 | 0.329 | 0.419 | 0.398 | 5 |
| 0.321 | 0.416 | 0.422 | 0.518 | 0.509 | 10 |
| 0.362 | 0.46 | 0.463 | 0.561 | 0.556 | 20 |
| 0.387 | 0.487 | 0.488 | 0.587 | 0.585 | 60 |

increases sharply when the secondary user tolerates a little more delay. When the sum $\sum_{i=1}^{N} x_i^S$ is reasonably large, it means the channels are more busy. As a consequence, the sum $\sum_{i=1}^{N} x_i^S$ goes up slowly to the maximum reachable value when $T_{\max}$ increases rapidly. And the fluctuation in quality of channels make the difference in the distribution of packets over $5$ channels which can be observed in Fig.3. When the delay constraint is tightly bound, it can be seen that all the packets are routed to the best channel. Table III shows that the channel $4$ is the best channel and the channel $1$ is the worst channel because the channel $1$ has the longest delay for both the primary users's and the secondary user's packet and the channel $4$ has the smallest delay for both primary and secondary packet.

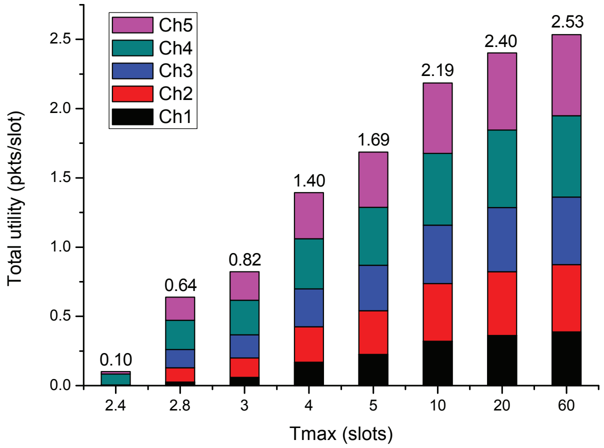

Fig. 3. The distribution of packet over the channels.

TABLE IV

THE AVERAGE WAITING TIME OF THE USERS' PACKETS

| $T_1^P$ | $T_2^P$ | $T_3^P$ | $T_4^P$ | $T_5^P$ | $\overline{T_i^P}$ |
|---|---|---|---|---|---|
| 2.275 | 1.742 | 1.850 | 1.479 | 1.367 | 1.742 |
| $T_1^S$ | $T_2^S$ | $T_3^S$ | $T_4^S$ | $T_5^S$ | $\overline{T_i^S}$ |
| 2.843 | 2.44 | 2.22 | 1.972 | 2.277 | 2.350 |

The average waiting time of both the primary users' and the secondary user's packets are given in Table IV. The last row in Table IV represents the minimum delay for secondary user packets on each channel. $\overline{T_i^P}$ and $\overline{T_i^S}$ denote the average delay of the packets of the primary users and the minimum average delay of the secondary user over $5$ channels respectively. If $T_{\max} \leq \overline{T_i^S} = 2.350$ then the optimization **P1** is infeasible. Therefore, we must run the adaptive algorithm to eliminate some "poor" channels. Table V shows the results of the adaptive algorithm in case of $T_{\max} = 2.3$. The algorithm eliminates channel 1,2,5 and 3 respectively. And the optimal way is to eliminate channel $1$ and $2$. The total transmitted packets rate can be reached is $0.166$ packets/timeslot.

TABLE V

ADAPTIVE ALGORITHM WITH $T_{\max} = 2.3$

| $x_1^S$ | $x_2^S$ | $x_3^S$ | $x_4^S$ | $x_5^S$ | Sum |
|---|---|---|---|---|---|
| E | 0 | 0.01 | 0.084 | 0.023 | 0.117 |
| E | E | 0.025 | 0.1 | 0.041 | 0.166* |
| E | E | 0.04 | 0.115 | E | 0.155 |
| E | E | E | 0.126 | E | 0.126 |

## V. CONCLUSION

In this paper, based on the analysis of average waiting time of the primary users' and the secondary user's packets, the distribution of the secondary user's packets over multi-channel has been derived. Moreover, an adaptive algorithm for delay sensitive cognitive radio networks has been proposed. Our numerical results show that the distribution of packets of the secondary user depends on the data traffic characteristics of the primary users. Furthermore, the performance of the secondary user is connected with the delay constraint.

## VI. ACKNOWLEDGEMENT

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## REFERENCES

[1] R. W. Broderson, A. Wolisz, D. Cabric, S. M. Mishra, and D. Willkomm, "CORVUS: a cognitive radio approach for usage of virtual unlicensed spectrum", white paper, submitted at the University of Berkeley, CA,July 2004.

[2] Federal Communications Commission, "Spectrum Policy Task Force", Rep. ET Docket no. 02-135, Nov. 2002.

[3] J. Mitola, "The software radio architecture", IEEE Commun. Mag., vol. 33, pp. 26-38, May 1995.

[4] S. Haykin, "Cognitive radio: Brain-empowered wireless communications", IEEE Journal on Selected Areas in Communications, vol. 23, pp. 201-220, 2005.

[5] I. Suliman and J. Lehtomaki, "Queueing analysis of opportunistic access in cognitive radios", International Workshop on Cognitive Radio and Advanced Spectrum Management, 2009.

[6] Shanshan Wang; Junshan Zhang; Lang Tong, "Delay Analysis for Cognitive Radio Networks with Random Access: A Fluid Queue View", in Proc. IEEE INFOCOM'10,San Diego, CA,2010,pp. 1-9.

[7] C. Zhang, X. Wang, and J. Li. "Cooperative cognitive radio with priority queueing analysis". In Proc. IEEE International Conference on Communications (ICC 09), pages 1 . 5, Dresden, Germany, 14.18, 2009.

[8] J. Heo, J. Shin, J. Nam, Y. Lee, J. G. Park, and H.-S. Cho. "Mathematical analysis of secondary user traffic in cognitive radio system". In Proc. IEEE 68th Vehicular Technology Conference (VTC 2008-Fall), pages 1 . 5, Calgary, Canada, Sept. 21.24, 2008.

[9] Bertsekas D, Gallager R. 1992, Data Networks (2nd edn). Prentice Hall.
[10] S. Boyd, L. Vandenberghe, Convex Optimization. Cambridge University Press, 2004
[11] http://cvxr.com/ cvx/
[12] H. -P. Shiang and M. van der Schaar, ”Queuing-Based Dynamic Channel Selection for Heterogeneous Multimedia Applications Over Cognitive Radio Networks”, IEEE Transactions on Multimedia, vol. 10, no. 5, pp. 896-909, Aug. 2008.