Resource Allocation for Virtualized Wireless Networks with Backhaul Constraints

Tuan LeAnh, Nguyen H. Tran, Duy Trong Ngo, and Choong Seon Hong

Abstract—In this letter, we study resource allocation for wireless virtualized networks considering both the backhaul capacity of the infrastructure provider (InP) and the users' quality-of-service (QoS) requirements. We focus on the profit gained by a mobile virtual network operator (MVNO), which is a middleman who buys physical resource from the InP, bundling them into virtual resources called slides before selling off the service providers. The objective of the MVNO is to maximize its profit while guaranteeing the backhaul constraint and users' QoS by jointly allocating the slices and the uplink transmit power to the users. To solve the formulated mixed integer non-convex problem, we propose a distributed solution framework based on Lagrangian relaxation to a find suboptimal decision about slice and transmit power allocations. We further propose a low-complexity solution based on the concept of a matching game that does not require any global information. Numerical results are provided to evaluate the performance of the proposed schemes.

Index Terms—Matching theory, optimization theory, resource allocation, virtualized wireless networks.

I. INTRODUCTION

THE wireless network virtualization has recently been considered as a promising solution to increase the spectrum and infrastructure efficiency [1]. Using virtualization, the same infrastructure can be shared for differentiated services. Moreover, it provides easier migration to newer technologies while supporting legacy technologies by isolating part of the network.

The design of an efficient virtualized resource allocation plays an important role in the virtualized cellular network deployment [2]–[4]. However, the works in [2] and [3] ignore *backhaul* link constraints that should be carefully studied for dense small-cell deployment scenarios with possible bottleneck in various backhaul solutions [5]. Unlike [2] and [3], the study of [4] incorporates a business model for the profit of MVNOs. In this model, the MVNOs rent the network resources from the wireless physical infrastructure providers (InPs) to create virtual resources. In turn, the service providers (SPs) rent virtualized resources from MVNOs to provide specific services to the end users. However, it is noted that works in [2]–[4] do not study virtual resource allocation for

uplink transmissions, which enables user equipments to use power more efficiently to meet their QoS requirements for the given scheduled radio resources. Besides, it should pay more attention to adapt the inevitable uplink traffic explosion in future mobile networks [6].

To fill the gap in the existing literature, we study the virtual resource allocation in an uplink virtualized cellular network. The resource allocation is formulated as an NP-hard optimization problem that jointly allocates power and slices in a business model. Here, the chunk-based radio resource allocation is used to isolate the slices [7]. The considered joint slice and power allocation complicate any optimization-based design due several coupled constraints: i) slice isolation, ii) backhaul limitation, and iii) chunk allocation for heterogeneous users' QoS. Our research contributions are summarized as follows:

- We propose a slice isolation approach for the uplink of a virtualized cellular network in which the *virtualized resources* or *slices* are isolated by base stations and chunk-based radio resources owned by different InPs.
- We propose a distributed algorithm based on Lagrangian relaxation to find suboptimal decision on slice and transmit *power* allocations. The problem is solved in two different phases of power allocation and slide allocation through updating the sequence of primal and dual variables. The optimal power is derived from Karush-Kuhn-Tucker (KKT) conditions, whereas the Hungarian method is applied to solve the slice allocation in a centralized manner.
- To circumvent the requirement of global information, we further propose a distributed algorithm based on the concept of the matching game. This algorithm is shown to converge to a suboptimal solution.
- Numerical results show that our proposed approaches require a small number of iterations to converge.

