Using Linear Programming to Solve the Power Control Problem in Cognitive Radio Networks.

Cuong T. Do, Choong Seon Hong
Department of Computer Engineering, Kyung Hee University
Emails: {dtcuong, cshong}@.khu.ac.kr

Abstract

In this paper, by using linear programming, we derive an optimal power control strategy under both the probability of dropping a packet due to buffer overflow constraints at the secondary users and the interference constraints to the primary users.

1. Introduction

Cognitive radio (CR) is the key enabling technology that enables the next generation communication networks to utilize the spectrum more efficiently in an opportunistic fashion without interfering with the primary users [1-3]. The main contributions of this paper is to minimize the total transmit power of all secondary user transmitter (SU-Tx) nodes under both the probability of dropping a packet due to buffer overflow constraints of each SU-Tx and the interference constraint at primary receiver (PU-Rx) [4]. Our interest is to design a power control (DPC) algorithm using linear programing in the SU networks.

2. System model

We consider a CR network where each SU node is powered by a battery and equipped with a finite buffer to store the data. There are M SU user links which are randomly distributed in an area that is away from the one primary user link just as the illustration in Fig. 1. There is no information exchange between primary users and SU users. The SU nodes access the spectrum in the underlay model, which means that SU nodes are allowed to work on the same frequency bands designated for primary users, so long as the interference generated by SU nodes to primary users should not exceed the maximum level that primary users can tolerate. Denote I as the maximum interference tolerance at PR, which characterizes the "worst case" of the RF environment. Then I can be given as $I=\xi T$ where ξ is the Boltzman's constraint and T is the interference temperature limit. So the main constraint of power control in CR networks is to make the interference caused by SUs to the PUs below I ,i.e.,

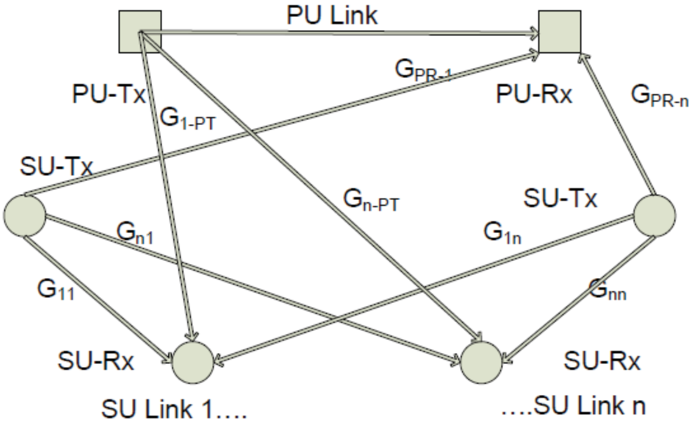

Fig.1.SU networks with M SU links and 1 PU link.

$$\sum_{i=1}^{M} G_{PR-i} P_i \le I \qquad (1)$$

Where G_{PR-i} is the channel gain between the SU-TX of link i and PU-RX, and P_i represents the transit power of SU-TX of link i. The signal-to-interference-noise-ratio (SINR) at the SU-Rx of link i is

$$SINR_i = \frac{G_{ii} P_i}{\sum_{j \neq i}^{M} G_{ij} P_j + G_{P-T_i} P_{PT} + \sigma^2} \qquad (2)$$

Where G_{ii} denotes the channel gain of the SU user link i, G_{ij} denotes the channel gain between SU-Tx of link j and SU-Rx of link i. The channel gain between primary transmitter (PU-Tx) and SU-Rx of link i is denoted by G_{PT-i}. P_{PT} is the transmit power of primary transmitter (PU-Tx), and σ^2 is the background noise at the SU-Rx of link i which is assumed to be AWGN.

The transmit rate R_i of SU-Tx is a variable which is correlated to the node modulation scheme and the simultaneous SINR at receiver. According to [4], we have:

$$R_i = \frac{1}{T}\log_2\left(1 + K \bullet SINR_i\right) \tag{3}$$

where T is the symbol period and K is a constant which is correlated to modulation scheme and bit error rate (BER).

The SU-Tx of link i first buffers the received packets in a queue and then transmits these packets at a rate R_i set by the $SNIR_i$ on the egress link, which is in turn determined by the transmit powers P_i [5]. A FIFO queuing discipline is used here for simplicity. The packets arrival process of link i is assumed to be a Poisson distribution with parameter λ_i. and to have an exponentially distributed length with parameter Γ. Using the model of an M/M/1 queue as in [6], the probability of transmitter i having a backlog of $N_i = k$ packets to transmit is well-known to be $\mathrm{Prob}\{N_i = k\} = (1 - \rho)\rho^k$ where $\rho = \lambda_i/\Gamma R_i(\mathbf{P}_i)$. The probability P_{BO} of dropping a packet due to buffer overflow at a node is also important in several applications. It is again a function of P and can be written as $P_{BO,i} = \mathrm{Prob}\{N_i > B\} = \rho^{B+1}$ where B is the buffer size. Setting an upper bound $P_{BO,i,max}$ on the buffer overflow probability also gives a posynomial lower bound constraint in P: $(T\lambda_i/\Gamma\log_2(1+K*SINR_i))^{B+1} \le P_{BO,i,max}$, or equivalently, $ISRi(\mathbf{P}) \le K/(2^{\Psi}-1)$ where $\Psi = (T\lambda_i)/(\Gamma(P_{BO,i,max})^{1/B+1})$ where ISRi is the inverse of the SINRi. For simplicity, we define $\pi_i \triangleq K/(2^{\Psi}-1)$, which is a constant when all the parameters of link i are fixed.

