Distributed Resource Allocation for Interference Management and QoS Guarantee in Underlay Cognitive Femtocell Networks

Tae Manh Ho, Nguyen H. Tran, S.M Ahsan Kazmi, Do Hyun Kim and Choong Seon Hwang
Department of Computer Science and Engineering, Kyung Hee University, 446-701, Republic of Korea
Email: {hmtai, nguyentn, ahsankazmi, doma, cshong}@khu.ac.kr

Abstract—Cognitive femtocell networks can opportunistically access the licensed spectrum to enhance spectrum utilization. However, interference management plays a crucial role to effectively utilize the spectrum. In this paper, we consider the joint resource allocation and power control problem for an uplink transmission for a network consisting of a licensed macrocell and multiple cognitive femtocells. Furthermore, our problem imposes crucial constraints of both cross-tier interference for macrocell base station and quality of service for femtocell user. The joint problem is shown to be mixed-integer nonlinear nonconvex optimization problem, which is NP-hard. To solve this problem efficiently, we employ a scheme consisting of two distributed algorithms. Numerical results show that the proposed scheme converges to the optimal power and resource allocation with a fast convergence speed. Additionally, our scheme guarantees the interference threshold at MBS and outage QoS for all cognitive femtocell users.

Index Terms—Resource Allocation, Power Control, Femtocell Networks, Interference Management.

I. INTRODUCTION

Resource allocation design including subchannel allocation and power allocation for efficient interference management is an important challenge in the cognitive femtocell networks [1], [2], [3]. There are some existing works on power allocation for the underlay cognitive femtocell networks in the literature [4]–[6]. Moreover, the problems of subchannel allocation have been studied in [3], [7]–[9]. In addition, the joint subchannel and power allocation issues have been addressed in [10]–[11]. A distributed power control and centralized matching algorithms for subchannel allocation are proposed in [10], which lead to fair resource allocation for uplink OFDMA femtocell networks. A distributed auction game is employed to design the joint power control and subchannel allocation in OFDMA femtocell networks [11]. In compare to the existing works, our main contributions can be summarized as follows:

- We first analyze the QoS requirement for cognitive femtocell users and interference protection constraint for MBS for the system in underlay spectrum sharing mode, and formulate the joint resource allocation and power control problem for the uplink transmission. This joint problem is shown to be mixed-integer nonlinear nonconvex optimization problem, which is NP-hard.
- We propose a scheme by two distributed algorithms using optimization theory that solve this joint problem efficiently. First algorithm solves the resource allocation problem for any given feasible power allocation.
- We develop the second algorithm to attain the local optimal transmit powers using a dual-based algorithm. This algorithm can be implemented in a distributed manner. Extensive numerical results show that the algorithm converge to the optimal power allocation with a fast convergence speed. In additionally, our scheme guarantees the interference threshold at MBS and outage QoS for all femtocell users.

II. SYSTEM MODEL

A. Network Model

We consider a two-tier Cognitive heterogeneous network consisting a single central MBS and a set of J cognitive femtocells implementing spectrum sharing in underlay mode in which spectrum is divided into several subbands, with a total of K subchannels across all subbands. Macrocell users are the primary user (or licensed users) of the system while femtocell users are the secondary users (or unlicensed users). FAPs are equipped with cognitive radio capability. The number of FUEs in the jth, j ∈ J femtocell is Nj. We assume that in each femtocell, at most one FUE to be allocated to subchannel k and at most one subchannel k to be allocated to one FUE at any time instant.

We model the uplink transmission for a given time slot, where hjkn and gjkn are the channel power gains of the links between FUE nth and FAP jth on subchannel kth and between FUE mth and FAP jth, respectively, and gjkn is the channel power gain between FUE nth in femtocell jth on subchannel kth and the MBS. Pk = [Pkn]j is the vector of power levels for FUEs in all femtocells which transmit on subchannel kth, and Pkn,min ≤ Pkn ≤ 1, ∀n, j, k.

