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Analysis of energy-tax for multipath routing in wireless sensor networks

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Abstract Recently, multipath routing in wireless sensor networks (WSN) has got immense research interest due to its capability of providing increased robustness, reliability, throughput, and security. However, a theoretical analysis on the energy consumption behavior of multipath routing has not yet been studied. In this paper, we present a general framework for analyzing the energy consumption overhead (i.e., energy tax) resulting from multipath routing protocol in WSN. The framework includes a baseline routing model, a network model, and two energy consumption schemes for sensor nodes, namely, periodic listening and selective wake-up schemes. It exploits the influence of node density, link failure rates, number of multiple paths, and transmission environment on the energy consumption. Scaling laws of energy-tax due to routing and data traffic are derived through analysis, which provide energy profiles of single-path and multipath routing and serve as a guideline for designing energy-efficient protocols for WSN. The crossover points of relative energy taxes, paid by single-path and multipath routing, reception, and transmission, are obtained. Finally, the scaling laws are validated and performance comparisons are depicted for a reference network via numerical results.

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1 Introduction

While energy minimization is the primary design driver of communication protocols for Wireless Sensor Networks (WSN), secondary metrics, such as reliability, throughput, security, and adaptivity to dynamic topology, are also very important in meeting certain application requirements. There has been a recent emergence of using multipath routing (MPR) in many applications of WSN; it provides increased reliability [1–3], end-to-end throughput [4], security [5], robustness [6, 7], etc. Even though the strategies of using multiple paths in them are different, it is intuitive that creation of multiple paths from each source node to the sink and their maintenance in MPR would drain more energy than for that in single-path routing (SPR). In this paper, our key objective is neither proposing new energy-aware heuristics nor new protocols aimed at increasing network lifetime or application reliability. Instead, it is to analyze and compare MPR and SPR routing mechanisms in WSN, where links are subject to failure, in terms of energy consumption overhead, namely, energy tax. More specifically, our goal is to investigate the interdependencies among network parameters and specify operation regions where MPR is more energy-efficient than SPR or vice versa.

There exist a number of research works that analyze the energy consumption, network lifetime, and tradeoff between energy and quality of service (QoS) in WSNs [8–12]. In [8], using Erlang distribution theory, a quantitative analysis on the tradeoffs between energy consumption and QoS in WSN is presented. A comparative study on the performances of single-session flow routing and multisession flow routing is carried out in [11]. The results show that an optimal single-session flow routing has less overhead than its counterpart while providing almost the same performance. The issue of node placement for desirable energy scaling has been considered in [13, 14], where it is argued that uniform node placement, routinely considered in the literature, has poor energy performance. In contrast to those, later, in [15], a theoretical proof was presented to show that, for sensor data collection networks, uniform node distribution is, in fact, optimal among a general class of distributions. However, none of them provide analytical results on the energy consumption for either SPR and MPR or for different multipath utility strategies in sensor networks.

While simulation-based studies comparing the above performances are available in the literature, the analytical approach is more desirable for decision making purposes in designing a new WSN protocol. In [4–7], we find simulation results showing the effectiveness of their proposed MPR protocols and comparative results with the SPR counterpart. However, they do not provide answers to the following questions:

- Does MPR always consume more energy than SPR? If not, when and why?
- Whether transmission or listening energy dominates the total energy tax due to route discovery and maintenance? Why and how much?
- What are the scaling laws of energy-tax payments? Among the network parameters, which are the most dominant?
- How much performance gain (in terms of energy consumption) can be achieved if a passive wake-up strategy is used in place of a periodic sleep/active schedule of sensor nodes?

The key contributions of this paper lie in answering the above questions. Using a general framework of network, routing, and node energy consumption models, we derive expressions for energy-tax payments due to routing and data forwarding through analytical analysis. These expressions provide the scaling laws and cross points, which would serve as design guidelines of a WSN and its protocols. The rest of the paper is organized as follows: In Section 2, we present a baseline model of MPR protocol. In Section 3, we describe two different energy consumption schemes for sensor nodes and derive expressions for average energy-tax payments per unit time due to routing messages and data traffic. The scaling laws of energy-tax and the procedure of determining system parameters are also stated in this section. Section 4 presents the numerical results, and the paper ends in Section 5 with a few concluding remarks and further research directions.

2 Baseline model of MPR protocol

Many of today's advanced MPR protocols [1–3] use geographical location information to reduce energy consumption. Other protocols compute a pathwise or link-by-link metric to derive the optimal routes. For example, interference minimized multiple paths (I2MR) [4], node-disjoint paths [7], and partially disjoint paths [6] are established between source and destination nodes. A taxonomy of MPR protocols is found in [16]. In this analysis, we are not interested in evaluating the performance of any specific energy-efficient MPR protocols. However, we do need a baseline routing model so that our analysis can serve as a benchmark for comparison with more sophisticated protocols. Our simple routing model is based on basic ad hoc on-demand distance vector protocol and lies in line with the many existing routing protocols for WSN [4, 6, 7]; the main modification made here is in the way of responding route reply packets from the destination node, and it works as follows.

When a source node has some data to send, it needs a route to the destination sink for data forwarding. If the route is not available, it broadcasts a route request message (RREQ). The packet contains the source ID and a sequence number that uniquely identify the packet. Each node that receives this message broadcasts it again. Intermediate nodes are not allowed to send route reply packets (RREPs) back to the source, even when they have route information to the destination, because, in the case where the sink node will not know all the available route information, it would not be possible to establish multiple paths as required. Eventually, RREQ messages are received by the sink node. In case of SPR, the sink responds only to the first RREQ message and discards others. However, in case of MPR, based on the policy of a certain routing protocol, the sink replies $P(P > 1)$ number of RREP messages, and when the source receives all these RREPs, multiple paths between the source and the sink are established.[1] The propagation procedures of RREQ and RREP messages are shown in Fig. 1. Now, when a link between two nodes fails, it affects only active paths using the link; protocol does not trigger any process action if

[1] These paths may be node-disjoint, link-disjoint, or high-energy path. A certain policy may need necessary modifications at the contents of RREQ and RREP messages.