3. Problem formulation

In this section, we will give the power control problem. Our optimization objective is to minimize the total SU-Tx transmit power of all links under both $P_{BO,i,max}$ constraints of each SU-Tx and the interference constraint at PU-Rx. The optimization problem can be written as:

$$\min\sum_{i=1}^{M} P_i \quad \text{s.t.} \quad \begin{aligned} & 0 \le P_i \le P_{\max}, \forall i \\ & \sum_{i=1}^{M} G_{PR-i}P_i \le I \\ & \frac{\sum_{j\ne i}^{M} G_{ij}P_j + G_{i-PT}P_{PT} + \sigma^2}{G_{ii}P_i} \le \Pi_i, \forall i \end{aligned} \tag{4}$$

The transmit power of PU-Tx can be assumed to be fixed and its interference to each SU-Rx is an approximate constant. So for simplicity, we combine the interference generated by PU-Tx with the background noise at SU-Rx of link i as $N_i \square G_{i-PT}P_{PT} + \sigma^2$. The power control problem (4) can be rewritten as the following linear problem:

$$\min\sum_{i=1}^{M} P_i \quad \text{s.t.}$$

$$\begin{aligned} & 0 \le P_i \le P_{\max}, \forall i \\ & \sum_{i=1}^{M} G_{PR-i}P_i \le I \\ & -\sum_{j\ne i}^{M} G_{ij}P_j + G_{ii}\Pi_i P_i \ge N_i, \forall i \end{aligned} \tag{6}$$

4. Power Control Algorithm

The power control problem (6) could be solved by simplex-II method by adding some slack variable and surplus variable. Therefore, the linear optimization can be written as a standard form as bellow:

$$\begin{aligned} & \max\sum_{i=1}^{M} -P_i \\ & 0 \le P_i, \forall i \\ & P_i + y_i = P_{\max}, \forall i \\ & \sum_{i=1}^{M} G_{PR-i}P_i + y_{M+1} = I \\ & -\sum_{j\ne i}^{M} G_{ij}P_j + G_{ii}\Pi_i P_i - y_{M+1+i} = N_i, \forall i \end{aligned} \tag{7}$$

The problem (7) can be solved using two phase simplex method [7] to solve the linear program maximize **cx** subject to the constraints **Ax = b** and **x** >= 0.

Here G_{ij} and G_{i-PT} can be estimated by SU-Rx node of the link i and broadcast G_{ij} and G_{i-PT} .In addition, each SU-Tx could estimate the channel gain G_{PR-i} by listening to the feedback signals, like ACK/NACK, which are transmitted by PU-Rx to PU-Tx. Then, each SU-Tx broadcasts channel gain G_{PR-i},N_i and π_i. Consequently, all SU-Tx nodes can learn all parameters in problem (7) and simplex-II algorithm can be carried out at each SU-Tx node to find out the optimal power P_i.

Proposed Algorithm

1. Initialization: $0 \le P_i(0) \le P_{\max}$
2. At the SU-Rx of link i: measure G_{ij} and G_{i-PT} and broadcast G_{ij}, G_{i-PT}
3. At the SU-Tx of link i: measure G_{PR-i} . broadcast

G_{PR-i},N_i and π_i. after receiving all parameter, run simplex-II algorithm to obtain optimal P_i

5. Numerical Results

Packet traffic at each SU-Tx node is assumed to be Poisson with intensity λ_i = 2000 pk/s, Γ = 30bits. T=1/10s, B=100. We set the delay bound as $P_{BO,i,max}$ = 0.05, i.e., $SINRi$=20dB. The distances between three SU-Tx nodes and PU-Rx are 1000m, 1300m and 1800m respectively. Transmit power of each SU-Tx is limited to 1W and background noise is assumed to be $N=1 \times 10^{-10}$W. Channel gains are defined using a simple path loss model, $G_{ij} = Ld^{-4}_{ij}$ and $G_{PR-i} = Ld^{-3}_{PR-i}$, where L is a constant. Matrix d=[80,650,480;700,70,470; 450,500,90]; represents the distance between SU-Txs and SU-Rxs. Matrix GPR and G_{ij} represent channel gain SU-Tx to PU-Rx and SU-Tx to SU-Rx respectively. π_i = ISR $=10^{-2}$.

With these parameters above, we have:

```
B = [1; 1; 1;I; N; N; N;];
c = [-1 -1  -1  0  0  0  0  0  0  0 ];
A = [ 1 0 0                         1 0 0  0  0 0 0 ;
      0 1 0                         0 1 0  0  0 0 0 ;
      0 0 1                         0 0 1  0  0 0 0 ;
    GPR(1) GPR(2) GPR(3)            0 0 0 1   0 0  0 ;
ISR(1)*Gij(1,1) -Gij(1,2) -Gij(1,3) 0 0 0 0 -1  0  0;
-Gij(2,1) ISR(2)*Gij(2,2) -Gij(2,3) 0 0 0 0  0 -1  0;
-Gij(3,1) -Gij(3,2) ISR(3)*Gij(3,3) 0 0 0 0  0  0 -1;]
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After run simplex-II algorithm we obtain the optimal power vector $\{P_1=0.4749,\ P_2=0.2823,\ P_3=0.7617\}$ and the optimal value $\sum_{i=1}^{M} P$ is 1.5189.

6. Conclusions

In this paper, we have studied the power control problem in CR networks aiming to minimize the total power consumption of SU-Tx nodes under both the interference and the probability of dropping a packet due to buffer overflow constraints. Based on linear programming, we can obtain the optimal power control.

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