

Joint Congestion Control and Power Allocation with Outage Constraint in Wireless Multihop Networks

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Abstract—We consider the problem of joint congestion control and power control with outage constraint in an interference limited multihop wireless network. We transform the original nonconvex problem into a convex programming problem and develop a message passing distributed algorithm that can attain the global optimal source rate and link transmit power. This algorithm however requires larger control message size than that of the conventional scheme, which increases network overheads. We continue developing a practical near-optimal distributed algorithm which only requires local SIR measurement to limit the size of the message. Numerical results show that both schemes have nearly identical performance and outperform the conventional scheme.

I. INTRODUCTION

In wireless multihop network, the congestion control and power control have a mutual relationship. The congestion control regulates the source rates to avoid overwhelming any link capacity which depends on interference levels, which in turn decided by link transmit power control. Based on this relationship, Chiang [3] characterized the first joint congestion control and power control (JCPC) problem via solving a transformed convex optimization problem. By using the gradient based algorithm, the author showed that the optimal source rate and link transmit power could be attained in a distributed fashion with message passing. However, the solutions were optimally achieved in a high SIR approximation sense (link capacity approximation); and they are suboptimal in a general sense [15].

Many works later considered different aspects of this JCPC problem. [6] studied a cross-layer problem of joint congestion, media contention and power control. Lee *et al.* [11] proposed a new window control algorithm for the congestion control and a new power control algorithm to improve throughput and power efficiency. Both of them, however, also employed link capacity approximation. Long *et al.* [13] considered a cross-layer design of random access and power control to adapt for the congestion states with a proposed optimal algorithm but their algorithm required a complicated convexification computation. Tran *et al.* [16] tackled the nonconvexity of JCPC using a successive approximation method, which also

requires a lot of computations due to the successive approximations. Donghahe *et al.* [5] considered queueing delay in JCPC problem, again with high SIR approximation.

The above works also assumed slowly varying wireless channels, implying such algorithms must attain optimal solutions before the fading state changes. In case of fast fading channel, the update rate must be fast enough to keep track of changing fading states. This leads to the extravagant overheads until the schemes collapse. One solution for this issue is allowing transmission outages to occur between successive updates; as a result, the updates can proceed on a much slower time scale. This idea was first employed in [9] to solve a centralized power control problem. [15] may be the first work using this idea to address the JCPC problem. However, firstly this work included outage constraint implicitly into the approximated outage capacity constraint. Compared with an explicit outage constraint, this approximated constraint results in suboptimal solutions. Secondly, one of the most important parameters of outage constraint, SIR threshold, was lost due to this approximation. Consequently, the network QoS control, which can be characterized by tuning this parameter, was lost.

To overcome the limitations of the above works, we study the JCPC problem without high SIR assumption and with explicit outage constraint in a fast fading environment. This model guarantees the true optimal solutions and keeps the SIR threshold alive for network QoS control. After formulating this JCPC as an optimization problem, we prove its convexity, which is not a trivial work due to the complex relationship between interfering powers in the explicit outage constraint. Next, we propose two message passing distributed algorithms that solve this convex problem. The first algorithm can attain the global optimal solutions using a dual gradient algorithm. However, this scheme requires larger size of the control message than that of conventional scheme [3], which increases the network overhead. To overcome this issue, we design a second algorithm which is near optimal yet practical due to its small size control messages as in [3]. Extensive numerical results show that the gap between the optimal and near optimal algorithms is almost indistinguishable, and the second design demonstrates a faster convergence rate than the first one. Both schemes also outperform the suboptimal conventional scheme [3].

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¹See [4], which showed that a minimum success frame rate can be converted to an appropriate SIR threshold for a specific modulation and coding scheme.

II. SYSTEM MODEL AND PROBLEM FORMULATION

A. Network Model

We consider a wireless multihop network with $\mathcal{L} = \{1, 2, \dots, L\}$ logical links shared by $\mathcal{S} = \{1, 2, \dots, S\}$ sources. We assume that each source s emits a flow using a fixed set of links $L(s)$ on its route. The set of sources using link l is denoted by $S(l) = \{s | l \in L(s)\}$. Each source s achieves a utility $U_s(x_s)$ when transmitting a flow at a data rate x_s . The utility function $U_s(x_s)$ is assumed to be twice continuously differentiable, non-decreasing and strictly concave in x_s .