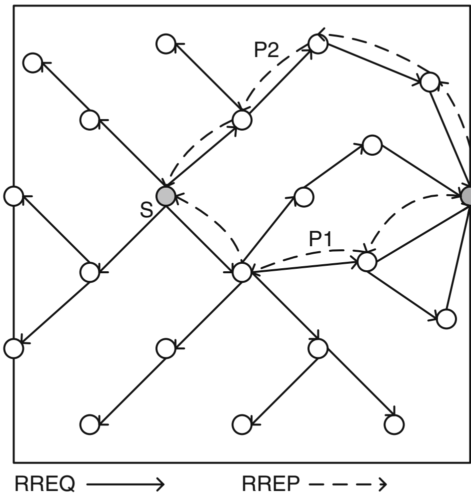


Fig. 1 Route discovery mechanism

the link is not used by any active path. However, if a node, involved in an active session, detects the failure of a link, it broadcasts a route error (RERR) packet in order to inform its precursor nodes about the failed link. This route failure information is propagated backwards until it reaches the source node. Then, the source may attempt to use other alternate routes, if available, or reinitiate the route discovery process. Also, each intermediate node, receiving RERR packets, updates its routing table accordingly.

The reason for making our routing very simple is that, frequently, network performance is compared for different levels of protocol overhead or complexity. Such overhead is generally the result of adapting to a particular requirement and its operating environment. As a higher level of overhead (or complexity) is added, the protocol generally becomes more adapted and performs better. Therefore, a baseline model should start with the lowest level of complexity.

3 Energy-tax analysis using analytical model

In this section, we first present two different energy consumption schemes for sensor nodes. Then, we present the network model used for the analytical analysis. Using both of the energy consumption schemes, we compute the total amount of energy-tax in one unit time for route establishment and data forwarding.

3.1 Node energy consumption schemes

Usually, the sensor node energy is consumed when performing sensing task, processing raw data, and transmitting and receiving of control and data packets for itself and other nodes. Since, in most types of sensor nodes, wireless transceivers dominate the energy consumption, in this paper, we focus on the energy consumed by communication modules and ignore energy consumed by the other modules, such as data processing, sensing, channel acquisition, etc. In most of the energy models in the literature [8, 9], the network interface has four possible energy consumption states: *transmit*, *receive*, *sleep*, and *listen*, as shown in Fig. 2a. When the transmitter circuitry is on (i.e., node is in active mode), node energy is consumed due to transmit and receive operations of data and control packets. At the end of transmiting/receiving packets, the node switches to sleep state. In this state, the radio remains turned off (but only sensors and other low-power circuitry may be on); hence, it consumes no energy (or very little energy). While in sleep state, the node wakes up periodically for short periods of time and listens to the medium to check whether any other nodes want to send their packets to it, and, if not, it goes back to sleep again. On the other hand, if the check returns yes, the node switches to active receive state. The node may change its state from sleep to active transmit if its local sensors detect some phenomena/events. We term this energy consumption scheme as a periodic listening (PL) scheme.

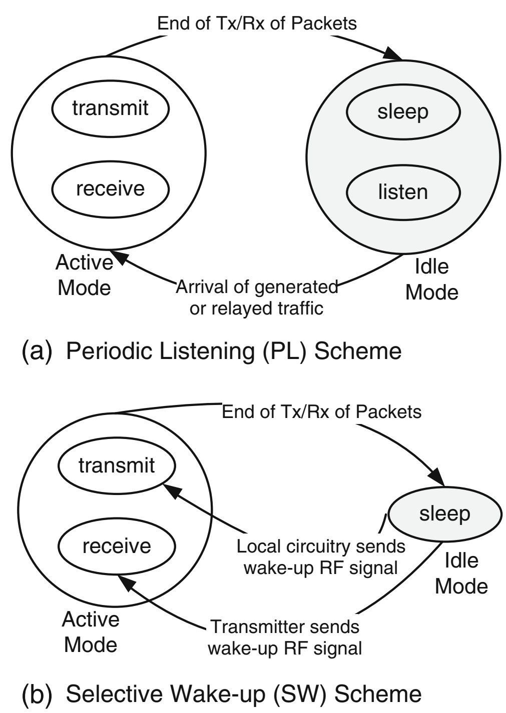


Fig. 2 Node energy consumption schemes (**a**, **b**)

The advent of remotely activated switch (RAS) [17] helps us to allow sensor radios to sleep most of the time and let them awaken precisely when they need to receive or transmit data packets. By sending an RF signal[2] with designated paging sequence (e.g., correspond to node ID) to the intended receiver node, equipped with a RAS, the latter can be woken up. This completely eliminates the unnecessary energy consumption due to listening. We term this as the selective wake-up (SW) energy consumption scheme; the state transition diagram is shown in Fig. 2b. Another advantage of using this scheme is that the maintenance of the RAS and RF transmitter consumes a very trivial amount of energy.

For the analysis of energy consumption using the above schemes, we assume that the energy consumption for receiving and transmitting are e_{re} J/b and e_{tx} J/b, respectively. The transmission energy of a node that covers a neighborhood of radius d is given by

$$e_{\text{tx}} = \max\{e_{\text{tx}}[\min], e_{\text{ta}} \times d^{\alpha}\} + e_{\text{te}}, \tag{1}$$

where e_{te} is the energy per bit needed by the transmitter electronics, e_{ta} is the energy consumption of the transmitting amplifier to send one bit over one unit distance, and $\alpha(1.6 \leq \alpha \leq 6)$ is the path loss factor depending on the radio frequency environment. For all nodes closer than $d_{\min} = (\frac{e_{\text{tx}}[\min]}{e_{\text{ta}}})^{\frac{1}{\alpha}}$, the energy requirement is constant at $e_{\text{tx}}[\min]$ J/b. Typical values of these parameters are $e_{\text{te}} = 50$ nJ/b, $e_{\text{re}} = 50$ nJ/b and $e_{\text{ta}} = 0.0013$ nJ/b/m^4 when $\alpha = 4$ [18]. There is no difference between listening and receiving in this radio transceiver model.

