Pricing Control for Hybrid Overlay/Underlay Spectrum Access in Cognitive Radio Networks

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***Abstract*—Recently overlay/underlay framework in Cognitive Radio (CR) have been studied and demonstrated the benefits such as spectrum efficiency and channel capacity maximization. We assume Secondary Users (SUs) can choose to either acquire a dedicated spectrum with constant payment or to use a Primary User (PU) band for free. However, using PU band yields delayed transmission cost. In this paper, we apply M/M/1 queueing model with heterogeneous service rate to derive the explicit expressions for the expected delays of an arbitrary SU data packets. Based on this, the SUs need to decide whether to use the Primary User (PU) band or to acquire the dedicated band. The interaction between selfish SUs is modeled as a noncooperative game. We prove the existence and uniqueness of a symmetric Nash equilibrium, and characterize the equilibrium behavior explicitly. Then an appropriate price of the dedicated band can be defined. Numerical analysis are used to prove a high degree of accuracy for the derived expressions.**

I. Introduction

Radio spectrum is one of the most scarce and valuable resources for wireless communications. Some surveys performing actual measurements have shown that most of the allocated spectrum is largely under-utilized [1]. Similar views on the under-utilization of allocated spectrum were reported by the Spectrum-Policy Task Force appointed by Federal Communications Commissions (FCC) [2]. Cognitive radio has been proposed as a way to improve spectrum efficiency by exploiting the unused spectrum in dynamically changing environments [3]. In a cognitive radio network, there are two types of users, namely, primary users and secondary users or cognitive users. In the cognitive radio concept, the transmission channel is licensed to the primary users while the secondary users opportunistically access the channel resources when it is not used by any primary users.

According to the access technology of the secondary users, spectrum sharing can be split into two groups: spectrum underlay and spectrum overlay [4], [5]. In spectrum underlay, the secondary users are permitted to transmit their data in the licensed spectrum band when the primary users are also transmitting. In order to protect the primary users, the interference temperature threshold is imposed on the secondary users' transmission power. However, due to the constraints on transmission power, the secondary users can not achieve the maximum throughput. On the other hand, the secondary users in spectrum overlay can only use the licensed spectrum when the primary users are not transmitting. Spectrum overlay is also referred to as opportunistic spectrum access. To avoid harmful interference to the primary users, the secondary users need to sense the licensed frequency band and detect the spectrum white space. In spectrum overlay, the secondary users can achieve the maximum throughput because there is no constraints on transmission power.

In recent years, the overlay/underlay framework in cognitive radio have been studied. In [6], authors showed that the primary user networks are benefitted by the relaying capability of the infrastructure based cognitive radio network that operates in overlay/underlay(hybrid) transmission mode. And in [7], the outage performance of relay assisted hybrid overlay/underlay cognitive radio systems were presented. Authors in [8] used Markov chain to find exact strategy switching threshold between overlay and overlay/underlay(hybrid) transmission mode. The inspiration of this paper came from [4], [5] where novel overlay/underlay waveforms have been proposed to exploit not only unused spectrum bands but also under-used spectrum bands in cognitive radio. In this paper, we analysis the average sojourn time of the secondary user in a overlay/underaly cognitive radio network by using M/M/1 queuing model where arrival and service rate are heterogeneous [9]. To the best of our knowledge, this is the first paper analyzing this kind of system.

Some recent works apply the queueing theory with server interruption model [10], priority queueing model [11], and server-breakdown queueing model [12] for controlling the queues in CR system. To our best knowledge, this paper is the first paper studying queueing control in overlay/underlay CR system. The main contributions of this paper are:

- We provides a first step towards a theoretical understanding of decision processes in the overlay/underlay CR system.
- We analysis the expected delay of SU customers in a overlay/underaly CR network by using M/M/1 queuing model where the service rates are heterogeneous.
- We consider the interaction between selfish SU customers as a noncooperative game. Based on the fixed dedicated band price policies, we then prove the existence and uniqueness of a symmetric Nash equilibrium, and characterize the equilibrium behavior explicitly.

The remainder of this paper is organized as follows. The system model is introduced in Section II. In Section III, the expected sojourn time is analyzed. Then, the equilibrium strategy of selfish SU is derived in Section IV. Finally, we draw conclusions in Section V.