B. Problem Formulation

FUE transmission rate. The instantaneous signal-to-interference-noise-ratio (SINR) of FUE nth at FAP jth on the kth subchannel with transmit power Pkn,j is

$$\gamma_{j,n}^k(P^k) = \frac{P_{j,n}^{k} h_{j,n}^k}{\sum_{m \neq j} P_{j,m}^{k} h_{j,m}^k} + \sigma_j^2 + 1,$$

where Pkn,j models the fast-fading channel from the FUE nth to FAP jth on subchannel kth, and $\sigma_j^2$ is the background noise at FAP jth on subchannel kth. Employing a Rayleigh fading model, we assume $P_{j,n}^k$ is i.i.d exponentially distributed with unit mean. Over the considered time slot, $h_{j,n}^k$ and $P_{j,m}^k$ are assumed constant. $\tau_j^{k}$ is the interference from MUE to FAP on subchannel kth. The certainty-equivalent SINR defined as follows:

$$\bar{\gamma}_{j,n}^k(P^k) = \frac{\sum_{m \neq j} P_{j,m}^{k} h_{j,m}^k} {\sum_{m \neq j} P_{j,m}^{k} h_{j,m}^k + \tau_j^{k} + \sigma_j^2}.$$

The transmission rate $R_n$ of FUE nth depends on many factors such as the modulation and coding schemes, bit-error rate (BER), and the instantaneous SINR at FAP nth, which can be modelled as $R_n(P^k) = \sum_k W_k \phi_{j,n}^k \log(1 + \bar{\gamma}_{j,n}^k(P^k))$, where $W_k$
is the subchannel bandwidth and $\phi_{j,n}^k$ denotes the subchannel allocation index for $n^{th}$ FUE in $j^{th}$ femtocell, $\phi_{j,n}^k = 1$ when $k^{th}$ subchannel is allocated to $n^{th}$ FUE and $\phi_{j,n}^k = 0$ otherwise, we should have $\sum_{n \in N_j} \phi_{j,n}^k \leq 1$, $\sum_{k \in K} \phi_{j,n}^k \leq 1$, $\forall n, j, k$.

Network throughput. The objective of the femtocell network is to maximize a system-wide efficiency metric, e.g., the total system throughput. Here, we assume that system operates in a high SINR regime, i.e., SINR is much larger than 1; thus, the data rate can be approximated as $R_j(P_k(t)) = \sum_k W_k \phi_{j,n}^k(t) \log(\gamma_{j,n}^k(P_k(t)))$. This approximation is reasonable when the signal level is much higher than the interference level. Therefore, in the high SINR regime, the aggregate data rate for the femtocell networks can be written as follows:

$$R_{sum}(P, \phi_{j,n}^k) = \sum_{j=1}^{J} \sum_{n=1}^{N_j} R_{j,n} = \sum_{j=1}^{J} \sum_{n=1}^{N_j} \sum_{k=1}^{K} \phi_{j,n}^k \log(\gamma_{j,n}^k(P_k)),$$

(3)

Without loss of generality we normalize subchannel bandwidth $W_j$ equal 1. The specific value of $W_j$ will be considered in the simulation section.

FUE QoS. In wireless networks, one of the most important QoS parameter for reliable communication is outage probability. A channel outage is declared (e.g., packets lost) when the received SINR falls below a given threshold SINR $\gamma_{th}$, often computed from the BER requirement. Using the close-form outage probability for Rayleigh fading channel, the outage probability is defined as

$$\Pr[\gamma_{j,n}^k \leq \gamma_{th}] = 1 - \exp\left(-\frac{\sigma_j^{2} \gamma_{k,th}^k}{P_j^k h_{jn}^k}\right) \prod_{m \in J \setminus j} \left(1 + \gamma_{j,n}^k \frac{P_m^k h_{jm}^k}{P_j^k h_{jn}^k}\right)^{-1}.$$

(4)

MBS interference protection. We assume that the maximum tolerable interference at MBS on subchannel $k$ is $I_{th}^k$, i.e., the interference constraint is used to assure that the aggregate interference from all FUEs who utilize subchannel $k$ to the MBS is less than $I_{th}^k$. Mathematically, this constraint can be written as

$$\sum_{j=1}^{J} \sum_{n \in N_j} \phi_{j,n}^k P_{jn}^k \leq I_{th}^k, \forall k.$$