II. SYSTEM MODEL AND PROBLEM FORMULATION

A. System Model

We consider an OFDMA-based virtualized cellular network for uplink transmissions as shown in Fig. 1. The MVNO rents network resources from a set of $\mathcal{B} = \{1, 2, ..., B\}$ InPs. For simplicity, we focus on a small area overlapped by multiple InPs, in which each InP provides infrastructure services including one small-cell base station (SBS), chunk-based wireless resources, and a wireless backhaul link underlay an InP's macro-cell base station (MBS). The MVNO then provides virtual resource services to a set of $I = \{1, 2, ..., I\}$ SPs. Each SP $i \in I$ has a set $\mathcal{U}^i = \{1, 2, ..., U_i\}$ subscribed users. Let $\mathcal{U}$ be the set of all users of all SPs. The radio resources owned by InP b consisting of a set of $\mathcal{L}_b = \{1, 2, ..., L_b\}$ subcarriers are divided into $\mathcal{C}_b = \{1, 2, ..., C_b\}$ chunks, each of
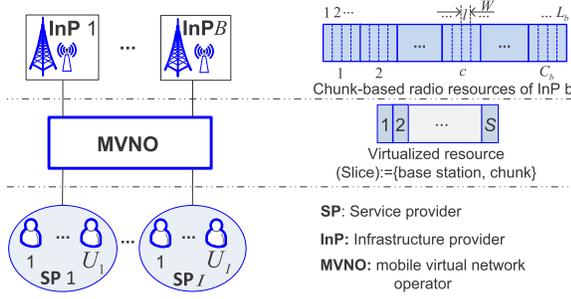
Fig. 1. Virtualized cellular network hierarchical model.

which is aggregated by $\mathcal{L}_{b,c} = \{1, 2, ..., L_b/C_b\}$ subcarriers. Additionally, we assume there is no interference among the small-cells from the same InP and among different InPs.

Virtualized resource. The infrastructure services are isolated by a set of $\mathcal{S}$ slices. Each slice is uniquely determined by a pair of base station and chunk. We denote $\boldsymbol{\alpha} = [\alpha_{b,c}^u]_{|\mathcal{U}| \times (|\mathcal{C}_b| \times B)}$ as the slice allocation matrix. Here, $\alpha_{b,c}^u$ is a binary indicator variable with $\alpha_{b,c}^u = 1$ if user u is allocated to slice $\{b, c\}$ ($b \in \mathcal{B}$, $c \in \mathcal{C}_b$), and $\alpha_{b,c}^u = 0$ otherwise.

B. Problem Formulation

1) Slice-Based Data Transmission Rate: When user u of the SP i transmits data with the slice-based allocation scheduled by the MVNO, its data rate is given by

$$R_u^i(\boldsymbol{\alpha}, \boldsymbol{P}) = \sum_{b \in \mathcal{B}} \sum_{c \in \mathcal{C}_b} \alpha_{b,c}^u r_{b,c}^u(\boldsymbol{P}_{b,c}^u), \tag{1}$$

where $r_{b,c}^u(\boldsymbol{P}_{b,c}^u) = \sum_{l \in \mathcal{L}_{b,c}} W \log_2(1 + \gamma_{b,c}^{u,l} P_{b,c}^{u,l})$ is the chunk-based data rate of user u associated to SBS b and chunk c; W is the bandwidth of each subcarrier $l \in \mathcal{L}_b, \forall b$; $\gamma_{b,c}^{u,l} = g_{b,c}^{u,l}/\sigma_b^2$ in which the additional interference from macrocell network is absorbed into the background noise σ_b^2; $g_{b,c}^{u,l}$ represents the instantaneous channel gain on subcarrier l of chunk c from user u to the SBS b; $\boldsymbol{P}_{b,c}^u = [P_{b,c}^{u,l}]_{1 \times |\mathcal{L}_{b,c}|}$ is the transmit power vector on all sub-carriers of chunk c; $\boldsymbol{P} = [\boldsymbol{P}_{b,c}^u]_{|\mathcal{U}| \times (|\mathcal{C}_b| \times B)}$ is the transmit power matrix of users on all slices of all subscribed users.