II. System Model

We consider a CR system with a single PU band. PUs have a license to use the band. And when the primary users wish to transmit, it is given a priority over SUs. This is implemented by having the secondary user performs spectrum sensing with perfect sensing assumed. If there is no signal of the primary users, the secondary user will operate under overlay framework. Otherwise if the band is occupied by the primary users, the secondary user will operate under underlay framework. The situation shown in Fig.1 can be interpreted to be an example of the hybrid ovelay/underlay spectrum access framework. We assume that PU sojourn time(i.e., the

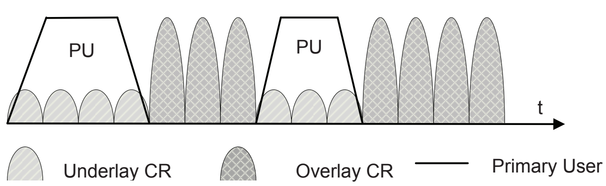

Fig. 1. Hybrid overlay/underlay spectrum access.

amount of time that PU band is in state ON) is random and exponentially distributed with mean $1/\eta$. And the amount of time that elapses between the end of a sojourn, and the starting of the next sojourn (i.e., the amount of time that PU band is in state OFF) is random and exponential with parameter ξ. The PU band can be considered as a server which oscillates between two feasible states ON/OFF which can be modeled by using Markov ON/OFF channel model [13]. Consequently, the secondary user operates also as a server that oscillates between two modes: underlay corresponding to state ON and overlay corresponding to state OFF. We denote underlay mode by 1 and overlay mode by 0. For both case, we assume that SU customers are served with the service times which are exponentially distributed with rate μ_0 in overlay mode and μ_1 in underlay mode respectively. We assume that $\mu_0 > \mu_1$. Because in spectrum overlay mode, the secondary user can achieve the maximum throughput while in the underlay mode the secondary user can not achieve the maximum throughput due to constraints on transmission power. Poisson process is used for packet arrivals so that the inter-arrival times are exponentially distributed with parameter λ. And we assume that the probability that two and more customers arrive at the same time is zero.

We consider an arrival process of SU customers, e.g., calls, packets, sessions or connections, arriving at the CR base station which is considered as the server. Each customer makes a decision whether to join or to leave the queue, i.e. discarding the packet. When a SU customer wants to be served, the SU (i.e., physical device) decides about whether to acquiring a dedicated band for a price or using the PU band for free. If

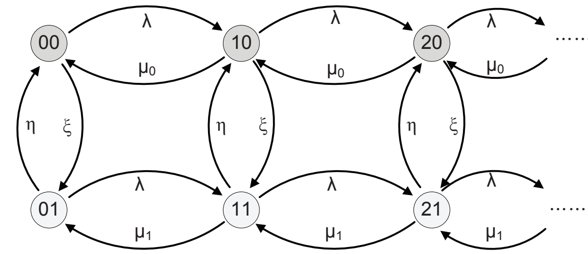

Fig. 2. Transition-rate diagram.

an SU chooses to acquire a dedicated band, it pays a fixed payment R. It consist of two part: the price of the dedicated band P and the average delay cost. We assume there exists a waiting cost of C units per time unit that is continuously accumulated from the time that the customer arrives at the system until the time it leaves after being served. In practical systems, the cost C represents the penalty for the delay or traffic congestion. Each SU customer using the dedicated band requires service for a random amount of time which is exponentially distributed with mean $1/\mu$. The expected cost when SU customer using the dedicated band is

$$R = P + \frac{C}{\mu}. \qquad (1)$$

If an SU chooses to use the PU band, it let the SU customer to joint the queue of SU customers who have chosen to use the PU band. A first-in-first-out (FIFO) queue is used for SU customers. SUs can utilize PU band without any admission free. Then, the expected cost of using PU band consist purely of a delay cost. Then, the payment of a SU customer that stays in the system for T time unit and completes the service successfully is CT. We assume that the customers' decisions are made only at their arrival instants, whereas a decision to join is irrevocable and reneging is not allowed.

III. Expected Sojourn Time Analysis

In this study, we use a M/M/1 queueing model with heterogeneous service rate to analyze the expected sojourn time of SU customers. Based on that, the strategies of customers are analyzed in the next sections.