We next consider the following sum-rate optimization problem in the high SINR regime, taking into account constraints on maximum tolerable interference, and outage probability:

$$\begin{align*}
\max_{\phi \in \mathbb{P}^N} & \quad R_{sum}(P, \phi_{j,n}^k) \\
\text{subject to} & \quad \sum_{n \in N_j} \phi_{j,n}^k \leq 1, \sum_{k \in K} \phi_{j,n}^k \leq 1, \forall n, j, k, \\
& \quad \sum_{n=1}^{N_j} \sum_{k=1}^{K} \phi_{j,n}^k P_{jn}^k \leq I_{th}^k, \forall k, \\
& \quad \Pr[\gamma_{j,n}^k \leq \gamma_{th}] \leq \bar{r}_{j,n}^k, \forall n, j, k, \\
& \quad \phi_{j,n}^k \in \{0, 1\}, \forall n, j, k,
\end{align*}$$

(5)

where $\bar{r}_{j,n}^k \in (0, 1)$ are the upper bound of the outage probability threshold of FUE $n$ in femtocell $j$, and $P_k^d = \{P_{jn}^k \mid P_{jn}^k \leq P_{jn}^{k,min} \leq P_{jn}^k \leq P_{jn}^{k,max}, \forall n, j, k\}$. Since the system objective is a non-convex function of the transmit powers and it also tightly coupled with integer variables of the resource allocation indicators, the problem (5) is a mixed-integer nonlinear and nonconvex program which is NP-hard, and there is no available efficient solution for such problem.

Algorithm 1 Distributed Subchannel Allocation

1: initialize: $t = 0$, $\alpha_{n}^{0} \geq 0$, step-size $\kappa_{n}^{0} > 0$;
2: repeat
3: $t \leftarrow t + 1$
4: Each FAP $j$ updates $x_{n}^{j,t}$ for its FUEs $k$ and $\alpha_{k}^{n}$ as follows:

$$\begin{align*}
\phi_{j,n}^k(t+1) &= \begin{cases} 1, & \text{if } k = k^*, \\
0, & \text{otherwise}
\end{cases} \\
\alpha_{k}^{n}(t+1) &= \alpha_{n}^{k}(t) - \kappa_{n}^{k(t)} \left(\phi_{j,n}^k(t) g_{j,n}^k P_{jn}^k - I_{th}^k\right),
\end{align*}$$

(6)

(7)

where, $\kappa_{n}^{k(t)} > 0$ is a step-size.
5: until $|\alpha_{n}^{k(t+1)} - \alpha_{n}^{k(t)}| \leq \epsilon$

III. JOIN RESOURCE ALLOCATION AND POWER CONTROL

A. Resource Allocation for Fixed Power Allocation

We now present our proposal to solve the problem given in the previous section. Note first that for any given feasible power allocation, the original problem can be decomposed into $J$ resource allocation problems in $J$ femtocells separately.

Lemma 1: For any fixed feasible power allocation $P_k^d$, the original problem (5) can be reduced to $J$ independent subproblems for each FUE $n$ and subchannel $k$ in each femtocell $j$ as follows:

$$\begin{align*}
\max_{\phi \in \mathbb{P}^N} & \quad R_{sum}(P, \phi_{j,n}^k) \\
\text{subject to} & \quad \sum_{n=1}^{N_j} \sum_{k=1}^{K} \phi_{j,n}^k \log(\gamma_{j,n}^k(P_k)) \\
& \quad \sum_{n \in N_j} \phi_{j,n}^k \leq 1, \forall n, j, k, \\
& \quad \sum_{j=1}^{J} \sum_{n=1}^{N_j} \phi_{j,n}^k P_{jn}^k \leq I_{th}^k, \forall k.
\end{align*}$$

(6)

The problem (6) is a combinatorial optimization problem that needs to be solved distributively at the FAPs side. In this section, we propose a distributed algorithm to optimally solve the problem (6) based on duality theory as shown in Algorithm 1. The convergence of Alg. 1 can be proved using a gradient-based standard technique [14].