2) User's QoS Demand: In order to guarantee the minimum rate requirement $R_{u,\min}^i$ for user u subscribed to the specific service of the SP i, the following constraint is imposed:

$$R_u^i(\boldsymbol{\alpha}, \boldsymbol{P}) \geq R_{u,\min}^i, \ \forall u \in \mathcal{U}, \forall i \in I. \tag{2}$$

3) Backhaul Link Constraint: In order to avoid congestion at the capacity-limited backhaul links of InPs' SBSs, the aggregated data rate aggregation from all the users needs to satisfy the following constraint:

$$\sum_{i \in I} \sum_{u \in \mathcal{U}_i : \alpha_{b,c}^u = 1} R_u^i(\boldsymbol{\alpha}, \boldsymbol{P}) \leq Z_{b,\text{bh}}, \ \forall b \in \mathcal{B}, \ \forall c \in \mathcal{C}_b, \tag{3}$$

where $Z_{b,\text{bh}} \geq 0$ is a predefined backhaul capacity of SBS b.

4) Network Utility Function: We consider the following network utility of the MVNO achieved from allocating the slice and transmit power to SPs' users:

$$U_{\text{MVNO}}(\boldsymbol{\alpha}, \boldsymbol{P}) = U^{\text{rev}}(\boldsymbol{\alpha}, \boldsymbol{P}) - U^{\text{cost}}(\boldsymbol{\alpha}, \boldsymbol{P}), \tag{4}$$

where $U^{\text{rev}}(\boldsymbol{\alpha}, \boldsymbol{P}) = \sum_{i \in I} \sum_{u \in \mathcal{U}^i} \varphi_i^{sp} R_u^i(\boldsymbol{\alpha}, \boldsymbol{P})$ is the MVNO network revenue resulting from providing virtual resources

to SPs; $U^{\text{cost}}(\boldsymbol{\alpha}, \boldsymbol{P}) = \sum_{b \in \mathcal{B}} \sum_{c \in \mathcal{C}_b} (\varphi_b^{\text{bh}} \alpha_{b,c}^u r_{b,c}^u(\boldsymbol{P}_{b,c}^u) + \varphi_{b,c}^{\text{slice}} \alpha_{b,c}^u)$ is total cost incurred as the MVNO leases physical resource from the InPs; φ_i^{sp} is the payment (in units/Mbps) of each SP i to the MVNO; φ_b^{bh} is the unit price (in units/Mbps) of the backhaul set by InP b for SBS b; $\varphi_{b,c}^{\text{slice}}$ is the unit price of the slice set by InP b for the chunk c of the SBS b.

The design problem is mathematically formulated as:

$$(\textbf{OP}): \ \max_{(\boldsymbol{\alpha}, \boldsymbol{P})} \ U_{\text{MVNO}}(\boldsymbol{\alpha}, \boldsymbol{P}) \tag{5}$$

$$\text{s.t.} \quad (2), (3),$$

$$\sum_{b \in \mathcal{B}} \sum_{c \in \mathcal{C}_b} \alpha_{b,c}^u \sum_{l \in \mathcal{L}_c} P_{b,c}^{u,l} \leq \bar{P}_u, \ \forall u \in \mathcal{U}, \tag{6}$$

$$P_{b,c}^{u,l} \geq 0, \ \forall b \in \mathcal{B}, \forall c \in \mathcal{C}_b, \forall u \in \mathcal{U}, \tag{7}$$

$$\alpha_{b,c}^u \in \Pi_{\boldsymbol{\alpha}}, \ \forall b \in \mathcal{B}, \forall c \in \mathcal{C}_b, \forall u \in \mathcal{U}, \tag{8}$$

where the total amount of transmit power ($\bar{P}_u$) of user is constrained by (6); $\Pi_{\boldsymbol{\alpha}}$ is the following non-convex set:

$$\sum_{u \in \mathcal{U}} \alpha_{b,c}^u \leq 1, \ \forall c \in \mathcal{C}_b, \ \forall b \in \mathcal{B}, \tag{9}$$

$$\sum_{b \in \mathcal{B}} \sum_{c \in \mathcal{C}_b} \alpha_{b,c}^u \leq 1, \ \forall u \in \mathcal{U}, \tag{10}$$

$$\sum_{u \in \mathcal{U}} \sum_{c \in \mathcal{C}_b} \alpha_{b,c}^u \leq 1, \ \forall b \in \mathcal{B}, \tag{11}$$

