Cross-Layer Design for Congestion, Contention, and Power Control in CRAHNs under Packet Collision Constraints

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***Abstract*—In this paper, we investigate the cross-layer design for congestion, contention, and power control in multi-hop cognitive radio ad-hoc networks (CRAHNs). In particular, we develop a unified optimization framework achieving flexible tradeoff between energy efficiency and network utility maximization where we design two novel cross-layer cognitive algorithms comprising efficient powered-controlled MAC protocols for CRAHNs based on the concepts of social welfare and net revenue in economics. The proposed framework can balance interference, collision, and congestion among cognitive users (CUs) including cognitive sources and cognitive links while utilizing stochastic spectrum holes vacated by licensed users (LUs). The former allows both cognitive sources and cognitive links to simultaneously adjust their transmission parameters (i.e., transmit power, persistence probability, and rate) following the law of diminishing returns whereas the latter forces cognitive links to control the persistence probability and transmit power in order to asymptotically balance the offered load regulated by cognitive sources. Our proposed protocols are then validated and their performance is compared with the existing MAC schemes in the literature via numerical studies.**

***Index Terms*—Cross-layer optimization, medium access control, CRAHNs, congestion control, power control.**

I. Introduction

COGNITIVE radio networks [1] are envisioned as a revolutionary communication paradigm that can resolve the radio spectrum scarcity by cleverly adapting its transmission into under-utilized and/or unoccupied spectral holes of incumbent systems. Many standardization projects such as IEEE 802.22, ECMA 392, IEEE SCC41, and IEEE 802.11af, IEEE 1900 have recently been evolved for potential applications of such cognitive radio networks. Similar to conventional wireless networks, the cognitive radio network can be implemented as either an infrastructure-based network or an ad hoc network. In multi-hop CRAHNs, collisions due to channel contention among different links at the MAC layer, congestion due to sharing of common links among greedy sources at the transport layer, and interference due to co-utilized dynamic spectrum sharing among several simultaneous transmissions at the physical layer are key obstacles that can fundamentally reduce the network performance. Hence, a concrete cross-layer framework is needed to harmonize these complex interactions at different layers of the wireless protocol stack.

The key principle of *opportunistic spectrum access* (OSA) is to seek spectrum opportunities in time and space over licensed channels through channel cognition, known as *spectrum interweave* [2]. Because of imperfect knowledge of primary channels and limited range of spectral hole sensing, CU transmissions are constrained on interference caused to LUs. In the literature, a large number of recent studies [3]–[11] address smart MAC protocol design in which spectrum sensing is integrated into some other network functionalities under *spectrum interweave* paradigm in order to avoid possible collisions with LUs. Dai *et al.* [3] proposed a beamforming-based cognitive cooperative communication protocol where CU-transmitter (CU-Tx) must regulate its power and employ the best relay selection algorithm while CU-receiver (CU-Rx) leverages beamforming to best-receive data from both its CU-Tx and relay node. Thereby, CUs can enhance the spectrum opportunities and reduce the outage probabilities while making no harmful interference to LUs. The authors in [4] proposed an opportunistic-sensing-based multi-channel MAC protocol where CU-Tx opportunistically senses the entire channel pool in collaboration with the others and regulates its power as low as possible so that it can utilize spectrum holes even in mis-detecting the LU's presence. As a result, CUs can fully utilize the spectrum opportunities without causing harmful interference to LUs. In [5], the authors developed an intelligent MAC protocol in which the SUs' spectrum sensing must be done simultaneously in cooperation with contention resolution task. Tan and Le [6] addressed the design of optimal contention window and the sufficient sensing time allocation to enhance CUs' throughput and protect active LUs. More specifically, the studies [7]–[10] concentrated on the design of opportunistic multi-channel MAC protocols which can operate in *ad-hoc* mode. The impact of hardware and physical layer