3.2 Network model

In an area of interest, a network consisting of a sink node and many tiny wireless sensor nodes is considered. The N number of sensor nodes are distributed uniformly with density ρ. We assume that the communication range of each sensor node is equal (d) and circular. Therefore, the number of nodes within the communication range of a sensor is calculated as $n = \pi d^2 \rho$. The average lengths of single paths and MPR paths are h_s and h_m hops, respectively, and obviously, $h_m \geq h_s$. We also assume that the communication link between two nodes may vary randomly and asynchronously due to mobility, block fading, presence of obstacles, etc. Each link has a failure rate of f, i.e., a link has an average lifetime of $\frac{1}{f}$ seconds on average. In addition, h_e is assumed to be the average length of route from the source to the node where the link failure occurs. Furthermore, the number of active connections per node is denoted by C for both routing mechanisms, P represents the number of different paths for each source-sink pair, and T stands for the time taken to find routes to the destination sink. Finally, L_{req}, L_{rep}, and L_{err} are, respectively, representing the sizes of RREQ, RREP, and RERR packets. The above notations are summarized in Table 1.

[2]The RF signal may be sent by a transmitter wishing to transmit data to a receiver node or by local circuitry when local sensors detect some phenomena or event.

Table 1 Notations

Notation	Meaning
N	Number of sensor nodes
ρ	Node density
d	Communication range of a node
n	Number of sensors within the communication range of a node
h_s and h_m	Hop lengths of SPR and MPR paths
f	Link failure rate
r_s and r_m	Route discovery rates of SPR and MPR paths
h_e	Hop length of failed link from the source
C	Number of active connections per node
P	Number of paths established by MPR
L_{req}, L_{rep}, and L_{err}	Sizes of RREQ, RREP and RERR packets, respectively

3.3 Energy tax in PL scheme

3.3.1 *Energy tax due to routing packets*

Considering that, during the path establishment phase, all nodes participate in the routing process and all routing packets are broadcasted (i.e., there is no retransmission), the energy consumption overhead due to routing packets is formulated as follows. The energy overhead due to RREQ and RREP packets mainly depends on the path creation frequency. Let r_s and r_m be the rates at which N nodes each broadcast RREQ message in SPR and MPR, respectively. They are related with the link failure rates as follows: $r_s = fh_s$ and $r_m = fh_m$. Therefore, the transmission energy overheads created by RREQ packets are $\{e_{\text{tx}} \times L_{\text{req}}\}r_s N^2$ and $\{e_{\text{tx}} \times L_{\text{req}}\}r_m N^2$ Joules for SPR and MPR, respectively. Similarly, the reception energy overheads due to RREQ packets are $\{e_{\text{re}} \times L_{\text{req}}\}r_s N^2(n-1)^2$ and $\{e_{\text{re}} \times L_{\text{req}}\}r_m N^2(n-1)^2$ Joules, for SPR and MPR, respectively. Notice here that both the routing paradigms incur almost the same energy overhead due to RREQ packets. However, for RREP packets, the destination node replies to only one RREQ packet in SPR and the corresponding RREP packet follows h_s hops to return back to the source node. On the other hand, in case of MPR, the destination node sends back P RREP packets to the source. Therefore, the transmission energy overhead created by RREP packets is calculated as $\{e_{\mathrm{tx}} \times L_{\mathrm{rep}}\} r_s h_s N$ and $\{e_{\mathrm{tx}} \times L_{\mathrm{rep}}\} r_m h_m N P$ Joules, for SPR and MPR, respectively. Similarly, the reception energy overhead due to RREP packets is $\{e_{\mathrm{re}} \times L_{\mathrm{rep}}\} r_s h_s N(n-1)$ and $\{e_{\mathrm{re}} \times L_{\mathrm{rep}}\} r_m h_m N (n-1) P$ Joules, for SPR and MPR, respectively.

The energy overhead due to RERR packets is largely influenced by the link failure rate (f) since an error packet is generated and sent back to the source whenever a link breakage is detected by an active node. The path failure rates for each node in SPR and MPR are found to be $f h_s C$ and $f h_m C$, respectively. Hence, in an N-node network, the average amount of transmission energy overheads due to RERR packets are $\{e_{\mathrm{tx}} \times L_{\mathrm{err}}\} f h_s C N h_e$ and $\{e_{\mathrm{tx}} \times L_{\mathrm{err}}\} f h_m C N h_e P$ Joules, for SPR and MPR, respectively. Similarly, reception energy overheads are $\{e_{\mathrm{re}} \times L_{\mathrm{err}}\} f h_s C N h_e (n-1)$ and $\{e_{\mathrm{re}} \times L_{\mathrm{err}}\} f h_m C N h_e (n-1) P$ Joules, respectively.

$$\begin{aligned} E_{\mathrm{SPR}}^{\mathrm{route}}(\mathrm{PL}) &= e_{\mathrm{tx}} \left\{ r_s N^2 L_{\mathrm{req}} + r_s h_s N L_{\mathrm{rep}} + f h_s C N h_e L_{\mathrm{err}} \right\} \\ &\quad + e_{\mathrm{re}} \left\{ r_s N^2 (n-1)^2 L_{\mathrm{req}} \right. \\ &\qquad + r_s h_s N (n-1) L_{\mathrm{rep}} \\ &\qquad \left. + f h_s C N h_e (n-1) L_{\mathrm{err}} \right\} \\ &= e_{\mathrm{tx}} \times O\left(r_s N^2\right) + e_{\mathrm{re}} \times O\left(r_s N^2 (n-1)^2\right) \end{aligned} \tag{2}$$

and similarly,

$$\begin{aligned} E_{\mathrm{MPR}}^{\mathrm{route}}(\mathrm{PL}) &= e_{\mathrm{tx}} \left\{ r_m N^2 L_{\mathrm{req}} + r_m h_m N L_{\mathrm{rep}} \right. \\ &\qquad \left. + f h_m C N h_e L_{\mathrm{err}} \right\} \\ &\quad + e_{\mathrm{re}} \left\{ r_m N^2 (n-1)^2 L_{\mathrm{req}} \right. \\ &\quad + r_m h_m N (n-1) L_{\mathrm{rep}} P \\ &\quad \left. + f h_m C N h_e (n-1) L_{\mathrm{err}} P \right\} \\ &= e_{\mathrm{tx}} \times O\left(r_m N^2 P\right) \\ &\quad + e_{\mathrm{re}} \times O\left(r_m N^2 (n-1)^2 P\right) \end{aligned} \tag{3}$$