Fig. 2 shows the Markov process corresponding to the system evolution. The steady state probability of the secondary user working under overlay mode is $P_0 = \eta/(\eta + \xi)$ and underlay mode is $P_1 = \xi/(\eta + \xi)$. The Markov process is positive recurrent if the average arrival rate is less than average service rate. The average service rate is defined by

$$\hat{\mu} = \mu_0 P_0 + \mu_1 P_1. \qquad (2)$$

Then the steady-state conditions of the system is $\lambda - \hat{\mu} < 0$.

Under the above condition, the set of balance equations is given by

$$(\lambda + \eta)P_{01} = \xi P_{00} + \mu_1 P_{11}, \qquad (3)$$

$$(\lambda + \eta + \mu_1)P_{n1} = \lambda P_{n-1,1} + \xi P_{n,0} + \mu_1 P_{n+1,1}, n > 0, \qquad (4)$$

$$(\lambda + \xi)P_{00} = \mu_0 P_{10} + \eta P_{01}, \qquad (5)$$

$$(\lambda+\xi+\mu_0)P_{n0} = \lambda P_{n-1,0} + \eta P_{n,1} + \mu_0 P_{n+1,0}, n > 0. \quad (6)$$

By employing vertical cuts in Fig. 2, we obtain

$$\lambda P_{n0} + \lambda P_{n1} = \mu_0 P_{n+1,0} + \mu_1 P_{n+1,1}, \ n \geq 0. \quad (7)$$

By summing above equation over n we get

$$\lambda = \mu_0(P_0 - P_{00}) + \mu_1(P_1 - P_{01}), \quad (8)$$

or

$$\mu_0 P_{00} + \mu_1 P_{01} = \hat{\mu} - \lambda. \quad (9)$$

where $P_i = \sum_{k=0}^{\infty} P_{n,i}, i = 0, 1$. By following similar approach as in [9], we define the partial generating function of the system as

$$G_i(z) = \sum_{n=0}^{\infty} z^n P_{n,i}\,, i = 0, 1, |z| \leq 1. \quad (10)$$

Multiplying each (3),(4), (5) and (6) by z^n and sum over n we obtain

$$\begin{aligned}&(\lambda+\eta+\mu_1)G_1(z) = \lambda z G_1(z) + \xi G_0(z) \\ &+\tfrac{\mu_1}{z}\left[G_1(z) - P_{01}\right] + \mu_1 P_{01},\end{aligned} \quad (11)$$

$$\begin{aligned}&(\lambda+\xi+\mu_0)G_0(z) = \lambda z G_0(z) + \eta G_0(z) \\ &+\tfrac{\mu_0}{z}\left[G_0(z) - P_{00}\right] + \mu_0 P_{00}.\end{aligned} \quad (12)$$

In order to calculate P_{01} and P_{00}, a polynomial of the third degree, g(z) is defined as follows

$$\begin{aligned}&g(z) = \lambda^2 z^3 - (\lambda^2 + \mu_0\lambda + \lambda\mu_1 + \xi\lambda + \eta\lambda)z^2 \\ &+(\mu_0\mu_1 + \mu_0\lambda + \lambda\mu_1 + \eta\mu_0 + \xi\mu_1)z - \mu_0\mu_1.\end{aligned} \quad (13)$$

(11), (12) and (13) are used to derive the bellow formula

$$\begin{aligned}&g(z)G_0(z) = P_{01}\lambda\eta\mu_1 z + \\ &P_{00}\mu_0\left[\eta z + \lambda z(1-z) - \mu_1(1-z)\right].\end{aligned} \quad (14)$$

Theorem 1: For positive μ_0 and μ_1 and finite η and ξ, the polynomial g(z) possesses a unique root z_0 in the open interval (0,1).