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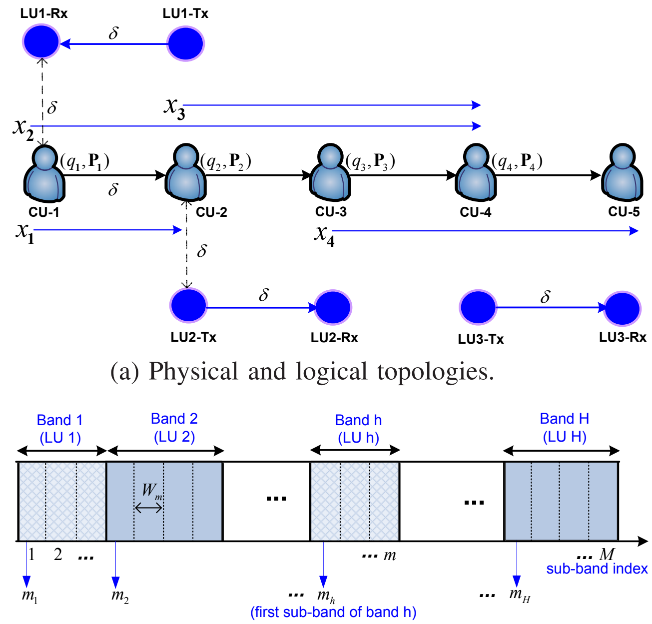
limitations on the contention control is practically considered for an efficient spectrum management [11].

Most existing OSA-based MAC studies ignore both stochastic spectrum utilization in space and fairness in channel contention resolution (see, for example, [3]–[11]). They only focus on how to best-exploit spectrum opportunities in time and minimize the number of collisions with LUs by proposing different transmission strategies associated with spectrum sensing policies. However, random interference due to fading and the limited sensing capability of CUs may seriously degrade the LUs' performance if there are no sufficient MAC mechanisms for LUs' protection. In addition, no CU's transmissions in the LUs' presence wastes the potential spectrum resource while the harmful interference caused by the CUs' concurrent transmissions to LU-Rx remains tolerable. To leverage these shortcomings, in this paper, we propose optimal cross-layer algorithms where congestion control and power control are taken into account jointly with contention resolution in an α-fair and distributed manner, where $\alpha \geq 0$ characterizes the user fairness [12]. The benefit of power control on OSA-based MAC protocol is that we can essentially reduce the contention density thanks to interference balance among CUs and get much more spectrum opportunities. By introducing a unique collision probability constraint and an acceptable interference threshold, we make sure that LUs' quality of service (QoS) is always maintained above the target thresholds in any fading channel conditions.

Although IEEE 802.22 based MAC protocols that take into account cognitive radio characteristics have been studied extensively in [13], [14], these efforts have not yet investigated the impact of contention resolution on the other layers of the protocol stack. In this paper, we focus on the OSA-based cross-layer MAC schemes for multi-hop CRAHNs, in which the interactions of three barriers (i.e., congestion, collision, and interference) are carefully considered to seek an optimal operating point for CUs. Also, different from its counterparts in traditional wireless ad hoc networks [15], [16], our OSA-based MAC protocols not only provide the good performance (e.g., throughput and fairness), but also perform interference-alignment tasks to best-utilize opportunistic spectrum in both space and time. Unlike the standardized MAC protocol in 802.11 [17] where the binary exponential back-off mechanism can result in inefficient and unfair contention resolution, our cross-layer MAC protocols aim to achieve energy efficiency and contention fairness through a unified cross-layer optimization framework. Our contributions can be summarized as follows:

1) *New Cross-layer MAC Problem:* A new cross-layer MAC framework in CRAHNs under spectrum interweave approach is proposed for interference-dependent contention resolution. By introducing a unique collision constraint and tolerable interference threshold to protect LUs' transmission, CUs can opportunistically adapt their transmissions into spectral holes even in the events of a false spectral hole detection or LU's presence.
2) *Optimal and Sub-optimal Algorithms:* We achieve the trade-off between efficiency and scalability through developing two cross-layer cognitive MAC protocols in stochastic spectrum medium from the optimal and sub-optimal distributed solutions with/without message passing. More importantly, both outperform the existing MAC schemes and move towards the stable state of lower potential energy as the link powers fall into contention-limited operation region whereas the performance of existing MAC schemes seriously degrade.
3) *Efficiency-Fairness Tradeoff:* We show that our cross-layer MAC protocols can achieve a high social welfare with a proportional fairness and yield much more considerable energy saving than the existing MAC schemes thanks to considering the interaction relationship of MAC and the other layers.