B. Power Control for Fixed Resource Allocation

For any given resource allocation $\phi$, the original problem reduces to the following power control problem:

$$\begin{align*}
\max_{P \in \mathbb{P}^N} & \quad \sum_{j=1}^{J} \sum_{n=1}^{N_j} \sum_{k=1}^{K} \log(\gamma_{j,n}^k(P_k)) \\
\text{subject to} & \quad \sum_{j=1}^{J} \sum_{n=1}^{N_j} \phi_{j,n}^k P_{jn}^k \leq I_{th}^k, \forall k, \\
& \quad \Pr[\gamma_{j,n}^k \leq \gamma_{th}] \leq \bar{r}_{j,n}^k, \forall n, j, k.
\end{align*}$$

(10)

The problem (10) is a nonliner nonconvex problem, hence cannot be solved via standard algorithms. In the next subsection, we first transform (10) into an equivalent convex problem and then propose a distributed algorithm to achieve the local optimal transmit powers.
1) Equivalent Convex Formulation: Using geometric programming, we define a new variable \( y^k_{j,n} = \log P^{k}_{j,n} \) and a new set \( \hat{P} = \{ y^k_{j,n} : n \in N_j \| P^{k}_{j,n} \leq y^k_{j,n} \leq log P^{k}_{j,n} \} \); thus, \( P^{k}_{j,n} = e^{y^k_{j,n}}. \) We also introduce an auxiliary variable \( \{ Z^k_{j,n} \} \) to show that every FUE has the capability to estimate the interference \( e^{Z^k_{j,n}} = \sum_{m \in j \neq j'} P^{k}_{j,m} h^{m}_{j,n}. \) We transform problem (5) into the following equivalent non-linear programming problem

\[
\begin{align*}
\min_{y^k_{j,n}} & \sum_{j=1}^{N_j} \sum_{n=1}^{N} \sum_{k=1}^{K} \log \left( \frac{e^{-y^k_{j,n}}}{h^{k}_{j,n}} (e^{Z^k_{j,n}} + \sigma^2_{j,k}) \right) \\
\text{s.t.} & \sum_{j=1}^{N_j} \sum_{n=1}^{N} g^h_{j,n} y^k_{j,n} - h^{k}_{j,n} \leq 0, \quad \forall k, \\
& \sum_{m \in j' \neq j} \log \left( 1 + e^{y^k_{j',m} - y^k_{j,n}} g^{k,th}_{j',j} h^{m}_{j',n} \right) \leq \log \Gamma_{j,n} (e^{y^k_{j,n}}), \\
& \sum_{m \in j' \neq j} h^{k}_{j,m} e^{y^k_{j,m}} - e^{Z^k_{j,n}} = 0, \quad \forall n, j, k.
\end{align*}
\]

(11)

where \( \Gamma_{j,n} (P^{k}_{j,n}) = \frac{\exp(-\sigma^2_{j,n}/P^{k}_{j,n} h^{k}_{j,n}^2)}{1 + \gamma_{j,n}}. \) We further assume that \( \gamma_{j,n} \) and \( c_{j,n} \) are chosen such that there exist feasible points in problem (11). It is straightforward to show that problem (11) is a convex problem [14]. The Lagrangian form of (11) can be decomposed into \( J \times N_j \times K \) subproblems as follows:

\[
L(y, Z, \lambda, \nu, \zeta) = \sum_{j=1}^{N_j} \sum_{n=1}^{N} \sum_{k=1}^{K} L^k_{j,n}(y^k_{j,n}, Z^k_{j,n}, \lambda, \nu^k_{j,n}, \zeta^k_{j,n}),
\]

(12)

where \( \lambda, \nu^k_{j,n}, \) and \( \zeta^k_{j,n} \) are Lagrange multipliers that represent interference price, outage price, and consistency price, respectively, and

\[
\begin{align*}
L^k_{j,n}(y^k_{j,n}, Z^k_{j,n}, \lambda, \nu^k_{j,n}, \zeta^k_{j,n}) &= \log \left( \frac{e^{-y^k_{j,n}}}{h^{k}_{j,n}} (e^{Z^k_{j,n}} + \sigma^2_{j,k}) \right) + \lambda g^h_{j,n} y^k_{j,n} - \nu^k_{j,n} \log \Gamma_{j,n} (e^{y^k_{j,n}}) \\
&+ \sum_{m \in j' \neq j} \nu^k_{j',m} \log \left( 1 + e^{y^k_{j',m} - y^k_{j,n}} g^{k,th}_{j',j} h^{m}_{j',n} \right) \\
&+ \sum_{m \in j' \neq j} \zeta^k_{j',m} h^{k}_{j,m} e^{y^k_{j,m}} - \zeta^k_{j,n} e^{Z^k_{j,n}}.
\end{align*}
\]