At the physical layer, we use the similar CDMA physical model of [3] where simultaneous communications can happen, which undergoes the multiple-access interference. The instantaneous capacity of link $l \in \mathcal{L}$ is a global and non-convex function of link power vector $\mathbf{P} = [P_1, \dots, P_L]$

$$c_l(\gamma_l(\mathbf{P})) = W \log(1 + K\gamma_l(\mathbf{P})), \quad (1)$$

where W is the baseband bandwidth and K is a constant depending on modulation, coding scheme and bit-error rate (BER) [7]. Unless otherwise stated, we assume $W = K = 1$ without loss of generality. $\gamma_l(\mathbf{P})$ is the *instantaneous* SIR of link l which is defined as

$$\gamma_l(\mathbf{P}) = \frac{P_l G_{ll} F_{ll}}{\sum_{k \neq l} P_k G_{lk} F_{lk} + N_0}, \quad (2)$$

where channel gain G_{lk} represents for slow-fading, F_{lk} models fast-fading channel from the transmitter on link k to the receiver on link l . N_0 is the thermal noise power at each receiver. Employing Rayleigh fading model, we assume F_{lk} is a i.i.d exponentially distributed with unit mean. Over the considered time scale, G_{lk} is assumed constant. Then we have the *average* SIR defined as follows

$$\bar{\gamma}_l(\mathbf{P}) = \frac{\mathbf{E}[P_l G_{ll} F_{ll}]}{\mathbf{E}\left[\sum_{k \neq l} P_k G_{lk} F_{lk} + N_0\right]} = \frac{P_l G_{ll}}{\sum_{k \neq l} P_k G_{lk} + N_0}$$

The operating range of source rates vector $\mathbf{x} = [x_1, x_2, \dots, x_S]$ and link power vector \mathbf{P} are denoted as follows

$$\mathcal{X} = \{x_s, s \in \mathcal{S} | x_s^{min} \leq x_s \leq x_s^{max}\} \quad (3)$$

$$\mathcal{P} = \{P_l, l \in \mathcal{L} | P_l^{min} \leq P_l \leq P_l^{max}\} \quad (4)$$

B. Problem Formulation: JCPC with Explicit Outage Constraint

We know that Chiang's work [3] with static fading state is impractical in a fast-fading channel. For example, when fading rate increases, the iteration rate must also increase in order to keep track of new channel states, which produces a considerable message-passing overhead until the scheme collapses. In order to alleviate this problem, some suggest that network can be allowed to suffer from a tolerable level of outage [9], [15] so that resources can be allocated on a much slower time scale. To take into account this issue, we

incorporate the outage constraint into the underlying NUM as follows

$$\begin{aligned} & \underset{\mathbf{x} \in \mathcal{X}, \mathbf{P} \in \mathcal{P}}{\text{maximize}} && \sum_s U_s(x_s) - \sum_l P_l && (5) \\ & \text{subject to} && \sum_{s \in S(l)} x_s \leq c_l(\bar{\gamma}_l(\mathbf{P})), \quad \forall l \\ & && Pr[\gamma_l \leq \gamma_l^{th}] \leq \xi_l, \quad \forall l \end{aligned}$$

where $c_l(\bar{\gamma}_l(\mathbf{P})) = \log(1 + \bar{\gamma}_l(\mathbf{P}))$, outage probability $Pr[\gamma_l \leq \gamma_l^{th}]$ is defined as a proportion of time that some SIR threshold γ_l^{th} is not met for sufficient reception at link l 's receiver and $\xi_l \in (0, 1)$ is the outage probability threshold on link l . The objective in this case is to maximize the network utility as well as minimize the total power. We further assume γ_l^{th} and ξ_l are chosen such that there exist feasible points in problem (5).

III. OPTIMAL ALGORITHM

A. Equivalent Convex Formulation

Using the close-form outage probability [9] for Rayleigh fading channel

$$Pr[\gamma_l \leq \gamma_l^{th}] = 1 - \exp\left(-\frac{N_0 \gamma_l^{th}}{P_l G_{ll}}\right) \prod_{k \neq l} \left(1 + \gamma_l^{th} \frac{P_k G_{lk}}{P_l G_{ll}}\right)^{-1},$$

and defining new variables $\hat{P}_l = \log P_l$, $\hat{x}_s = \log x_s$, and new sets $\hat{\mathcal{X}} = \{\hat{x}_s, s \in \mathcal{S} | \log x_s^{min} \leq \hat{x}_s \leq \log x_s^{max}\}$, $\hat{\mathcal{P}} = \{\hat{P}_l, l \in \mathcal{L} | \log P_l^{min} \leq \hat{P}_l \leq \log P_l^{max}\}$, we transform (5) into the following equivalent non-linear programming problem

$$\begin{aligned} & \underset{\hat{\mathbf{x}} \in \hat{\mathcal{X}}, \hat{\mathbf{P}} \in \hat{\mathcal{P}}}{\text{maximize}} && \sum_s U_s(e^{\hat{x}_s}) - \sum_l e^{\hat{P}_l} && (6) \\ & \text{subject to} && \log\left(\sum_{s \in S(l)} e^{\hat{x}_s}\right) \leq \log c_l(\hat{\gamma}_l), \quad \forall l \\ & && \sum_{k \neq l} \log\left(1 + e^{\hat{P}_k - \hat{P}_l} \frac{\gamma_l^{th} G_{lk}}{G_{ll}}\right) \leq \log \Omega_l(e^{\hat{P}_l}), \quad \forall l. \end{aligned}$$

where $\Omega_l(P_l) = \frac{\exp(-N_0 \gamma_l^{th} / P_l G_{ll})}{1 - \xi_l}$ and in order to simplify the notation, we denote $\bar{\gamma}_l = \bar{\gamma}_l(\mathbf{P})$ and $\hat{\gamma}_l = \bar{\gamma}_l(e^{\hat{\mathbf{P}}})$.