The proof was presented in [9]. By substituting $z = z_0$ in (14) we can eliminate the left hand side and obtain

$$\begin{aligned}&0 = P_{01}\lambda\eta\mu_1 z_0 + \\ &P_{00}\mu_0\left[\eta z_0 + \lambda z_0(1-z_0) - \mu_1(1-z_0)\right].\end{aligned} \quad (15)$$

Using (9), we eliminate P_{01} in the above equation and get

$$P_{00} = \frac{\eta(\hat{\mu}-\lambda)z_0}{\mu_0(1-z_0)(\mu_1 - z_0\lambda)}, \quad (16)$$

and, similarly we obtain

$$P_{01} = \frac{\xi(\hat{\mu}-\lambda)z_0}{\mu_1(1-z_0)(\mu_0 - z_0\lambda)}. \quad (17)$$

Then, we get

$$G_0(z) = \left[\eta(\hat{\mu}-\lambda)z + \mu_0 P_{00}(1-z)(\lambda z - \mu_1)\right]/g(z), \quad (18)$$

$$G_1(z) = \left[\xi(\hat{\mu}-\lambda)z + \mu_1 P_{01}(1-z)(\lambda z - \mu_0)\right]/g(z). \quad (19)$$

Define $E[L_i] = \sum_{n=0}^{\infty} nP_{n,i}$ as the contribution of state i to the mean queue size. Using (12) we have

$$E[L_i] = (d/dz)G_i(z)|_{z=1}. \quad (20)$$

Then, the (unconditional) expected queue size $E[L]$ can be obtained as

$$\begin{aligned}&E[L] = (d/dz)G_0(z)|_{z=1} + (d/dz)G_1(z)|_{z=1} \\ &= \tfrac{\lambda}{\hat{\mu}-\lambda} + \tfrac{\mu_1(\mu_0-\lambda)P_{01} + \mu_0(\mu_1-\lambda)P_{00} - (\mu_0-\lambda)(\mu_1-\lambda)}{(\eta+\xi)(\hat{\mu}-\lambda)}.\end{aligned} \quad (21)$$

Applying Little's formula to $E[L]$, we get the expected sojourn time of an arbitrary SU customer in the system as

$$\overline{T}(\lambda) = \frac{E[L]}{\lambda}. \quad (22)$$

A. Special case: Opportunistic spectrum access network

A special case of a model is that the service station is incapacitated from time to time and resumes its operation after a random time. This system can be modeled by using queueing model with heterogeneous arrivals and service but can be considered as a special case $\mu_1 = 0$ which was described in [9], [14]. In particular, this special case interprets the overlay mode of cognitive radio network which is also referred to as opportunistic spectrum access(OSA). Fig. 3 shows the Markov process corresponding to the system evolution.

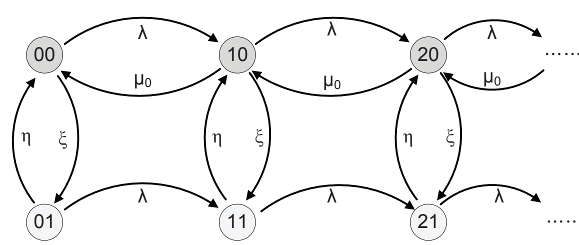

Fig. 3. Transition-rate diagram for OSA network.

A slotted OSA cognitive network was modeled by M/G/1 queueing model [15]. In contrast, we are concerned with modeling and analysis of an OSA cognitive network by M/M/1 queueing model. For simplicity, we assume that $\lambda_1 = \lambda_0 = \lambda$. This assumption was mentioned in the past literature. By substituting $\mu_1 = 0$ in (9), we obtain

$$P_{00} = P_0 - \frac{\lambda}{\mu_0}. \quad (23)$$

Substituting this value in (21) we have the expected queue size $E[L]$ as

$$E[L] = \frac{\lambda + P_1\frac{\lambda\mu_0}{\eta+\xi}}{\mu_0 P_0 - \lambda}. \quad (24)$$

For steady-state condition, the relation $\mu_0 P_0 > \lambda$ must hold. Then we get the expected sojourn time of an arbitrary packets in the system as

$$\overline{T} = \frac{E[L]}{\lambda} = \frac{1 + P_1\frac{\mu_0}{\eta+\xi}}{\mu_0 P_0 - \lambda}. \quad (25)$$

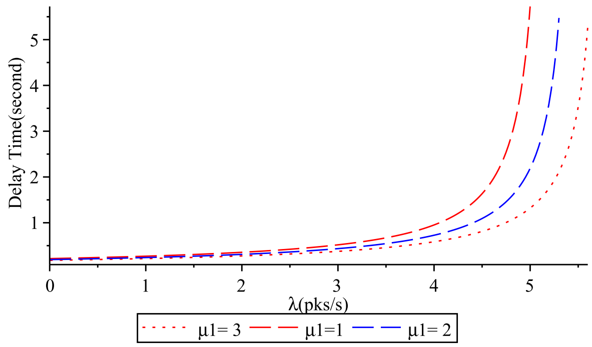

Fig. 4. Expected Sojourn Time under Hybrid Overlay/Underlay CR.