(13)

The dual problem is then given as

\[
\begin{align*}
\max_{\lambda, \nu, \zeta} & \quad D(\lambda, \nu, \zeta) \\
\text{s.t.} & \quad \lambda, \nu, \zeta \geq 0,
\end{align*}
\]

(14)

where \( D(\lambda, \nu, \zeta) = \min_{y^k_{j,n}, Z^k_{j,n}} L(y^k_{j,n}, Z^k_{j,n}, \lambda, \nu^k_{j,n}, \zeta^k_{j,n}) \) is the dual function. Problem (11) is convex; hence, there exists a strictly feasible point so Slater’s condition holds, leading to strong duality [14]. This allows us to solve the primal (11) via the dual (14). The dual problem (14) can be solved using the sub-gradient method, which updates the Lagrange multipliers as follows:

\[
\lambda^{(t+1)} = \lambda^{(t)} + \kappa^{(t)} \left( \sum_{j=1}^{N_j} \sum_{n=1}^{N} g^h_{j,n} P^{k}_{j,n} (t) - h^{k}_{j,n} \right),
\]

(15)

Algorithm 2 Distributed Power Allocation - DPC

Initialization:
- Set \( t = 0, P^{k}_{j,n} (0) \) be any feasible point in feasible set \( P^{k}_{j,n} \leq P^{k}_{j,n} (0) \leq \Gamma_{j,n}, \forall n, j, k; \)
- Set \( \lambda^{(0)} \geq 0, \nu^{k}_{j,n} (0) \geq 0, \zeta^{k}_{j,n} (0) \geq 0, \forall n, j, k; \)
- Set step size \( \kappa^{(t)}, \kappa^{(t)}_{\nu}, \kappa^{(t)}_{\zeta} > 0; \)

Algorithm at FAP \( j \)

1) Measure the interference \( \sum_{m \in j' \neq j} P^{k}_{j,m} h^{m}_{j,n} \) generated by all other FUEs in other femtocells which transmit on sub-channel \( k, \) and the noise power level \( \sigma_{j,k}; \)
2) Calculate \( \zeta^{k}_{j,n} (t+1) \) according to (19);
3) Update the consistency price \( \zeta^{k}_{j,n} (t+1) \) according to (17);
4) Transmit \( h^{k}_{j,n} \) to FUE \( n \) and broadcast \( c^{k}_{j,n} (t+1); \)

Algorithm at FUE \( n, n \in N_j \)

1) Estimate the channel gain \( g^h_{j,n} \) and receive \( \{ g^{k}_{j,m}, P^{k}_{j,m} \} \) to calculate the total interference at the MBS; Receive \( c^{k}_{j,n} (t+1), \) and estimate \( \{ h^{k}_{j,n} \} \) according to (15) and (16), respectively;
2) Update the interference price \( \lambda^{(t+1)} \) and the outage price \( \nu^{k}_{j,n} (t+1) \) according to (15) and (16), respectively;
3) Receive \( \{ \nu^{k}_{j,m} (t+1), c^{k}_{j,m} (t+1), \Lambda^{k}_{j,m} (t+1) \} \) \( m \neq j, \) and then calculate the power value according to (18);
4) Broadcast \( g^{k}_{j,n} (t+1) \), \( \nu^{k}_{j,n} (t+1), \) \( \zeta^{k}_{j,n} (t+1), \) and \( \Lambda^{k}_{j,n} (t+1); \)

\[
\nu^{k}_{j,n} (t+1) = \left[ \nu^{k}_{j,n} (t) + \kappa^{(t)}_{\nu} \left( \sum_{m \in j' \neq j} \log \left( 1 + \frac{P^{k}_{j,m} (t)}{h^{m}_{j,n}} \right) \right) \right]^{+},
\]

(16)

\[
\zeta^{k}_{j,n} (t+1) = \left[ \zeta^{k}_{j,n} (t) + \kappa^{(t)}_{\zeta} \left( \sum_{m \in j' \neq j} P^{k}_{j,m} (t) h^{m}_{j,n} - e^{Z^k_{j,n}} \right) \right]^{+},
\]