We assume that $U_s(\exp(\cdot))$ is a concave function with mild conditions [12]. After all we have the following theorem.

Theorem 1: Problem (6) is a convex optimization problem.

Proof: The proof is provided in Appendix A ■

B. Dual Decomposition and Optimal Solution

Thanks to the separable nature of the problem (6), its Lagrangian can be decomposed into two separate partial functions as followings

$$L(\hat{\mathbf{x}}, \hat{\mathbf{P}}, \boldsymbol{\lambda}, \boldsymbol{\nu}) = L_{\hat{\mathbf{x}}}(\hat{\mathbf{x}}, \boldsymbol{\lambda}) + L_{\hat{\mathbf{P}}}(\hat{\mathbf{P}}, \boldsymbol{\lambda}, \boldsymbol{\nu}) \quad (7)$$

and

$$L_{\hat{x}}(\hat{\mathbf{x}}, \boldsymbol{\lambda}) = \sum_s U_s(e^{\hat{x}_s}) - \sum_l \lambda_l \log \left(\sum_{s \in S(l)} e^{\hat{x}_s} \right) \quad (8)$$

$$L_{\hat{P}}(\hat{\mathbf{P}}, \boldsymbol{\lambda}, \boldsymbol{\nu}) = \sum_l \lambda_l \log c_l(\hat{\gamma}_l) + \nu_l \log \Omega_l(e^{\hat{P}_l}) - e^{\hat{P}_l} - \sum_l \nu_l \sum_{k \neq l} \log \left(1 + e^{\hat{P}_k - \hat{P}_l} \frac{\gamma_l^{th} G_{lk}}{G_{ll}} \right) \quad (9)$$

where $\boldsymbol{\lambda} = (\lambda_1, \lambda_2 \dots \lambda_L)$ and $\boldsymbol{\nu} = (\nu_1, \nu_2 \dots \nu_L)$, the Lagrange multipliers of the first and second constraint, are considered as the link *congestion price* and *outage price* respectively following the spirit of [10]. The dual problem of (6) is

$$\min_{(\boldsymbol{\lambda}, \boldsymbol{\nu}) \geq 0} [\max_{\hat{\mathbf{x}} \in \hat{\mathcal{X}}} L_{\hat{x}}(\hat{\mathbf{x}}, \boldsymbol{\lambda}) + \max_{\hat{\mathbf{P}} \in \hat{\mathcal{P}}} L_{\hat{P}}(\hat{\mathbf{P}}, \boldsymbol{\lambda}, \boldsymbol{\nu})] \quad (10)$$

From Theorem 1 we know the objective of the primal problem (6) is a concave function and the constraints are convex. There exists a strictly feasible point in its domain so the Slater's constraint qualification holds leading to strong duality [2, Prop.5.3.1]. This allows us to solve the primal (6) via the dual (10) through the following iterative algorithm.

Algorithm 1 Optimal JCPC with Outage Constraint

All primal and dual variables update iteratively as following until the termination criterion is satisfied

Congestion control: The source rate updates

$$x_s^{(t+1)} = \left[U_s'^{-1} \left(\lambda_s^{(t)} \right) \right]_{x_{min}}^{x_{max}} \quad (11)$$

, where $U_s'^{-1}$ is the inverse of the first derivative of utility and $\lambda_s^{(t)} = \sum_{l \in L(s)} \frac{\lambda_l^{(t)}}{\sum_{f \in S(l)} x_f^{(t)}}$.

Power control: The link power updates

$$P_l^{(t+1)} = \left[\frac{\delta_l^{(t)} - \nu_l^{(t)} \tilde{m}_l^{(t)} \frac{N_0}{\log(1-\xi_l)}}{1 + \sum_{k \neq l} G_{kl} m_k^{(t)} + \nu_k^{(t)} \frac{G_{kl} \tilde{m}_k^{(t)}}{1 + G_{kl} \tilde{m}_k^{(t)} P_l^{(t)}}} \right]_{P_{min}}^{P_{max}} \quad (12)$$

with

$$\delta_k^{(t)} = \lambda_k^{(t)} \frac{1}{\log(1 + \tilde{\gamma}_k^{(t)})} \frac{\tilde{\gamma}_k^{(t)}}{(1 + \tilde{\gamma}_k^{(t)})}, \quad (13)$$

$$m_k^{(t)} = \delta_k^{(t)} \frac{\tilde{\gamma}_k^{(t)}}{G_{kk} P_k^{(t)}}, \quad \tilde{m}_k^{(t)} = \frac{\gamma_k^{th}}{G_{kk} P_k^{(t)}} \quad (14)$$