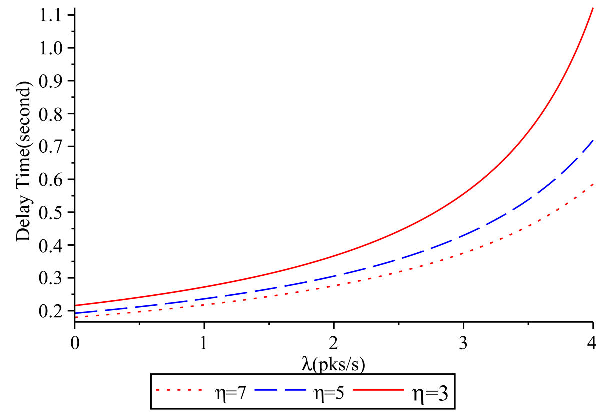

Fig. 5. Expected Sojourn Time under Hybrid Overlay/Underlay CR with different value of η.

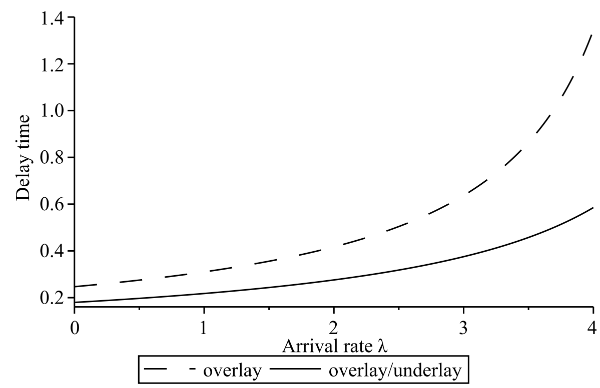

Fig. 6. Expected Sojourn Time under Overlay CR vs Hybrid Overlay/Underlay CR.

If we consider $\xi = 0$ it means that the primary user is always absent. Substituting $\xi = 0$ in (25) then the expected sojourn time can be expressed as

$$\overline{T} = \frac{1}{\mu_0 - \lambda}. \tag{26}$$

It is plausible because the system now can be modeled by using the normal M/M/1 queue, and the (26) is similar to the previous work in the literature [16].

B. Numeric Analysis

The numeric analysis are performed by "Maple". Fig.4 presents the expected sojourn time for secondary user under hybrid overlay/underlay framework. The parameters are set as $\eta = 7; \xi = 3; \mu_0 = 7$; and three value of μ_1 are $\mu_1 = 3$, $\mu_1 = 2$ and $\mu_1 = 1$. Then the maximum value of λ (i.e., $\hat{\mu}$) to guarantee the system stable are $29/5$, $11/2$ and $26/5$ respectively. As can be observed, when the arrival rate is small, the expected sojourn time for secondary user's packet is comparable. However, when the arrival rates increase to the maximum value then the delay rise up sharply. Fig.5 shows the sojourn time with different value of η. We vary η with 3, 5 and 7. As can be seen, when η increases, the sojourn time of the secondary user decreases. The reason is the transition rate from state OFF to state ON is faster. Consequently, the secondary user operates under overlay mode longer, then the expected sojourn time will decrease.

Fig.6 shows the expected sojourn time under overlay mode vs overlay/underlay mode. The parameters are given as $\eta = 7; \xi = 3; \mu_0 = 7$; and the maximum value of λ is $\mu_0 P_0 = 4.9$. We can see that the delay in only overlay is much higher than in hybrid overlay/underlay mode when the arrival rate approach λ the maximum value. Further mode, under hybrid overlay/underlay mode, the arrival rates can reach the higher value than overlay mode.

IV. Equilibrium Strategy Analysis

We consider the customers' strategies described by a probability q which is the probability an SU customer decides to use the PU band (thus, with probability $1 - q$ it decides to use the dedicated band). Because the customers are allowed to take their own decisions, then the system can be modeled as a noncooperative and symmetric game among the customers. Note that the fundamental problem in a game is to identify the Nash equilibrium point. From the game theory point of view, a Nash equilibrium stands for an operating point (a collection of strategies) where no user can improve its cost by unilaterally deviating from its current strategy. Moreover, a strategy is dominant if it is the best response against any strategy.