$$\sum_{u \in \mathcal{U}} \sum_{b \in \mathcal{B}} \alpha_{b,c}^u \leq 1, \ \forall c \in \mathcal{C}_b, \tag{12}$$

$$\alpha_{b,c}^u = \{0, 1\}, \forall u, b, c. \tag{13}$$

Here, constraint (9) implies that each slice is allocated to at most one user. Constraint (10) indicates that each user is allocated at most one slice (i.e., at most one chunk and one SBS). Constraints (11) and (12) represent the slice isolation, which is uniquely determined in the MVNO.

The problem (**OP**) is a mixed integer non-convex optimization problem, which is computationally intractable. In next section, a suboptimal solution of the problem (**OP**) is proposed using the Lagrangian relaxation.

III. PROPOSED JOINT SLICE AND POWER ALLOCATION

The partial Lagrangian of problem (**OP**) is obtained by augmenting its objective function with a weighted sum of constraints (2), (3), (6) as follow:

$$L(\boldsymbol{\alpha}, \boldsymbol{P}, \boldsymbol{\lambda}, \boldsymbol{\beta})$$
$$= U_{\text{MVNO}}(\boldsymbol{\alpha}, \boldsymbol{P}) + \sum_{i \in I} \sum_{u \in \mathcal{U}^i} \lambda_u (R_u^i(\boldsymbol{\alpha}, \boldsymbol{P}) - R_{u,\min}^i)$$
$$- \sum_{b \in \mathcal{B}} \beta_b \left(\sum_{i \in I} \sum_{u \in \mathcal{U}_i} R_u^i(\boldsymbol{\alpha}, \boldsymbol{P}) - Z_{b,\text{bh}} \right)$$
$$- \sum_{i \in I} \sum_{u \in \mathcal{U}_i} \mu_u \left(\sum_{b \in \mathcal{B}} \sum_{c \in \mathcal{C}_b} \alpha_{b,c}^u \sum_{l \in \mathcal{L}_c} P_{b,c}^{u,l} - \bar{P}_u \right), \tag{14}$$

where $\boldsymbol{\lambda} = [\lambda_u]_{1 \times (|\mathcal{U}|)}$, $\boldsymbol{\beta} = [\beta_b]_{1 \times B}$ and $\boldsymbol{\mu} = [\mu_u]_{1 \times (|\mathcal{U}|)}$ are Lagrangian nonnegative multipliers associated with constraints (2), (3) and (6), respectively.

Then, the Lagrangian dual function of the dual problem for the problem (**OP**) is

$$(\textbf{D}) \ \max_{(\boldsymbol{\alpha}, \boldsymbol{P})} \ \sum_{i \in I} \sum_{u \in \mathcal{U}^i} \sum_{b \in \mathcal{B}} \sum_{c \in \mathcal{C}_b} \alpha_{b,c}^u \left[\Omega_{b,c}^u(\boldsymbol{P}_{b,c}^u) - \varphi_{b,c}^{\text{slice}} \right]$$

$$\text{s.t.} \quad (7), (8), \tag{15}$$

Algorithm 1 JSPA-HSA: JSPA With H̲ungarian-Based S̲lice Allocation.

Initialization: I, $\mathcal{B}$, $\mathcal{C}_b$, $\mathcal{U}_i$, $\boldsymbol{P}^{(0)}$, $\boldsymbol{\lambda}^{(0)}$, $\boldsymbol{\mu}^{(0)}$, and $\boldsymbol{\beta}^{(0)}$.
Repeat:

Power allocation phase:
  At the subscribed user u:
1: Update λ_u as:

$$\lambda_u(t+1) = [\lambda_u(t) - s_1(t)(R_u^i(\boldsymbol{\alpha}, \boldsymbol{P}) - R_u^{\min})]^+; \tag{18}$$