Link Congestion Price Update:

$$\lambda_l^{(t+1)} = \left[\lambda_l^{(t)} - \kappa^{(t)} \left(\log c_l(\tilde{\gamma}_l) - \log \left(\sum_{s \in S(l)} x_s^{(t)} \right) \right) \right]^+ \quad (15)$$

Link Outage Price Update:

$$\nu_l^{(t+1)} = \left[\nu_l^{(t)} - \kappa^{(t)} \left(\log \Omega_l(P_l^{(t)}) - \sum_{k \neq l} \log \left(1 + \gamma_l^{th} \frac{G_{lk} P_k^{(t)}}{G_{ll} P_l^{(t)}} \right) \right) \right]^+ \quad (16)$$

where $[a]^+ = \max\{a, 0\}$, $[a]_b^c = \max\{\min\{a, c\}, b\}$ and $\kappa(t)$ is the step-size.

Proposition 1: The source rate update (11) solves the maximization problem $\max_{\hat{\mathbf{x}} \in \hat{\mathcal{X}}} L_{\hat{x}}(\hat{\mathbf{x}}, \boldsymbol{\lambda})$ for a fixed $(\boldsymbol{\lambda}, \boldsymbol{\nu})$.

Proof: Due to $L_{\hat{x}}(\hat{\mathbf{x}}, \boldsymbol{\lambda})$ is a strictly concave function with respect to $\hat{\mathbf{x}}$ for a fixed $\boldsymbol{\lambda}$, by the first-order optimal condition, we have

$$\frac{\partial L_{\hat{x}}(\hat{\mathbf{x}}, \boldsymbol{\lambda})}{\partial \hat{x}_s} = 0 = e^{\hat{x}_s} \left(U_s'(e^{\hat{x}_s}) - \sum_{l \in L(s)} \frac{\lambda_l^{(t)}}{\sum_{f \in S(l)} e^{\hat{x}_f}} \right)$$

Transforming back to the \mathbf{x} -space we have the claimed result. \blacksquare

Proposition 2: The power update (12) solves the maximization problem $\max_{\hat{\mathbf{P}} \in \hat{\mathcal{P}}} L_{\hat{P}}(\hat{\mathbf{P}}, \boldsymbol{\lambda}, \boldsymbol{\nu})$ for a fixed $(\boldsymbol{\lambda}, \boldsymbol{\nu})$.

Proof: The partial Lagrangian $L_{\hat{P}}(\hat{\mathbf{P}}, \boldsymbol{\lambda}, \boldsymbol{\nu})$ can be recast as follows

$$L_{\hat{P}}(\hat{\mathbf{P}}, \boldsymbol{\lambda}, \boldsymbol{\nu}) = \sum_l \lambda_l \log c_l(\hat{\gamma}_l) + \nu_l \log \Omega_l(e^{\hat{P}_l}) - e^{\hat{P}_l} - \sum_l \sum_{k \neq l} \nu_k \log \left(1 + e^{\hat{P}_l - \hat{P}_k} \frac{\gamma_l^{th} G_{kl}}{G_{kk}} \right) \doteq A + B - e^{\hat{P}_l} - C$$

And we have these following derivatives

$$\begin{aligned} \bullet \quad \frac{\partial}{\partial \hat{P}_l} (A) &= \lambda_l \frac{\partial (\log c_l(\hat{\gamma}_l))}{\partial \hat{\gamma}_l} \frac{\partial \hat{\gamma}_l}{\partial \hat{P}_l} + \sum_{k \neq l} \lambda_k \frac{\partial (\log c_k(\hat{\gamma}_k))}{\partial \hat{\gamma}_k} \frac{\partial \hat{\gamma}_k}{\partial \hat{P}_l} \\ &= \sum_{k \neq l} \lambda_k \frac{1}{\log(1 + \hat{\gamma}_k)} \frac{1}{1 + \hat{\gamma}_k} \left(-\frac{\hat{\gamma}_k^2}{G_{kk} e^{\hat{P}_k}} G_{kl} e^{\hat{P}_l} \right) \\ &\quad + \lambda_l \frac{1}{\log(1 + \hat{\gamma}_l)} \frac{\hat{\gamma}_l}{1 + \hat{\gamma}_l} \end{aligned} \quad (17)$$

$$\bullet \quad \frac{\partial}{\partial \hat{P}_l} (B) = -\nu_l^{(t)} \frac{\gamma_l^{th}}{G_{ll} e^{\hat{P}_l}} \frac{N_0}{\log(1 - \xi_l)} \quad (18)$$

$$\bullet \quad \frac{\partial}{\partial \hat{P}_l} (C) = \sum_{k \neq l} \nu_k \frac{e^{\hat{P}_l - \hat{P}_k} \gamma_l^{th} G_{kl}}{1 + e^{\hat{P}_l - \hat{P}_k} \frac{\gamma_l^{th} G_{kl}}{G_{kk}}} \quad (19)$$