The expected sojourn time of SU customer $\bar{T}(\lambda)$ is continuous and monotonely increasing in the interval $(0, \hat{\mu})$. To avoid a trivial solution, we make the following assumption: $C\bar{T}(0) < R$. Then, we consider two cases:

1) $C\bar{T}(\lambda) \leq R$. All choosing PU band customers pay less than customer choosing the dedicated band. Therefore, the strategy of joining with probability $q_e = 1$ is an equilibrium strategy and no other equilibrium is possible. Moreover, in this case, joining is a dominant strategy.
2) $C\bar{T}(0) < R < C\bar{T}(\lambda)$. If $q_e = 1$ then a customer who chooses PU band pay more than customer acquiring the dedicated band. Hence, this cannot be an equilibrium

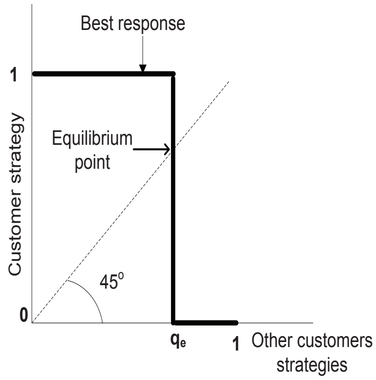

Fig. 7. The best response strategy correspond to other customers' strategy.

strategy. At the same way, if $q_e = 0$, a customer who acquires the dedicated band pay more than customer choosing PU band. Therefore, this too cannot be an equilibrium. Therefore, there exists a unique equilibrium strategy q_e satisfies q_e and $q_e\lambda$ solves $C\bar{T}(q_e\lambda) = R$.

The equilibrium strategy in this model is straightforward as follows. Suppose that the probability of choosing PU band is q. Because the customers's behavior are modeled as a symmetric game between customers then we have statement as follows: if $q < q_e$ then the unique best response is 1, if $q = q_e$ then any strategy between 0 and 1 is a best response, and if $q > q_e$ then the unique best response is 0. Since, a strategy is a symmetric equilibrium if it is a best response against itself, we conclude that q_e is a equilibrium point. Fig.7 depicts the best response function and the equilibrium point.

A. Numeric Analysis

The numeric analysis are performed by "Maple". The parameters are given as $\eta = 7$, $\xi = 3$, $\mu_0 = 7$, $\mu_1 = 3$, $\lambda = 7$. Fig. 8 shows the probability of SU customer using PU band respect to different value of R. As one can see when brand price R is cheap (less than 0.5), no SU customers choose PU band. If brand price R increase, there are more SU customer choosing PU band. If we know the potential arrival rate λ and the capacity of the dedicated band, we will define a appropriate brand price R to balance traffic between the dedicated band and free band.

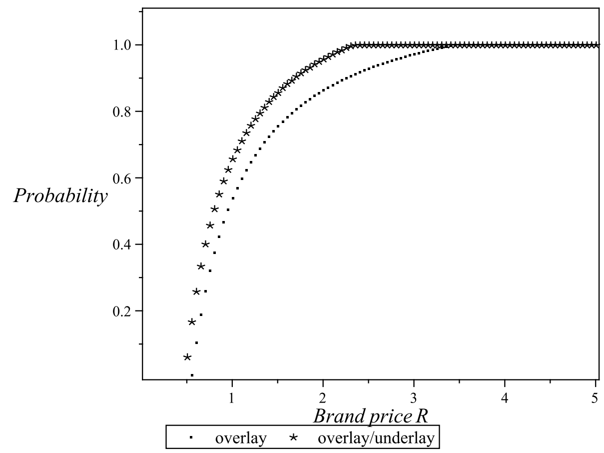

Fig. 8. Probability of choosing the PU band vs band price R.

V. Conclusion

In this paper, we used a M/M/1 queueing model with heterogeneous arrivals and service to model overlay/underlay cognitive radio networks. And we modeled opportunistic access in cognitive radio network as a special case. Sojourn time and expected queue size were theoretically derived. And numerical results demonstrate a high degree of accuracy for the derived expressions. In addition, a symmetric Nash equilibrium has been analyzed.

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