By summarizing all the above energy expenditures, we can express the total amount of energy tax due to routing packets as of Eqs. 2 and 3. The simplifications are made under the assumption that, for a particular network, the variables L_{req}, L_{rep}, and L_{err} have some constant values; we also omitted h_s, h_m, h_e, and C since they make very little difference in energy taxes incurred by SPR and MPR, especially for small networks. From Eqs. 2 and 3, we observe the following energy-tax scaling laws:

- The energy consumption overhead in MPR is increased roughly by a factor of P compared to its counterpart, SPR.
- In both the routing mechanisms, the energy consumption overhead is increased exponentially with N and linearly with route discovery rates r_s or r_m.
- As n increases (either by increasing the node density, ρ, or transmission range, d), the reception energy overhead is increased exponentially, and at some point, it may cross the transmission energy overhead.

The above scaling laws are validated and comparisons are depicted via numerical results in Section 4.

3.3.2 Energy tax due to data packet headers

The energy consumption overhead created during data transmission is due to the overhead part of data packets, i.e., headers. Unlike routing packets, in this case, we have to consider that a packet may not be reached at the next hop node successfully in a single transmission attempt due to collision or link error. We consider that a link layer ARQ mechanism is adopted at each sensor node and the maximum retry limit is set to t. If all t transmission attempts fail, then a node drops the packet.

Now, let p_s^i be the probability that a transmission attempt of node i is successful. The probabilistic distribution of getting first success in a repeated finite number of trials (t) is truncated geometric random [19]. Let q represent the number of transmission attempts required to transfer the packet successfully to the next hop. Using [19], we get the conditional probability mass function (*pmf*) of q, with the condition that the transmission attempt of a node is successful, as

$$p^i(q) = \frac{\left(1 - p_s^i\right)^{q-1} \times p_s^i}{1 - \left(1 - p_s^i\right)^t} \tag{4}$$

Using geometric series equations [20], we find the expected number of transmission attempts required for single hop, $E[q]$, as

$$E[q] = \frac{1 - \left(1 - p_s^i\right)^t \left(1 + t p_s^i\right)}{p_s^i \left\{1 - \left(1 - p_s^i\right)^t\right\}} \tag{5}$$

The value of $E[q]$ in SPR might be somewhat greater than that in MPR[3] as argued and results shown in previous simulation-based works [2, 3]. To make a difference, we use notations $E_s[q]$ and $E_m[q]$ for SPR and MPR, respectively, to denote the expected number of transmissions of a data packet in one hop.

Now, we consider that each source sensor node transmits data packets at the rate of λ packets/second once the route establishment is completed. Since the route discovery rate is r_s and each route discovery takes, on average, T seconds, the actual time for data transmission is $(\frac{1}{r_s} - T)$, and thus, the data packets are sent with an average rate of $\lambda r_s(\frac{1}{r_s} - T)$. Therefore, in an N-node network, the transmission and reception energy overheads for data packet headers in SPR are calculated as follows:

$$E^{\text{data}}_{\text{SPR}}(\text{PL}) = e_{\text{tx}} \times L_{hdr}\lambda r_s \left(\frac{1}{r_s} - T\right) E_s[q]h_s N + e_{\text{re}} \times L_{hdr}\lambda r_s \left(\frac{1}{r_s} - T\right) E_s[q]h_s N(n-1) \quad (6)$$

Using a similar derivation as above, the total amount of energy overhead for data packet headers in MPR can be expressed as Eq. 7, if we consider that source nodes continue to transfer data packets over only one path and use alternate paths only when the primary one fails.

$$E^{\text{data}}_{\text{MPR}}(\text{PL}) = e_{\text{tx}} \times L_{hdr}\lambda r_m \left(\frac{1}{r_m} - T\right) E_m[q]h_m N + e_{\text{re}} \times L_{hdr}\lambda r_m \left(\frac{1}{r_m} - T\right) \times E_m[q]h_m N(n-1) \quad (7)$$

$$E^{\text{route}}_{\text{SPR}}(\text{SW}) = e_{\text{tx}} \left\{r_s N^2 L_{\text{req}} + r_s h_s N L_{\text{rep}} + f h_s C N h_e L_{\text{err}}\right\} + e_{\text{re}} \left\{r_s N^2 (n-1)^2 L_{\text{req}} + r_s h_s N L_{\text{rep}} + f h_s C N h_e L_{\text{err}}\right\} = e_{\text{tx}} \times O\left(r_s N^2\right) + e_{\text{re}} \times O\left(r_s N^2 (n-1)^2\right) \quad (8)$$

and similarly

$$E^{\text{route}}_{\text{MPR}}(\text{SW}) = e_{\text{tx}} \left\{r_m N^2 L_{\text{req}} + r_m h_m N L_{\text{rep}} + f h_m C N h_e L_{\text{err}}\right\} + e_{\text{re}}\left\{r_m N^2 (n-1)^2 L_{\text{req}} + r_m h_m N L_{\text{rep}} P + f h_m C N h_e L_{\text{err}} P\right\} = e_{\text{tx}} \times O\left(r_m N^2 P\right) + e_{\text{re}} \times O\left(r_m N^2 (n-1)^2 P\right) \quad (9)$$

Apparently, comparing Eqs. 6 and 7, we see that, excepting route discovery rates and the number of expected transmissions, there is no difference between energy taxes paid by SPR and MPR due to data packets. In fact, the exact value of r_m as compared to r_s depends on the choice of a multipath utility strategy, and it determines which routing is more energy-efficient. By multipath utility strategy, we mean how a source node uses the paths after the establishment phase completes. The first and most-used category is the MPR for fault tolerance (MPR-FT), where source nodes use only one path at a time for data forwarding and choose alternate paths when the primary one fails. In this case, the route discovery is initiated whenever all available paths fail. Hence, in MPR-FT, the mean route discovery rate is bounded by $\frac{r_s}{P} \leq r_m \leq r_s$. The second category is the MPR for maximizing the end-to-end throughput or reliability (MPR-TR), where the source nodes either split traffic over the available paths [4, 5] or send duplicate packets over the different paths [2, 3]. Here, failure of any one of the paths initiates the route discovery process because the protocol needs to use them simultaneously. Hence, the route creation rate is the same as that of SPR, $r_m = r_s$.