Substituting (17), (19), (18) and $\frac{\partial (e^{\hat{P}_l})}{\partial \hat{P}_l} = e^{\hat{P}_l}$ into the first-order optimal condition $\frac{\partial L_{\hat{P}}(\hat{\mathbf{P}}, \boldsymbol{\lambda}, \boldsymbol{\nu})}{\partial \hat{P}_l} = 0$, we will have the result after transforming back to \mathbf{P} -space. \blacksquare

Both partial Lagrangian $L_{\hat{x}}(\hat{\mathbf{x}}, \boldsymbol{\lambda})$ and $L_{\hat{P}}(\hat{\mathbf{P}}, \boldsymbol{\lambda}, \boldsymbol{\nu})$ are strictly concave, hence their optimal solutions are unique for a specific $(\boldsymbol{\lambda}, \boldsymbol{\nu})$. We applied the gradient (subgradient) method to solve the dual problem (10) through the dual variable

updates (15) and (16). Then the convergence of Algorithm 1 can be proved using gradient-based standard technique [1]. Due to the limited space, we skip the proof here. Choosing the step-size satisfying $\kappa(t) > 0$, $\sum_{t=0}^{\infty} \kappa(t)^2 < \infty$ and $\sum_{t=0}^{\infty} \kappa(t) \rightarrow \infty$, we have the following result.

Theorem 2: For any initial power $\mathbf{P}^{(0)} \in \mathcal{P}$, source rate $\mathbf{x}^{(0)} \in \mathcal{X}$ and prices $(\boldsymbol{\lambda}^{(0)}, \boldsymbol{\nu}^{(0)}) \geq 0$, the sequence of $\{\mathbf{x}^{(t)}, \mathbf{P}^{(t)}, \boldsymbol{\lambda}^{(t)}, \boldsymbol{\nu}^{(t)}\}$ generated by Algorithm 1 converges to the global optimal point.

Remarks:

- 1) The congestion control can be implemented distributively using message passing. The destination sends a message back to the source for adjusting its rate with (11).
- 2) Link power update is analogous to the algorithm of [3]. Each receiver of link k broadcasts its control message containing three real-value fields reserved for $m_k^{(t)}$, $\tilde{m}_k^{(t)}$ and $\nu_k^{(t)}$. Each transmitter of link l then receives them, estimate G_{kl} through training sequence and update its power as (12).
- 3) The link congestion price update (15) only needs link's local information: ingress rate and SIR measurement.
- 4) The link outage price update (16) requires the receiver can measure separately each interfering power, which might be impractical. Another way to solve this issue is reserving another field (i.e. the fourth field) containing $P_k^{(t)}$ in the control message.
- 5) The overhead increases because the messages broadcast by receivers contain large information. Next, we will eliminate this problem by proposing a near-optimal scheme.

IV. NEAR-OPTIMAL ALGORITHM

In Algorithm 1, the messages broadcast by receivers contain much information causing the overhead and energy consumption increase for decoding at transmitters. In this section, we eliminate this issue by proposing a near-optimal scheme.

Using the upper and lower bounds on the outage probability derived in [9], we apply them to the outage constraint as $Pr[\gamma_l \leq \gamma_l^{th}] \leq 1 - \exp\left(-\frac{\gamma_l^{th}}{\bar{\gamma}_l}\right) \leq \xi_l$ and $\frac{\gamma_l^{th}}{\bar{\gamma}_l + \gamma_l^{th}} \leq Pr[\gamma_l \leq \gamma_l^{th}] \leq \xi_l$, which corresponds to these SIR constraints $\bar{\gamma}_l \geq -\frac{\gamma_l^{th}}{\log(1-\xi_l)}$ (upper bound) and $\bar{\gamma}_l \geq \gamma_l^{th} \left(\frac{1}{\xi_l} - 1\right)$ (lower bound), respectively. Hence the second constraint of problem (5) can be approximately replaced by $\bar{\gamma}_l \geq \eta_l$, where η_l is either of those two constant. We have a new optimization problem after changing variables

$$\begin{aligned} & \underset{\hat{\mathbf{x}} \in \hat{\mathcal{X}}, \hat{\mathbf{P}} \in \hat{\mathcal{P}}}{\text{maximize}} && \sum_s U_s(e^{\hat{x}_s}) - \sum_l e^{\hat{P}_l} && (20) \\ & \text{subject to} && \log\left(\sum_{s \in S(l)} e^{\hat{x}_s}\right) \leq \log c_l(\hat{\gamma}_l), \quad \forall l \\ & && -\log \hat{\gamma}_l \leq -\log \eta_l, \quad \forall l. \end{aligned}$$

This problem is also a convex programming problem. While the objective function and the first constraint is the same as the convex problem (6), at the second constraint $-\log \hat{\gamma}_l =$