2: Update μ_u as:

$$\mu_u(t+1) = \left[\mu_u(t) \right.$$
$$\left. -s_2(t)\left(\sum_{b\in\mathcal{B}}\sum_{c\in\mathcal{C}_b} \alpha_{b,c}^u \sum_{l\in\mathcal{L}_c} P_{b,c}^{u,l} - \bar{P}_u\right)\right]^+; \tag{19}$$

3: Update transmit power $P_{b,c}^{u,l}(t+1)$ by (16);
  At the SBS b:
4: Update congested backhaul link price $\beta_b(t+1)$:

$$\beta_b(t+1) = \left[\beta_b(t) + s_3(t)\left(\sum_{i\in I}\sum_{u\in\mathcal{U}_i} R_u^i(\boldsymbol{\alpha}, \boldsymbol{P}) - Z_{b,\text{bh}}\right)\right]^+; \tag{20}$$

  Slice allocation phase:
  At the MVNO:
5: Update $\alpha_{b,c}^u(t+1)$ using the Hungarian algorithm to maximize (17).
Until $|\lambda_u(t+1) - \lambda_u(t)| \le \epsilon_1$, $|\mu_u(t+1) - \mu_u(t)| \le \epsilon_2$, and $|\beta_b(t+1) - \beta_b(t)| \le \epsilon_3$ are simultaneously satisfied.

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where $\Omega_{b,c}^u(\boldsymbol{P}_{b,c}^u) = (\varphi_i^{\text{sp}} - \varphi_b^{\text{bh}} + \lambda_u - \beta_b)r_{b,c}^u(\boldsymbol{P}_{b,c}^u) - \mu_u \sum_{l\in\mathcal{L}_{b,c}} P_{b,c}^{u,l}$.

A. Power Control (PA) Phase

Regardless of slice allocation $\boldsymbol{\alpha}$ and Lagrangian multiplier values, the optimal power can be determined based on the KKT condition for optimality [8] by taking the first derivative of $\Omega_{b,c}^u(\boldsymbol{P}_{b,c}^u)$ with respect to $P_{b,c}^{u,l}$ as:

$$P_{b,c}^{u,l*} = \left[\frac{\varphi_i^{\text{sp}} - \varphi_b^{\text{bh}} + \lambda_u - \beta_b}{(\ln 2/W)\mu_u} - \frac{1}{\gamma_{b,c}^{u,l}}\right]^+, \tag{16}$$

where $(x)^+ = \max(x, 0)$.

B. Slice Allocation (SA) Phase

Given the power allocation in (16), problem (**D**) reduces to a maximum weighted matching problem as:

$$\textbf{(D-1)} \quad \max_{(\boldsymbol{\alpha}, \boldsymbol{P})} \sum_{i\in I}\sum_{u\in\mathcal{U}^i}\sum_{b\in\mathcal{B}}\sum_{c\in\mathcal{C}_b} \alpha_{b,c}^u \left[\Omega_{b,c}^u(\boldsymbol{P}_{b,c}^{u*}) - \varphi_{b,c}^{\text{slice}}\right]$$
$$\text{s.t.} \quad (9), (10), (13). \tag{17}$$

Here, an optimal slice allocation can be obtained using the Hungarian algorithm [9] in which each slice $\{b,c\}$ is weighted by $[\Omega_{b,c}^u(\boldsymbol{P}_{b,c}^{u*}) - \varphi_{b,c}^{\text{slice}}]$ for user u.

In Algorithm 1, we propose a distributed algorithm for the JSPA problem, which is referred to as the JSPA-HSA algorithm. Given the allocations $\boldsymbol{\alpha}$ and $\boldsymbol{P}$, the optimal value

Algorithm 2 MSA: Matching-Based Slice Allocation

1: **while** $\sum_{\forall u,\{b,c\}} b_{u\to\{b,c\}} \ne 0$ or convergence not achieved **do**
2:   *At the subscribed users*:
3:     Send a bid for the slice $\{b,c\}^* = \arg\max_{\{b,c\}\in\succ_u} \phi_u(\{b,c\})$.
4:   *At the MVNO*:
5:     Construct $\succ_{\{b,c\}}$ based on (21).
6:     Update $\{b,c\}^* = \mu(\{b,c\})|u^* = \arg\max_{u\in\succ_{\{b,c\}}} \phi_{\{b,c\}}(u)\}$.
7:     Update the rejected user lists on the slices and the preference $\succ_u$.
8: **end while**