3.4 Energy-tax in SW scheme

As described in Section 3.1, the main advantage that springs from using the SW scheme is that it saves energy by reducing the energy expenditures incurred by overhearing. In this scheme, only the forwarding node listens to the transmission. Therefore, the energy consumed in one hop comes from one transmission and one reception. For example, the RREP and RERR packets are sent towards the upstream direction of the route, and thereby, they consume energy due to one transmission and one reception at each hop. However, during the flooding of route request packets (RREQ), all nodes have to participate in the route discovery process and, hence, consume much energy due to listening. Therefore, Eqs. 2 and 3 can be rewritten for the SW scheme as Eqs. 8 and 9, respectively.

[3]The main reason behind achieving this result is that the data traffic is diversified in MPR, which reduces the collision probability and, hence, the number of retransmission attempts at each hop.

Due to the same reasons, in SW scheme, the total amount of energy tax due to data packet headers for SPR and MPR can be rewritten as Eqs. 10 and 11, respectively.

$$E_{\text{SPR}}^{\text{data}}(\text{SW}) = e_{\text{tx}} \times L_{hdr}\lambda r_s \left(\frac{1}{r_s} - T\right) E[q]h_s N + e_{\text{re}} \times L_{hdr}\lambda r_s \left(\frac{1}{r_s} - T\right) E[q]h_s N \quad (10)$$

$$E_{\text{MPR}}^{\text{data}}(\text{SW}) = e_{\text{tx}} \times L_{hdr}\lambda r_m \left(\frac{1}{r_m} - T\right) E[q]h_m N + e_{\text{re}} \times L_{hdr}\lambda r_m \left(\frac{1}{r_m} - T\right) E[q]h_m N \quad (11)$$

Comparing Eqs. 2 and 8, we see that the transmission energy cost due to routing packets is the same for both PL and SW schemes; however, as node density ρ increases, the reception energy cost with PL scheme increases slightly faster than that with SW scheme. The reason for this is that the listening energy dominates when ρ increases. Therefore, the use of SW scheme leads to a different scaling law. The SW scheme brings more benefits over PL scheme in energy consumptions for data traffic, since it has zero energy overhead due to listening. Comparing Eqs. 6 and 10 or Eqs. 7 and 11, we see the aforementioned differences in scaling laws.

3.5 Determination of the parameter values

In this subsection, we discuss the process of determining the values of parameters r_s, r_m, h_s, h_m, and C used in the analysis. The results in [12] indicate that the route creation rate of MPR strategy is much lower than it is for SPR. This reduction is because, in MPR, route discovery is only initiated when all the routes to the destination sink fail or are broken, whereas in SPR, it is done on the breakage of single route only. We assume that a link's lifetime is independent and identically distributed exponential random variables with mean $\frac{1}{f}$. Since a route fails when any one of the wireless links in its path breaks, the lifetime of a route, consisting of h wireless links, is also an exponential distributed random variable with a mean of $\frac{1}{fh}$. Let X_i represent this random variable for route i and let T_r denote the time between successive route discoveries. Hence, the probability density function of T is given by

$$f_{T_r}(t) = \prod_{i=1}^{P}\left(1 - e^{-\frac{1}{fh_i}t}\right)\sum_{i=1}^{P}\frac{1}{fh_i}\left(\frac{e^{-\frac{1}{fh_i}t}}{1 - e^{-\frac{1}{fh_i}t}}\right) \quad (12)$$

The expected value of T_r can be derived from Eq. 12 by knowing the link failure rate and hop-wise lengths of the route(s), which in turn provides the values of r_s and r_m.

For densely deployed networks, the average number of hops in SPR paths can be calculated as $h_s = \lceil \frac{\bar{x}}{d} \rceil$, where $\bar{x}$ is the average distance of sensor nodes from the destination sink and d is the transmission or communication range of a node. In this case, SPR paths form almost straight lines from the source node to the sink. As node density ρ decreases, the value of h_s may increase. However, the average route length of MPR paths is $h_m > h_s$. The value of h_m is determined by the node density and the policy of MPR protocol employed. For example, in I2MR [4], h_m can be as long as $2h_s$ and in [6, 7], h_m is either equal to or slightly greater than h_s. Detailed analysis on path lengths can be found in Appendix A of [21]. For the sake of unbiased evaluation, we consider the worst case and set $h_m = 2h_s$ in the following performance evaluation section. Similarly, the average hop lengths of the failed link from the source node is given by $h_e = \frac{h_s}{2}$ and $h_e = \frac{h_m}{2}$, for SPR and MPR, respectively. The number of active connections C maintained by a node depends on its relative location from the sink. Nodes closer to the sink have to maintain more active connections than the far nodes; a related analysis can be found in [12]. In the following performance evaluation section, we take an average value of C for all nodes.

4 Numerical results

In this section, through numerical results, we validate the energy-tax scaling laws derived in the previous section and depict the performance comparisons between SPR and MPR, PL and SW schemes, and MPR-FT and MPR-TR strategies. For the numerical results, we consider a network of an area of 120 × 120 m, where the sensor nodes are randomly distributed. As few as 25 nodes (having sensing radius 20 m) are required to ensure the sensing coverage of the whole area. Unless otherwise specified, MPR-FT is used as multipath utility strategy and the default values of parameters are listed in Table 2.