$-\log(G_{ll}e^{\hat{P}_l}) + \log\left(\sum_{k \neq l} G_{lk}e^{\hat{P}_k} + N_0\right)$ is a convex function of $\hat{\mathbf{P}}$ (recall that log-sum-exponent is convex). Using the same approach as in Section III, the partial Lagrangians of (20) are

$$L_{\hat{\mathbf{x}}}(\hat{\mathbf{x}}, \boldsymbol{\lambda}) = \sum_s U_s(e^{\hat{x}_s}) - \sum_l \lambda_l \log\left(\sum_{s \in S(l)} e^{\hat{x}_s}\right) \quad (21)$$

$$L_{\hat{\mathbf{P}}}(\hat{\mathbf{P}}, \boldsymbol{\lambda}, \boldsymbol{\nu}) = \sum_l \lambda_l \log c_l(\hat{\gamma}_l) + \nu_l \log \hat{\gamma}_l - e^{\hat{P}_l} \quad (22)$$

Making use of gradient-descent algorithm to solve its dual problem analogous to previous section, we design the second iterative algorithm as follows

Algorithm 2 Near-Optimal JCPC with Outage Constraint

All primal and dual variables update iteratively as following until the termination criterion is satisfied

Congestion control: The source rate updates

$$x_s^{(t+1)} = \left[U_s'^{-1}\left(\lambda_s^{(t)}\right) \right]_{x_{min}}^{x_{max}} \quad (23)$$

Power control: The link power updates

$$P_l^{(t+1)} = \left[\frac{\delta_l^{(t)} + \nu_l^{(t)}}{1 + \sum_{k \neq l} G_{kl} m_k^{(t)}} \right]_{P_{min}}^{P_{max}} \quad (24)$$

with

$$\delta_k^{(t)} = \frac{\lambda_k^{(t)}}{\log(1 + \bar{\gamma}_k^{(t)})} \frac{\bar{\gamma}_k^{(t)}}{(1 + \bar{\gamma}_k^{(t)})}, \quad m_k^{(t)} = \frac{(\delta_k^{(t)} + \nu_k^{(t)}) \bar{\gamma}_k^{(t)}}{G_{kk} P_k^{(t)}}$$

Link Congestion Price Update:

$$\lambda_l^{(t+1)} = \left[\lambda_l^{(t)} - \kappa^{(t)} \left(-\log\left(\sum_{s \in S(l)} x_s^{(t)}\right) + \log c_l(\bar{\gamma}_l^{(t)}) \right) \right]^+ \quad (25)$$

Link Outage Price Update:

$$\nu_l^{(t+1)} = \left[\nu_l^{(t)} - \kappa^{(t)} \left(\log \bar{\gamma}_l^{(t)} - \log \eta_l \right) \right]^+ \quad (26)$$

From (21), we see that the congestion control mechanism would be exactly the same as Algorithm 1. We focus on the power control in the following result.

Proposition 3: The power update (24) solves the maximization problem $\max_{\hat{\mathbf{P}} \in \hat{\mathcal{P}}} L_{\hat{\mathbf{P}}}(\hat{\mathbf{P}}, \boldsymbol{\lambda}, \boldsymbol{\nu})$ for a fixed $(\boldsymbol{\lambda}, \boldsymbol{\nu})$.

Proof: Taking the derivatives and using first-order optimal condition similarly to Algorithm 1. ■

The convergence of Algorithm 2 also can be proved using gradient-based standard techniques.

Theorem 3: For any initial power $\mathbf{P}^{(0)} \in \mathcal{P}$, source rate $\mathbf{x}^{(0)} \in \mathcal{X}$ and prices $(\boldsymbol{\lambda}^{(0)}, \boldsymbol{\nu}^{(0)}) \geq 0$, the sequence of $\{\mathbf{x}^{(t)}, \mathbf{P}^{(t)}, \boldsymbol{\lambda}^{(t)}, \boldsymbol{\nu}^{(t)}\}$ generated by Algorithm 2 converges to the global optimal point.

Remarks:

- 1) The congestion control mechanism is the same as Algorithm 1.

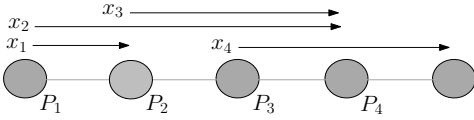
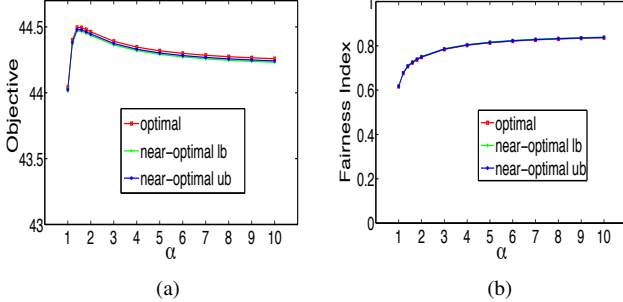


Fig. 1. Network Topology

Fig. 2. Impact of varying utility parameter α on network efficiency and fairness. (a) Objective value. (b) Jain's fairness index.

