Joint Rate Control, Routing, and Scheduling for Inelastic Traffic in Multihop Wireless Networks

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Abstract

In this paper, we consider the network utility maximization (NUM) problem for the inelastic traffic in multihop wireless networks using a node-centric formulation. The NUM is a nonconvex problem and cannot be solved by canonical methods. To address the nonconvexity, we approximate its equivalent problem to a convex one which can be solved by the dual-based decomposition approach. By successively solving the approximations, the stationary point of the proposed algorithm converges to a Karush-Kuhn-Tucker solution of the original problem.

1. Introduction

In multihop wireless networks, the network utility maximization (NUM) framework using the node-centric formulation has been thoroughly studied in the literature, e.g., [1,2]. A jointly control rate, scheduling and routing algorithm is proposed. With the strictly concave utilities of elastic traffic, the NUM is a convex problem. Therefore, the dual approach in those works always results to an optimal solution.

In this paper, we address the above NUM problem as it applies to the inelastic traffic of the real-time applications. The utility of inelastic flow is often modeled by the sigmoidal functions $\frac{1}{1+e^{-a(x-b)}}$, $a,b > 0$, which is a nonconcave function, [3,4].

Due to the nonconcave objective, the NUM problem becomes a nonconvex problem and cannot be solved by canonical methods as in [1,2]. In order to address the nonconvexity, we transform the original NUM to an equivalent problem P2. Then we approximate P2 to a new convex problem P3 which is efficiently solved by the dual-based subgradient approach. By successively approximating and solving P3, the solution sequence is improved monotonically and converges to a KKT solution of the original problem.

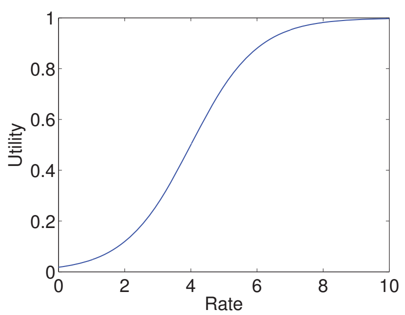


Fig. 1. Sigmoidal utility function $\frac{1}{1+e^{-(x_s-4)}}$.

2. Formulation

Consider a wireless network with the set of nodes N and the set of directed links L. Let S and D be the sets of source nodes and destination nodes, respectively, $S, D \in N$. Each flow in network maps a unique pair of nodes (s,d) where $s \in S$ and $d \in D$. Let $\boldsymbol{x} = \{x_s\}_{s \in S}$ be the rate vector of the flows. Assume that $\boldsymbol{x}$ is bounded in $[m, M]$. The utility function of flow s is given by

$$U_s(x_s) = \frac{1}{1+e^{-a(x_s-b)}} \tag{1}$$

Let $\boldsymbol{f} = \{f_{ij}^d\}_{(i,j)\in L, d\in D}$ be the link-rate vector where f_{ij}^d is the rate at link (i,j) when transmitting the data to destination d. Let Π be the feasible set of $\boldsymbol{f}$. For each destination d, the total outgoing traffic at node $i \in \{N \backslash D\}$ is $\sum_{j:(i,j)\in \boldsymbol{L}} f_{ij}^d$ and the total incoming traffic at node $i \in \{N \backslash S\}$ is $\sum_{j:(j,i)\in \boldsymbol{L}} f_{ji}^d$. For all $i \in N$ and $d \in D$, we have the conservation constraint

$$\sum_{s:b(s)=i,e(s)=d} x_s + \sum_{j:(j,i)\in \boldsymbol{L}} f_{ji}^d \le \sum_{j:(i,j)\in \boldsymbol{L}} f_{ij}^d, \tag{2}$$

in which $b(s)$ is the source node of s and e(s) is the destination of the flow from s. Note that the link-rate vector $\boldsymbol{f}$ and user-rate vector $\boldsymbol{x}$ are different. Link-rate is the transmitting rate that is allocated on the links for endogenous traffic (traffic among nodes), whereas user-rate is the rate of the exogenous traffic injected to the system at the nodes (traffic from the transport layer). We assume that the link capacities are not varied in this paper.