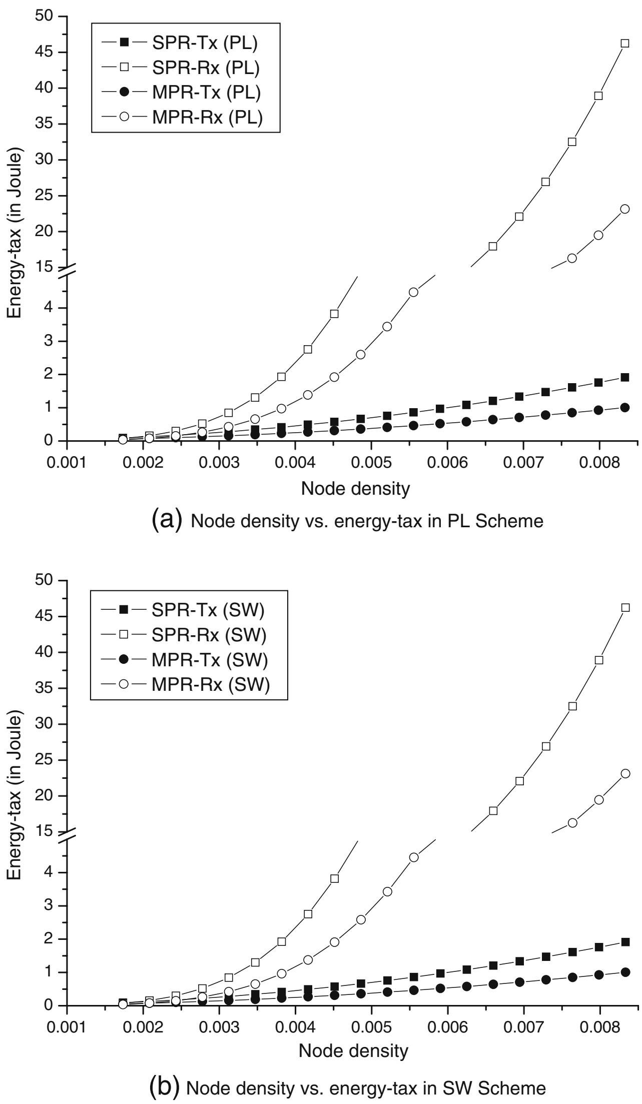
For different node densities, Fig. 3 uses Eqs. 2 and 8 for SPR, and Eqs. 3 and 9 for MPR to plot corresponding energy taxes paid by routing packets in PL and SW schemes, respectively. We see that, for routing packets, the difference between energy taxes paid in PL and SW schemes is insignificant. This is because all nodes have to participate in the route discovery process irrespective of the schemes. We also notice that, at

Table 2 Parameter values

Parameter	Value
N	$25 \sim 120$
ρ	$0.001736111 \sim 0.008333333$
d	40 m
n	$8.726388889 \sim 42.88666667$
h_s, h_m, h_e	$h_s = 2, h_m = 4, h_e = 2$
f, C, P	$f = 0.1, C = 3, P = 2$
$L_{\text{req}}, L_{\text{rep}}, L_{\text{err}}$	$L_{\text{req}} = 192\text{b}, L_{\text{rep}} = 160\text{b}, L_{\text{err}} = 160\text{b}$
r_s and r_m	$r_s = 0.2, r_m = 0.1$
α, p_s^i, t	4, 0.8, 7
L_{hdr}	320b
λ	2 packets/s

minimum node density, the transmission and reception energy costs are almost the same. However, as node density ρ increases, the number of neighbor nodes that overhear transmissions also increases, which in turn results in an exponential growth of reception energy cost. Therefore, for routing packets, the reception energy cost is greater than or equal to the transmission energy cost in WSN, where we must maintain at least a minimum level of node density required for sensing coverage.

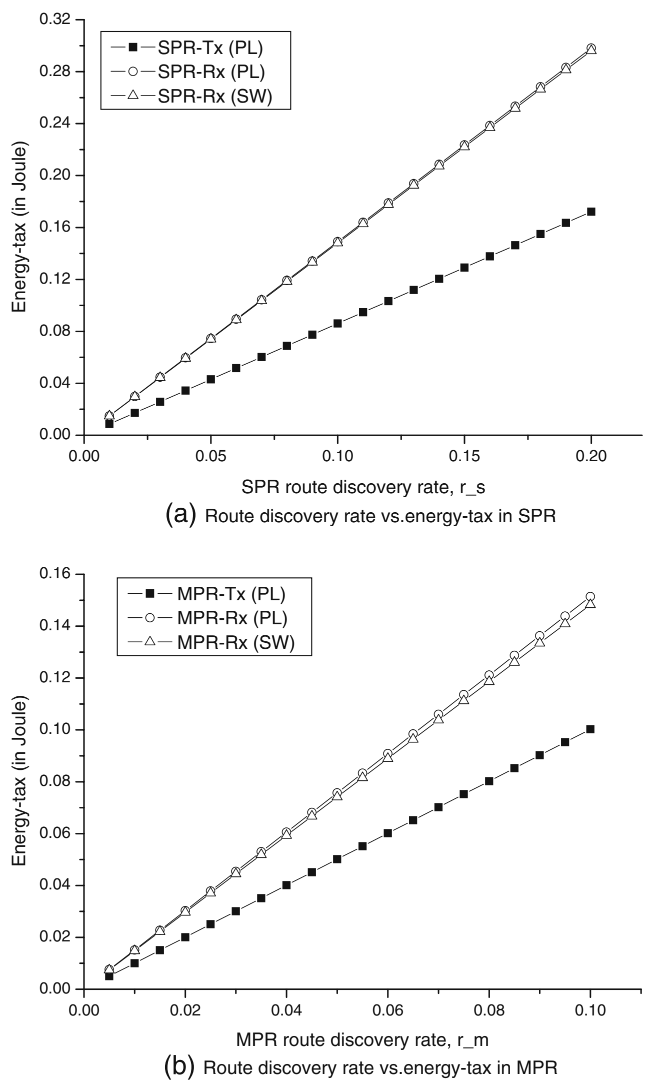
Like Fig. 3, Fig. 4 uses the same equations to plot energy taxes for different route discovery rates. From the graphs of Fig. 4, we see that energy taxes increase linearly as route discovery rate increases. The gap between the reception energy costs in PL and SW schemes is very low, and it slightly widens at higher route discovery rates. This is because of increasing the listening energy costs due to RREP and RERR packets in PL scheme. Again, the reception energy cost is much higher than the transmission energy cost in both SPR



(a) Node density vs. energy-tax in PL Scheme

(b) Node density vs. energy-tax in SW Scheme

Fig. 3 Effect of node density on energy tax due to route discovery and maintenance (**a**, **b**)