- 2) Link power control is analogous to that of [3] where the control message broadcasted by each receiver of link k only contains $m_k^{(t)}$ with locally measurable quantities. Link power update (24) also uses both link congestion price and outage price.
- 3) The link outage price update requires only its link's local SIR measurement.
- 4) We note that Algorithm 2 converges to global optimal point of (20), which is an approximated problem of (6). Hence the optimal solution of (20) is considered as a near-optimal solution of (6) (due to the tightness of the bounds).

V. NUMERICAL RESULTS

A. Simulation Setting

We consider a network topology as in Fig. 1 with 4 flows and 5 nodes placed equally at a distance d . The baseband bandwidth W is set to 32 kHz, and we use $K = -1.5/\log(5\text{BER})$ with $\text{BER} = 10^{-3}$ for MQAM modulation [7]. The slow-fading channel gain is assumed to be $h(d) = h_o(\frac{d}{100})^{-4}$ where h_o is a reference channel gain at a distance 100 m. The step sizes of both algorithms are chosen to be $0.01/t$. We use the following α -fair utility function [14]

$$U_s^\alpha(x_s) = \begin{cases} (1-\alpha)^{-1}x^{1-\alpha}, & \text{if } \alpha \geq 0, \alpha \neq 1 \\ \log x, & \text{if } \alpha = 1. \end{cases} \quad (27)$$

Since inner links suffer more interference than outer links, we set the outage probability threshold ξ_l of link 2 and link 3 to 0.3 and that of link 1 and link 4 to 0.2. The SIR thresholds γ_l^{th} of four links are set to [0.6 0.2 0.2 0.6] dB, which can be mapped to the respective minimum success frame rates as in [4].

B. Optimal Gap

We investigate the impact of the utility parameter α on the network performance. This parameter can act as a *knob* to

TABLE I
CONVERGENCE RATE COMPARISON

ϵ	10^{-3}	10^{-4}	10^{-5}
Algorithm 1	93.97	141.26	189.01
Algorithm 2	33.34	58.29	105.23

control the tradeoff between network efficiency and fairness in general NUM problem [12]. We fix $d = 80$ m and vary α from 1 to 10 to compare the network efficiency (objective value) and fairness where we use the Jain's fairness index [8] as the standard fairness measurement: $(\sum_s x_s)^2 / (S \sum_s x_s^2)$. As the results shown in Fig. 2, when α increases the objective value achieves maximum value at $\alpha = 1.5$ and then becomes less efficient. The system always becomes fairer when α increases. The performances between Algorithm 1 and Algorithm 2 with upper and lower bounds are almost indistinguishable.

C. Algorithm Convergence

The criterion used to evaluate the convergence speed is $\max_l \frac{|P_l^{(t)} - P_l^{(t-1)}|}{P_l^{(t-1)}} < \epsilon$ where ϵ is a small number. We fix $\alpha = 1$ and $d = 80$ m for all scenarios. Table I shows the average number of iterations over 100 realizations with various values of ϵ . The convergence speed of near-optimal algorithm is the same for both upper bound and lower bound and uniformly presented as Algorithm 2. We see that near-optimal scheme converges faster than the optimal scheme. This is a significant point as Algorithm 2, which can achieve a close-to-optimal solution with less control message size and faster convergence, would be efficiently practical.

Fig. 3 shows the comparison between Algorithm 1 and 2 about the convergence of source rates, link powers and outage probabilities. We only choose the lower-bound SIR constraint for Algorithm 2 to present. It can be seen that the source-rate allocations of both schemes are the same while the power allocation of Algorithm 2 is somewhat more aggressive due to the constraint approximation. The outage probabilities also converge to the desired values for both schemes in Fig. 3.

VI. CONCLUSION

We design two algorithms for joint rate control and power allocation in wireless multihop networks. The first algorithm is the optimal scheme yet impractical. The second design is a near-optimal scheme based on a tight bound approximation on outage probability. Numerical experiments show that the both schemes' performance are almost indistinguishable and the near-optimal scheme has faster convergence speed.