Our goal is to maximize the total utility of the flows

P1: $\max \sum_{s\in S} U_s(x_s)$
$\quad$ st. (2) and $f \epsilon \Pi$.

P1 is a nonconvex problem due to the nonconcave objective.

3. Proposed algorithm

We replace P1 by the equivalent problem as follows:

P2: $\max \log(\sum_{s\in S} U_s(x_s))$

st. (2) and $f\epsilon\Pi$.

P2 is still a nonconvex optimization problem. We now approximate P2 to a convex problem P3 using the arithmetic-geometric mean inequality. The following inequality is obtained:

$$\log(\sum_{s\in S} U_s(x_s)) \geq \sum_{s\in S} \theta_s \log\left(\frac{U_s(x_s)}{\theta_s}\right) \quad (3)$$

for any $\theta = [\theta_1, \dots, \theta_{|S|}] > 0$. The equality holds iff.

$$\theta_s = \frac{U_s(x_s)}{\sum_{i\in S} U_i(x_i)} \quad (4)$$

From (3), we approximate P2 to a new problem

P3: $\max \sum_{s\in S} \tilde{U}_s(x_s;\theta_s)$

st. (2) and $f\epsilon\Pi$,

where $\tilde{U}_s(x_s;\theta_s) = \theta_s \log\left(\frac{U_s(x_s)}{\theta_s}\right)$, a function characterized by θ_s. With the sigmoidal utility (1), $\tilde{U}_s(x_s;\theta_s)$ is a strictly concave function. Therefore, given $\boldsymbol{\theta}$, P3 becomes a canonical NUM with a strictly concave objective. Hence, we can apply the dual-based subgradient approach to solve the NUM. The detail analysis is omitted due to the space constraint. The interest readers can refer to [1,2] for the details. The following joint scheduling, routing and rate control algorithm is proposed.

Algorithm 1.

1: Initialize from any feasible point

2: $\tau := 0$;

3: **loop** {outer-iteration}

4: $\tau := \tau + 1$;

5: $\theta_s^\tau = \frac{U_s(x_s^\tau(0))}{\sum_{i\in S} U_i(x_i^\tau(0))}$

6: **repeat** {inner-iteration}

7: each node update the prices

$$\lambda_i^{d,\tau}(t+1) = \left[\lambda_i^{d,\tau}(t) + \gamma(t)\Big(\sum_{s:b(s)=i,e(s)=d} x_s^\tau(t) + \sum_{j:(j,i)\in L} f_{ji}^{d,\tau}(t) - \sum_{j:(i,j)\in L} f_{ij}^{d,\tau}(t)\Big)\right]^+$$

8: choose the link schedule according to max-weight scheduling $\boldsymbol{f}^\tau(t) = \arg\max_{f\epsilon\Pi} \sum_{(i,j)\in L} f_{ij} w_{ij}^\tau(t)$, where $w_{ij}^\tau(t) = \max_{d\in D}\left(\lambda_i^{d,\tau}(t) - \lambda_j^{d,\tau}(t)\right)$.

9: source update the rates $x_s^\tau(t) = \left[\tilde{U}_s'^{-1}(\lambda_s^{d,\tau}(t);\theta_s^\tau)\right]_m^M$

10: **until** convergence to a stationary point, i.e., $\boldsymbol{x}^{\tau*}$.

11: $x_s^{\tau+1}(0) = x_s^{\tau*}$.

12: broadcast $\sum_{s\in S} U_s(x_s^{\tau*})$ to all the sources.

13: **end loop**

In step 11 of Algorithm 1, $\boldsymbol{x}^{\tau*}$ is the stationary value of the τ-th outer-iteration. So, the initial value of a new outer-iteration is the stationary value of the previous outer-iteration. Also the new value $\boldsymbol{\theta}$ is calculated by the stationary rate of previous outer-iterations at step 5.

Using the diminishing step-size, the solution of P3 improves the objective of P1 in every outer-iteration. At the stationary point, Alg. 1 converges to a KKT solution of P1. The proof is very similar to [5,6].