(a) Physical and logical topologies.

(b) Spectrum allocation diagram.

Fig. 1. An example of OSA-based CRAHN system.

The rest of this paper is organized as follows. Section II presents the system model and assumptions. Section III introduces the optimal cross-layer cognitive radio MAC design using the methodology of dual decomposition. In Section IV, we present the sub-optimal distributed strategy to make our proposed MAC protocol implementable and scalable. We present numerical results to illustrate the performance of our proposed protocols in Section V, and conclusion is presented in Section VI.

II. System Model and Network Assumptions

We consider a multi-hop CRAHN modeled by a *directed* graph $G(\mathcal{N}, \mathcal{L})$ as exampled in Fig. 1, where $\mathcal{N}$ is the set of cognitive nodes, $\mathcal{L}$ is the set of *unidirectional* cognitive links; N and L are their corresponding cardinalities. In contrast to [15], in this paper, the set of M orthogonal spectrum sub-bands, denoted by $\mathcal{M}$, is opportunistically exploited from primary system consisting of H pairs of LUs by all CUs using spectrum pooling [18], [19]. Note that each pair of

IMPORTANT NOTATION

Symbol	Definition
N, M	Number of cognitive users, number of licensed sub-bands
H	Number of licensed users or number of licensed bands
$\mathcal{M}$	Set of licensed bands, $M = \mid \mathcal{M} \mid$
m_h	Index of the first sub-band in the h^{th} band
$\mathcal{S}$	Set of sources with infinite amount of data to send
$\mathcal{L}$	Set of logical secondary links
$\mathcal{L}_s$	Set of links on the path of source s
$\mathcal{S}_l$	Set of sources using link l
$\mathcal{N}_{h,0}^I$	Set of nodes interfere LU h
$\mathcal{N}_l^I$	Set of nodes interferes with cognitive link l
$L_{out}(n)$	Set of outgoing links from node n
$L_{out}^l(n')$	Set of outgoing links from node n' interferes with link l
$L_{out^-}(n)$	Set of links is interfered by node n
Υ_n^l	Interference weights in which node n interferes link l
Υ_n^h	Interference weights in which node n interferes LU h
P_l^m	Power of link l on sub-band m
γ_l^m	SIR at link l on sub-band m
I_{lk}^m	Interference power caused by link k to l on sub-band m
I_l^{th}, I_h^{th}	Interference power threshold at link l and LU h
CPP	Cost per unit of consumed power
ζ_h^{col}	Collision probability of LU h
ζ_h^{busy}	Busy probability of LU h
λ_l	Congestion price at link l
ν^h	Spectrum price at on band h
x_s	Transmission rate of cognitive source s
q_l	Persistence Probability of link l
q_l^{suc}	Transmission Probability of success of link l
q_l^{idle}	Probability of ilde channel sensed by link l
Υ_n^l, Υ_n^h	Interference weights caused by node n to link l and LU h
$c_l(\mathbf{q}, \mathbf{P})$	Achievable Shannon capacity at link l

LUs h is licensed a non-overlapped sub-set of contiguous sub-bands $\mathcal{M}_h \in \mathcal{M} \mid \cup_h \mathcal{M}_h = \mathcal{M}$. For ease of exposition, we use the same index h to show both the hth spectrum band and the hth LU. The availability of each spectrum band h is characterized as a two-state ergodic Markov Chain [20] with the idle probability π^h. We also assume that π^h is obtained by CUs through a knowledge of the traffic statistics and/or the channel sensing.