APPENDIX A PROOF OF THEOREM 1

The convexity of the first constraint can be shown similarly as in [15]. The remaining task is to show the convexity of the second constraint. In particular, we will prove that $f_i(\hat{\mathbf{P}}) \triangleq \sum_{k \neq i} \log \left(1 + e^{\hat{P}_k - \hat{P}_i} \frac{\gamma_l^{\text{th}} G_{lk}}{G_{li}} \right)$ is a convex function with respect to $\hat{\mathbf{P}}$ because it is straightforward that

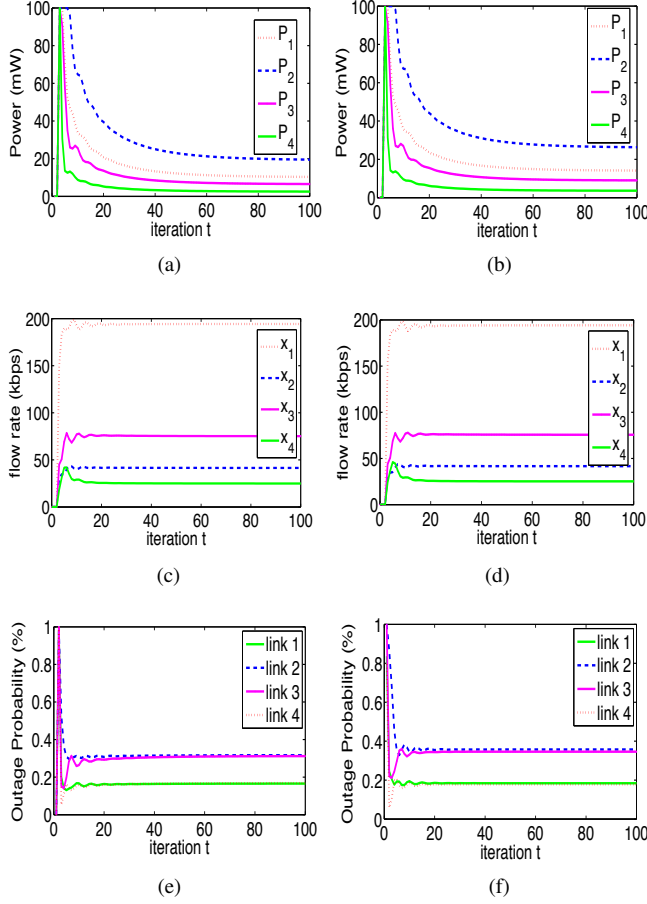


Fig. 3. Convergence of the source rates, link powers and outage probabilities for the Algorithm 1 (respectively (a), (c) and (e)) on the left and Algorithm 2 with lower-bound (respectively (b), (d) and (f)) on the right.

$\log \Omega_l (e^{\hat{P}_l})$ is a concave function of \hat{P}_l . First we have

$$\frac{\partial^2 f_l(\hat{\mathbf{P}})}{\partial \hat{P}_l^2} = \sum_{k \neq l} \frac{e^{\hat{P}_k - \hat{P}_l} \gamma_l^{th} G_{lk}}{(1 + e^{\hat{P}_k - \hat{P}_l} \gamma_l^{th} G_{lk}})^2}, \text{ and for any link } j \neq l$$

$$\frac{\partial^2 f_l(\hat{\mathbf{P}})}{\partial \hat{P}_j \partial \hat{P}_j} = -\frac{e^{\hat{P}_j - \hat{P}_l} \gamma_l^{th} G_{lj}}{(1 + e^{\hat{P}_j - \hat{P}_l} \gamma_l^{th} G_{lj})^2}, \quad \frac{\partial^2 f_l(\hat{\mathbf{P}})}{\partial \hat{P}_j \partial \hat{P}_i} = 0, \quad i \neq l, j$$

$$\frac{\partial^2 f_l(\hat{\mathbf{P}})}{\partial \hat{P}_j^2} = \frac{e^{\hat{P}_j - \hat{P}_l} \gamma_l^{th} G_{lj}}{(1 + e^{\hat{P}_j - \hat{P}_l} \gamma_l^{th} G_{lj})^2}, \quad \frac{\partial^2 f_l(\hat{\mathbf{P}})}{\partial \hat{P}_j \partial \hat{P}_l} = \frac{\partial^2 f_l(\hat{\mathbf{P}})}{\partial \hat{P}_l \partial \hat{P}_j}$$

Denote $z_k = \frac{e^{\hat{P}_k - \hat{P}_l} \gamma_l^{th} G_{lk}}{(1 + e^{\hat{P}_k - \hat{P}_l} \gamma_l^{th} G_{lk})^2} > 0$, the Hessian matrix

$\nabla^2 f_l(\hat{\mathbf{P}})$ can be shown as follows

$$\nabla^2 f_l(\hat{\mathbf{P}}) = \begin{pmatrix} \sum_{k=1, k \neq l}^{L-1} z_k & -z_1 & -z_2 & \cdots & -z_{L-1} \\ -z_1 & z_1 & 0 & \cdots & 0 \\ -z_2 & 0 & z_2 & \cdots & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ -z_{L-1} & 0 & 0 & \cdots & z_{L-1} \end{pmatrix}$$

Then, with all $\mathbf{v} \in \mathbb{R}^L$, we have $\mathbf{v}^T \nabla^2 f_l(\hat{\mathbf{P}}) \mathbf{v} = \sum_{k=1}^{L-1} z_k (v_1 - v_{k+1})^2 \geq 0$, which shows that $\nabla^2 f_l(\hat{\mathbf{P}})$ is

a positive semidefinite matrix.

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