4. Numerical results

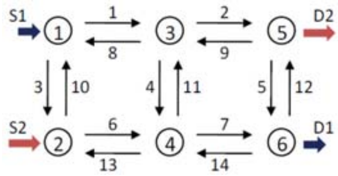
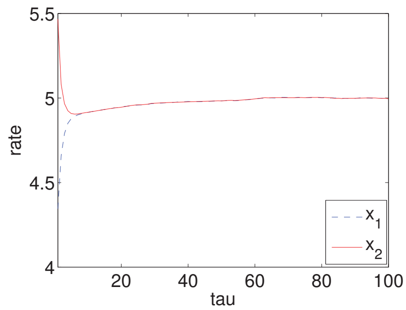
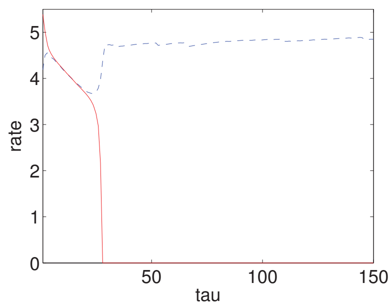


Fig. 2. 2x3 grid wireless network.



a) $c = 10$ Mbps



b) $c = 5$ Mbps

Fig. 3. Rate evolutions of Algorithm 1 *wrt.* outer iterations.

We consider a 2x3 grid wireless network shown in Fig.2. There are two inelastic flows with same utility $\frac{1}{1+e^{-(x_s-4)}}$. Let $m = 0$ and $M = 20$ Mbps. Assume node-exclusive interference model is used. That is the links that share a same node cannot be active at the same time. For example in Fig. 1, the links 1, 8, 4, 11, 3, 10, 2, 9 interfere each other; the independence sets are (1,5,6), (1,6,12), (1,7),… The diminishing step-size is $0.002/t$.

When the link capacities are all 10 Mbps, Alg. 1 results the user-rates $x_1 = x_2 = 5$ Mbps as shown in Fig. 3a. When we reduce the capacity to 5 Mbps, although two flows are symmetric, only one flow is allocated with 5 Mbps, whereas the other one is not (see Fig. 3b). In this case, we can see that the sigmoidal utilities do not guarantee the fairness among inelastic flows as the well-known alpha-fair family does among elastic flows. Intuitively, when many real-time users access the network simultaneously, it is better to drop some users to guarantee the good quality of the remaining ones than to allow all users access the network with bad quality. Therefore, with the use of sigmoidal utilities, the admission control is naturally integrated. Tables 1a,b show the link rates when $c = 10$ and 5 Mbps respectively.

Link no.	*1*	*2*	*3*	*4*	*5*	*6*	*7*
Flow 1	2.5	2.5	2.5	0	2.5	2.5	2.5
Flow 2	2.5	2.5	0	0	0	2.5	2.5
Link no.	*8*	*9*	*10*	*11*	*12*	*13*	*14*
Flow 1	0	0	0	0	0	0	0
Flow 2	0	0	2.5	0	2.5	0	0

a) $c = 10$ Mbps

Link no.	*1*	*2*	*3*	*4*	*5*	*6*	*7*
Flow 1	2.5	2.5	0	0	0	2.5	2.5
Flow 2	0	0	0	0	0	0	0
Link no.	*8*	*9*	*10*	*11*	*12*	*13*	*14*
Flow 1	0	0	2.5	0	2.5	0	0
Flow 2	0	0	0	0	0	0	0

b) $c = 5$ Mbps

Table 1. Average link rates of two flows with a) $c = 10$ Mbps and b) $c = 5$ Mbps.

5. Conclusions

We used a node-centric formulation to address the nonconvex NUM problem for inelastic traffic in multihop wireless networks. After a series of under-approximations, the solution to the convex-approximation problem, as well as the aggregate utility, monotonically increases and converges.

The simulation results show the convergence and the integrated admission control scheme of Alg. 1. A more practical heuristic algorithm which leads to similar results to Algorithm 1 in many experiments has been also proposed. Our analysis can be generalized to multiclass traffic which has log-concave utilities.

Acknowledgment

This research was supported by the MSIP (Ministry of Science, ICT&Future Planning), Korea, under the ITRC (Information Technology Research Center) support program supervised by the NIPA (National IT Industry Promotion Agency). Dr. CS Hong is the corresponding author.

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