(a) Route discovery rate vs.energy-tax in SPR

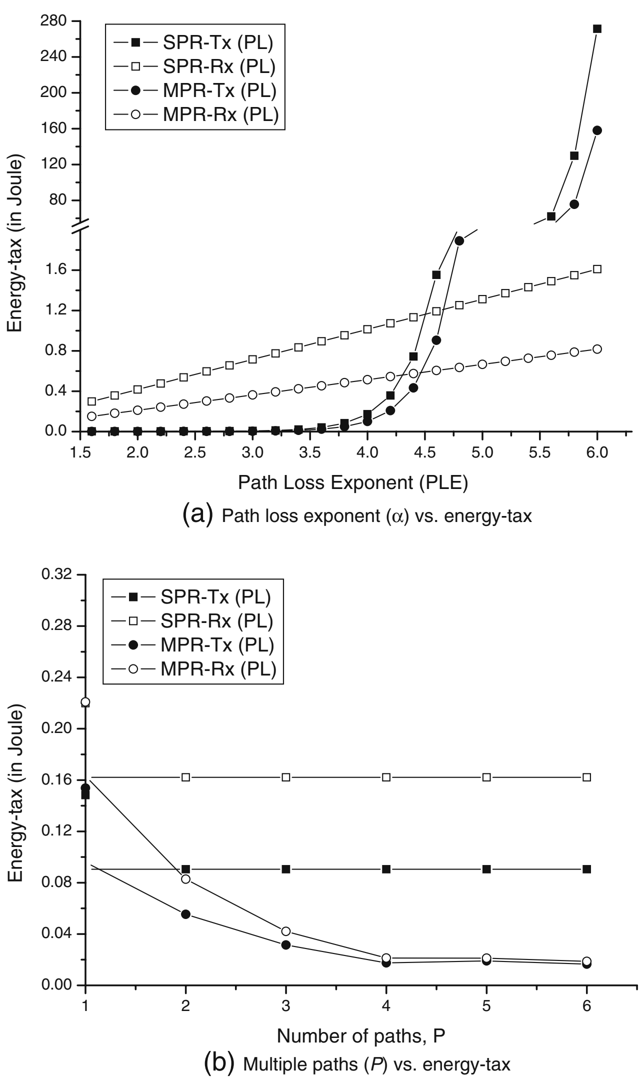
(b) Route discovery rate vs.energy-tax in MPR

Fig. 4 Effect of route discovery rate on energy tax (**a**, **b**) and MPR. Comparing the Y axis of Fig. 4a, b, we see that energy tax in MPR is almost half of that in SPR. This is due to a reduced route discovery rate of MPR.

We plot energy taxes paid by SPR and MPR in PL scheme for varying path loss exponents (α) and the number of multiple paths (P) in Fig. 5a and b, respectively. Here, we keep the node density at its minimum value. In Fig. 5a, we find a crossover point at around a path loss exponent value of 4.5, above which the transmission energy cost becomes higher than the reception energy cost. If α is increased further, the transmission energy cost rises up to a very high value, and it becomes incomparable with the reception energy cost. This is because of an exponential increase of transmission energy in the order of d^{α}, as depicted in Eq. 1. On the other hand, when the quality of the transmission environment is not so bad, i.e., $\alpha \leq 4.0$, transmission energy cost is much less than its counterpart.

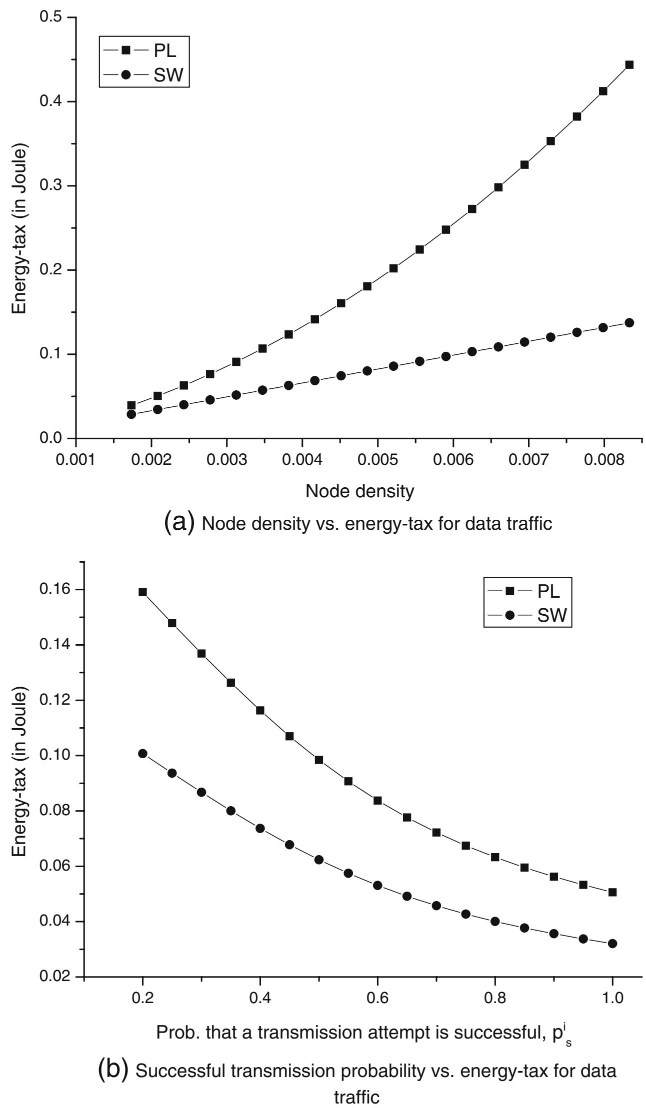
As expected theoretically, the route discovery rate is decreased with the increased number of multiple paths in MPR-FT, which in turn decreases the per-unit time energy cost for routing packets. The graphs of Fig. 5b depict the above result. However, the number of paths greater than four cannot gain further reductions in energy cost since, in these cases, the cost for each route establishment is increased significantly.