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of Lagrangian multipliers can be obtained by the projected gradient-descent method [8] according to (18), (19) and (20) with positive step sizes $s_1(t)$, $s_2(t)$ and $s_3(t)$. The convergence of the JSPA-HSA algorithm can be proved using the gradient-based standard technique [8]. An optimal solution in the SA phase is found by the Hungarian method with a computation complexity of $O(|\mathcal{U}| \times |\mathcal{S}|)^3$. Moreover, the MVNO needs global information about $\Omega_{b,c}^u(\boldsymbol{P}_{b,c}^u)$ on all the slices from the users via dedicated reliable feedback channels [1]. Due to the high complexity of this algorithm, we next propose a low-complexity distributed algorithm in the SA phase.

IV. MATCHING-BASED LOW-COMPLEXITY ALGORITHM

Herein, we present a low-complexity solution for the problem (**OP**) in which the SA phase is formed as a two-side matching game [10] including subscribed users and slices to maximize objective function (17).

We consider a two-side matching game $(\mathcal{U}, \mathcal{S}, \succ_{\mathcal{U}}, \succ_{\mathcal{S}})$ for the slice allocation. Here, $\succ_{\mathcal{U}} = \{\succ_u\}_{u\in\mathcal{U}}$ and $\succ_{\mathcal{S}} = \{\succ_{b,c}\}_{\{b,c\}\in\mathcal{S}}$ denote the preference relations of the users and slices, respectively. The two-side matching game is defined as a function $\mu: \mathcal{U} \mapsto \mathcal{S}$ such that:

(i) $u = \mu(\{b,c\}) \leftrightarrow \{b,c\} = \mu(u), \forall u \in \mathcal{U}, \{b,c\} \in \mathcal{S}$;
(ii) $|\mu(\{b,c\})| \le 1$ and $|\mu(u)| \le 1$, $u \in \mathcal{U}, \{b,c\} \in \mathcal{S}$.

In the matching μ, user u prefers slice $\{b,c\}$ to $\{b,c\}'$ is denoted by $\{b,c\} \succ_u \{b,c\}'$ ($\{b,c\}, \{b,c\}' \in \mathcal{S}$). Additionally, slice $\{b,c\}$ prefers user u to u' is represented by $u \succ_{\{b,c\}} u'$ ($u, u' \in \mathcal{U}$). A pair $(u, \{b,c\})$ is a blocking pair for μ if there exists $\{b,c\} \succ_u \{b,c\}'$ or $u \succ_{\{b,c\}} u', \forall u, b, c, i$.

In the matching μ, utility functions $\phi_u(\{b,c\})$ and $\phi_{\{b,c\}}(u)$ form the preference relations $\succ_u$ and $\succ_{\{b,c\}}$ of the users and the MVNO, respectively. In the proposed two-side matching game, user u always seeks to maximize its utility value ($\phi_u(\{b,c\}) = \Omega_{b,c}^u(\boldsymbol{P}_{b,c}^u)$), which means that it will bid the slice $\{b,c\}^* := \arg\max_{\{b,c\}\in\mathcal{S}} \Omega_{b,c}^u(\boldsymbol{P}_{b,c}^u)$ in its preference list. In response to the request, the MVNO wishes to maximize a utility function on each slice defined as follows:

$$\phi_{\{b,c\}}(u) = \Omega_{b,c}^u(\boldsymbol{P}_{b,c}^u) - \varphi_{b,c}^{\text{slice}}. \tag{21}$$