(17)

where \( \kappa^{(t)}_{\nu}, \kappa^{(t)}_{\zeta}, \) and \( \kappa^{(t)}_{\lambda} \) are positive step sizes, and \([X]^+ = \max\{X, 0\}. Based on the KKT condition [14], the optimal transmit power \( P^{k}_{j,n} \) of each FUE \( n \) can be obtained through

\[
P^{k}_{j,n} (t+1) = e^{b^{k}_{j,n} (t+1)} \left[ 1 + \nu^{k}_{j,n} (t) \Lambda^{k}_{j,n} (t) \frac{\sigma^2_{j,k}}{\log (1 - \frac{\sigma^2_{j,k}}{h^{k}_{j,n}})} \right],
\]

(18)

where \( \Lambda^{k}_{j,n} (t) = \frac{\gamma_{j,n}}{P^{k}_{j,n} (t) h^{k}_{j,n}}. \) The auxiliary variable \( Z^k_{j,n} \) can be achieved according to the KKT condition

\[
e^{Z^k_{j,n} (t+1)} = \frac{1}{\zeta^{k}_{j,n} (t) - \sigma^2_{j,k}}.
\]

(19)

Based on above optimization analysis, we present the optimal distributed power control algorithm as shown in Algorithm 2.
We consider a cellular network with a MBS and three femtocells consisting of 3 FUEs distributed randomly inside a circle of radius of \( r_1 = 1000 \) m. The distances between the three FUEs and the MBS are 150 m, 300 m and 500 m, respectively. \( P_{\text{max}} \) and \( P_{\text{min}} \) are set to 1 mW and 0 mW, respectively. Background noise is assumed to be \( 1 \times 10^{-10} \) W. The channel gains are defined using a simple path loss model, \( h_{k,n,m} = L d_{k,n,m}^{-4} \), where \( L \) is a constant. The respective outage probability thresholds \( \xi_n \) of the three FUEs are [0.3, 0.2, 0.1]. The SINR thresholds \( \gamma_{th}^n \) are [30, 20, 10] dB. The maximum interference tolerance threshold for each subchannel \( k \) is \( I_{th}^k = 10 \times 10^{-10} \) W.

Fig. 1 illustrates the power convergence of FUEs, respectively. It can be seen that for the farthest FUE who generates lowest interference to the MBS, can transmit with the highest power level, i.e., denoted as FUE 3. Meanwhile, for the nearest FUE who generates highest interference to the MBS, it will transmit with the lowest power level in comparison to the others FUEs. Furthermore, it can be seen that the convergence occurs under all scenarios in a limited iterations, i.e., less than 40.

The outage probabilities of our scheme also converge to the desired values, i.e., [0.3, 0.2, 0.1] as shown in Fig. 2. This implies that under all conditions our proposed scheme maintains the QoS in terms of predefined outage set for each FUE once the scheme converges. In Fig. 3 we show the transmission power levels of FUEs versus the maximum tolerable interference margin \( I \) at the MBS. It is observed that for small \( I_{th}^k \), i.e., \( I_{th}^k \leq 10 \times 10^{-10} \) W, all FUEs transmit with low power level to guarantee the interference threshold constraint at MBS. When the interference tolerance at MBS is sufficiently large, i.e., \( I_{th}^k > 20 \times 10^{-10} \) W, all FUEs transmit at the maximum power. This is a key challenge which needs to be addressed for the CRN to operate in an underlay spectrum sharing. Our scheme guarantees that the unlicensed FUE do not violate the interference constraint given by the licensed MBS.

**V. Conclusion**

We have designed two distributed algorithms to solve the joint resource allocation and power control problem for interference management and network QoS guarantee in two-tier heterogeneous cognitive radio networks. The first algorithm is to solve the resource allocation for given feasible transmit power. The second algorithm can obtain the local optimal transmission power for all cognitive femtocell users and can be implemented in a distributed manner. Numerical results show that the distributed power control algorithm can obtain local optimal transmission power with a fast convergence speed. In additionally, our scheme guarantees the interference threshold at macro base station and outage QoS for all femtocell users.

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