For different node densities and successful transmission probabilities, Fig. 6 uses Eqs. 6 and 10 to plot energy taxes paid by SPR in PL and SW schemes, respectively. The graphs of Fig. 6a show that the energy tax due to data traffic in PL scheme is higher than that in SW scheme and the gap widens significantly at



(a) Path loss exponent (α) vs. energy-tax
(b) Multiple paths (P) vs. energy-tax

Fig. 5 Effect of α and P on energy tax due to route discovery and maintenance (**a**, **b**)

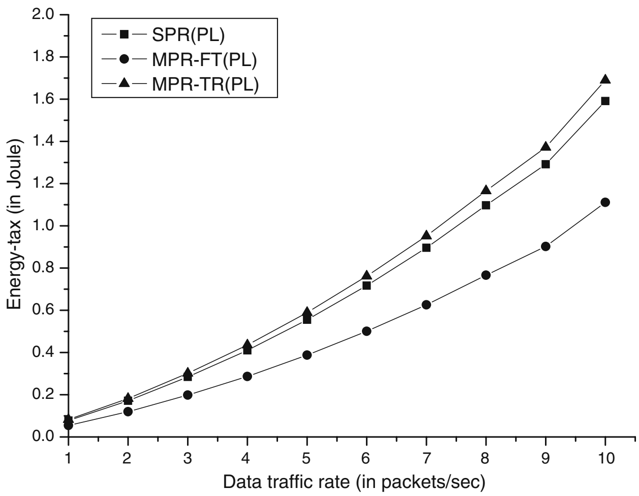


(a) Node density vs. energy-tax for data traffic
(b) Successful transmission probability vs. energy-tax for data traffic

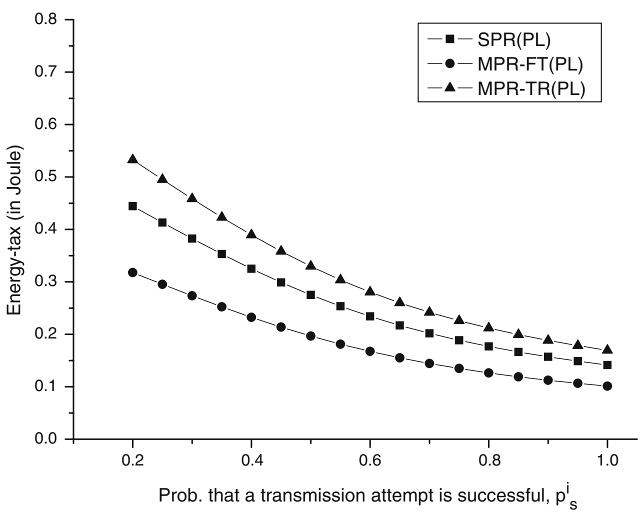
Fig. 6 Energy tax due to data traffic (**a**, **b**) higher node densities, as much as five times increase is observed. Therefore, the use of SW scheme results in a substantial reduction in energy consumption, especially at high node-density deployment. This is caused by an increasing amount of listening energy.

As depicted in Eq. 5, the number of transmission attempts at each hop increases when the successful transmission probability decreases, which in turn increases both the transmission and listening energy costs. The graphs of Fig. 6b comply with the same trend of energy tax. We also see that the SW scheme experiences much less energy tax than the PL scheme since it is able to cut down the huge amount of listening energy cost.

Figure 7 uses Eqs. 6 and 7 to plot energy taxes paid by SPR, MPR-FT, and MPR-TR due to data traffic in



(a) Data traffic rate vs. energy-tax inPL scheme



(b) Successful transmission probability vs.energy-taxes for multi path utility strategies

Fig. 7 Effect of multipath utility strategy on energy tax (**a**, **b**)

the PL scheme. In Fig. 7a, we see that MPR-FT gains significant reduction in energy tax over SPR, and this is mainly caused by a reduced route discovery rate in the former one. As expected theoretically, it is also seen that the MPR-TR has to pay more energy-tax per unit time than the SPR. However, the increased amount is not so high due to its capability of reducing the expected number of transmissions of each packet in one hop, as discussed in Subsection 3.3.2.

In Fig. 7b, we notice the similar comparative relations among the different strategies in their energy-tax payments for varying successful transmission probabilities. The additional amount of energy tax paid by the MPR-TR strategy is actually compensated by its achievement of throughput or reliability gains. Almost the same results in simulation environments can be found in [1–3].

Notice that our analysis does not take HELLO messages into consideration since the power consumption due to these periodic messages is the same for SPR and MPR. Note also that the analysis is not precise for any particular MPR protocol, rather it is based on a general routing model, which does not impose any restrictions on the methodology of MPR in WSN. Thus, it can be used as a basis for analyzing a broad class of networking protocols. Also, the techniques used in the analysis may be applied to specific improvements on SPR and MPR routing on local route repair, probabilistic flooding, routing caches, and backup paths, which is out of the scope of this work.

5 Conclusions

In this paper, we analyzed the influence of node density, link failure rates, number of multiple paths, and transmission environment on the energy-tax payments in sensor networks. The analytical results, presented here, are based on generic routing protocol using two different energy consumption schemes and an idealized network model. Our model clearly captures the essential elements of the network properties, which allow the overall network performance (in terms of energy consumption) to be predicted. The results also focus on the asymptotic behavior of the SPR and MPR routing strategies and serve as an analytical supplement to the existing practical approaches. The following observations are the outcome of our analysis:

– The per-unit time energy-tax payment of a MPR protocol that uses only one path at a time (and uses the alternate path on failure of a primary one) is

much smaller (1.5 times) than that of SPR when $P = 2$ and the ratio increases with P.
- On the other hand, a MPR protocol that uses all the available paths simultaneously has about 1.2 times more energy overhead than its single path counterpart when $P = 2$.
- At minimum node density, required for the sensing coverage of a designated area, the transmission and reception energy overheads for routing packets are almost the same; however, as node density increases, the reception energy overhead rises up exponentially. Hence, extending network lifetime by increasing the node density is not a good approach either.
- Whenever the transmission environment is very bad, we find a cross-over point at about $\alpha \geq 4.5$, where transmission energy cost starts to supersede the reception energy cost.
- In case of data traffic, the use of the SW scheme of sensor nodes can save a significant amount of energy, and the energy consumption gap between SW and PL schemes widens with increased node density.

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