To maximize the objective function (17), the distributed slice allocation strategy is presented in Algorithm 2, which is referred to as the MSA algorithm. The MSA algorithm operates based on the conventional deferred acceptance algorithms [10]. This algorithm always converges to the stable matching μ^* with a computation complexity of $\mathcal{O}(|\mathcal{U}|^2(|\mathcal{S}|-1))$. Since the MSA algorithm execution is stopped at the stable matching μ^*, only a suboptimal solution is achieved.
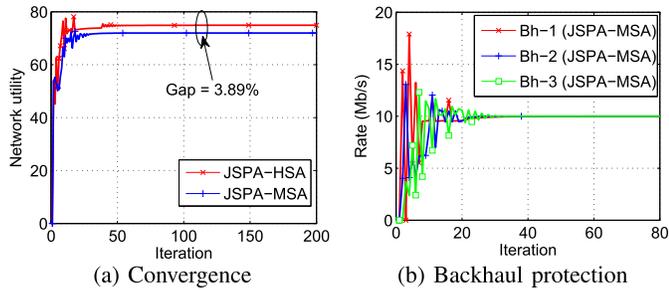
(a) Convergence (b) Backhaul protection

Fig. 2. Evaluation results of the proposed algorithms with $B = 3$ InPs, $I = 2$ SPs, $U_i = 10$ users, $S = 30$ slices.

(a) CDF of network utilities. (b) Network utility versus.

Fig. 3. Evaluation results with $Z_{b,\mathrm{bh}} = 10$ Mbps.

From above analysis, we now develop a low-complexity distributed algorithm to solve problem (**OP**). Referred to as JSPA-MSA, this algorithm is formed by substituting the Hungarian method by the MSA algorithm in Step 5 of the JSPA-HSA algorithm. In this algorithm, the slice and power allocations are assumed to be performed in different timescales. In the SA phase, the MVNO need not share the cost information of the slices to the users, whereas users only share the slice information with the most preferred slice in its preference list to the MVNO. The convergence of the JSPA-MSA algorithm can be proved using a gradient-based standard technique [8]. The duality gap is nonzero because the low-complexity algorithm MSA is suboptimal.

V. NUMERICAL RESULTS

We consider $B = 3$ InPs each having an SBS with a coverage radius of 100 m and signal bandwidth of 3 MHz. Each SBS contains 10 chunks and each chunk contains 12 sub-carriers. The bandwidth of each subcarrier is $W = 15$kHz. The MVNO rents InPs' network resources to serve two SPs, each of which has 10 users. The SP i has the minimum target rate of $200 \times i$ kbps ($i = 1, 2$). The small-scale channel gains are assumed to be independent and identically distributed Rayleigh random variables with unit mean. The large-scale path loss in dB for distance d (between a user and an SBS) is assumed to be $L_d = 38.46 + 20\log_{10}(d)$. The noise power is set to -174 dBm/Hz. Each user u has $P_u^{\max} = 100$ mW. We set $\varphi_1^{\mathrm{sp}} = 2.5$ and $\varphi_2^{\mathrm{sp}} = 3.5$units/ Mbps for SPs 1 and 2, respectively. The backhaul prices for InPs 1, 2 and 3 are 0.2, 0.4 and 0.6 units/Mbps, respectively. The slices price of InPs 1, 2, and 3 are 0.1, 0.2, and 0.3 units/slice, respectively. Moreover, we set the error tolerance as $\epsilon = 10^{-3}$ for all concerned algorithms.

Fig. 2a show that two proposed algorithms converge in a few iterations, whereas backhaul links are protected by the JSPA-MSA scheme as shown in Fig. 2b. In Fig. 2a, the JSPA-MSA scheme approaches the JSPA-HSA scheme at a gap of 3.98%.

Fig. 3a compares the profits attained by the MVNO with the proposed algorithms where we also include the baseline algorithm Max-Rate. Similar to the JSPA-HSA, this Max-Rate algorithm allocates the virtual resources without considering the business model as in [2] and [3]. However, this benchmark algorithm only focuses on maximizing the sum rate $\sum_{i \in I} \sum_{u \in U^i} R_u^i(\boldsymbol{\alpha}, \boldsymbol{P})$ in problem (**OP**). As seen, our proposed algorithms outperform the Max-Rate solution. The gain is observed for the JSPA-MSA scheme with an
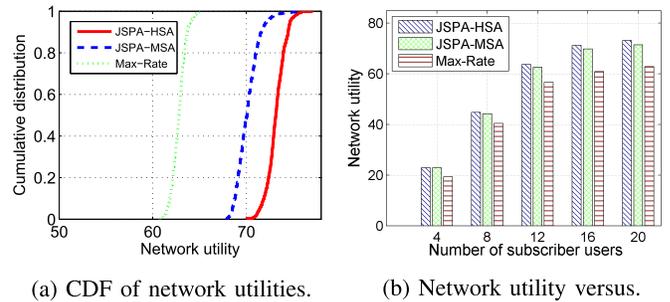
improvement of up to 9.8% over the baseline, whereas 4.1% over the JSPA-HSA scheme. The gain is slightly less for the JSPA-HSA scheme at the benefit of carrying out the computation in a distributed fashion.

In Fig. 3b, we show the network utility versus the number of SPs' users for different schemes with backhaul rate of 10 Mbps. The number of subscribed users $|U|$ increases from 4 to 20 users, and each SP has $|U|/2$ users. As the number of users increases, the network utilities of all schemes are improved since data traffic grows. Moreover, the proposed schemes outperform the Max-Rate solution in terms of network utility. The results of the low-complexity JSPA-MSA scheme follow those of the JSPA-HSA counterpart.

VI. CONCLUSIONS

In this letter, we have proposed efficient virtual resource allocations in the uplink of a virtualized cellular network with backhaul constraints. An algorithm based on Lagrangian relaxation has been proposed to solve the formulated problem. Additionally, a low-complexity distributed algorithm based on the concept of the matching game has been developed to reduce computation complexity. Numerical results have confirmed that the devised algorithms quickly converge and guarantee higher profits for the MVNO compared to those of the existing designs.

REFERENCES

[1] C. Liang and F. R. Yu, "Wireless network virtualization: A survey, some research issues and challenges," *IEEE Commun. Surveys Tut.*, vol. 17, no. 1, pp. 358–380, 1st Quart., 2015.

[2] M. I. Kamel *et al.*, "LTE multi-cell dynamic resource allocation for wireless network virtualization," in *Proc. IEEE Wireless Commun. Netw. Conf. (WCNC)*, Mar. 2015, pp. 966–971.

[3] S. Parsaeefard *et al.*, "Joint resource provisioning and admission control in wireless virtualized networks," in *Proc. IEEE Wireless Commun. Netw. Conf. (WCNC)*, Mar. 2015, pp. 2020–2025.

[4] C. Liang and F. R. Yu, "Distributed resource allocation in virtualized wireless cellular networks based on ADMM," in *Proc. IEEE Conf. Comput. Commun. Workshops (INFCOMW)*, Apr./May 2015, pp. 360–365.

[5] R. Baldemair *et al.*, "Ultra-dense networks in millimeter-wave frequencies," *IEEE Commun. Mag.*, vol. 53, no. 1, pp. 202–208, Jan. 2015.

[6] J. Oueis and E. C. Strinati, "Uplink traffic in future mobile networks: Pulling the alarm," in *Proc. Int. Conf. Cognit. Radio Oriented Wireless Netw.*, 2016, pp. 583–593.

[7] H. Zhu and J. Wang, "Chunk-based resource allocation in OFDMA systems—Part I: Chunk allocation," *IEEE Trans. Commun.*, vol. 57, no. 9, pp. 2734–2744, Sep. 2009.

[8] S. Boyd and L. Vandenberghe, *Convex Optimization*. Cambridge, U.K.: Cambridge Univ. Press, 2004.

[9] H. W. Kuhn, "The hungarian method for the assignment problem," *Naval Res. Logistics*, vol. 2, nos. 1–2, pp. 83–97, 1955.

[10] D. Gale and L. S. Shapley, "College admissions and the stability of marriage," *Amer. Math. Monthly*, vol. 69, no. 1, pp. 9–15, 1962.