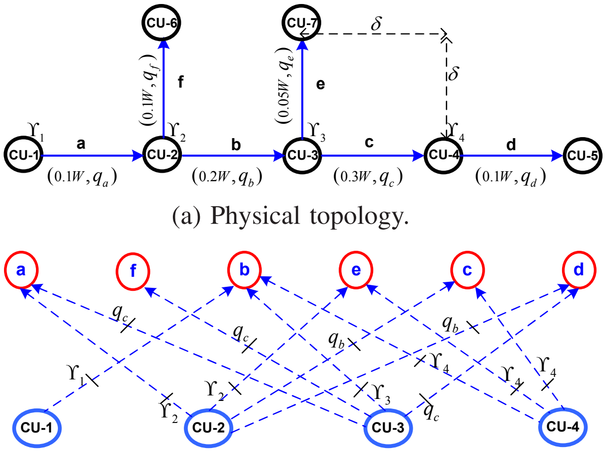
We make an assumption that time is divided into the fixed length intervals, so-called time slots. All cognitive nodes are synchronized and start their transmissions only at the beginning of each time slot. Similar to [19], we further assume that the cognitive nodes' queue is infinite and sources always have data to send. We don't study delay due to packet buffering and source vacation during unscheduled time slots. In our proposed protocols, all CUs are supposed to have two transceivers, one is used for control signalling and the other is specifically designed to opportunistically access all licensed sub-bands for data transmission. We further assume that each source $s \in \mathcal{S}$ emits one flow traveling through a pre-defined set of links, $\mathcal{L}_s \subseteq \mathcal{L}$ at rate $x_s \in \mathcal{X}_s = [x_s^{min}, x_s^{max}]$ and attains an individual utility $U_s(x_s)$ [12], so-called welfare. Then, in this paper, *social welfare* refers to the aggregated utility of secondary system (i.e., $\sum_{s\in\mathcal{S}} U_s(x_s)$) and it is the first term in the objective function of our following cross-layer MAC optimization framework. Specifically, we focus on a slotted random access system in which the contention resolution among links at each time slot is performed on the basis of transmission persistence probability $q_l \in \mathcal{Q}_l = [q_l^{min}, q_l^{max}]$, where $0 \le q_l^{min} \le q_l^{max} \le 1$.

Similar to [15], [16], each cognitive node n with a random-access-based contention resolution protocol transmits data with a probability Υ_n, and it can not transmit or receive simultaneously. However, it is different from [15], [16] in which *location-dependent contention* among links forces all outgoing links from one node to have the same transmit power, our proposed protocols are performed on the basis of *interference-dependent contention*. Accordingly, when cognitive node n determines to transmit, to avoid collisions among its outgoing links (denoted by $L_{out}(n)$), it therefore chooses link $l \in L_{out}(n)$ with a probability q_l/Υ_n, such that $\sum_{l\in L_{out}(n)} q_l = \Upsilon_n \le 1$, and transmit data on the chosen link l at a power level per sub-band P_l^m. The chosen link then transmits data on all spectrum bands.

To characterize interference relationship among cognitive links in CRAHNs, we use a *mixed interference model* [21] in which physical model and protocol model are reconciled to not only decrease the computational complexity of physical model but also fulfill the correctness of protocol model. Thereby, the link k is supposed to cause a harmful interference to the link l if its total interference power $\sum_m^M I_{lk}^m$ at the lth link's receiver is greater than the lth link's acceptable interference threshold I_l^{th}, where $I_{lk}^m = G_{lk}^m P_k^m$ is the kth link's interference power per sub-band m at the lth link's receiver. We assume that the channel fading changes very slowly so that the channel gain between the kth link's transmitter and the lth link's receiver on band m, G_{lk}^m, remains constant during time slot, but changeable over time slots. Then, we define $\mathcal{N}_l^I = \cup_{k\neq l}\{Tx_k : \sum_m^M I_{lk}^m \ge I_l^{th}\}$ as the set of other nodes whose transmissions generate a considerable interference to the receiver of link l. For a successful transmission of link l within a time slot, two following constraints must be satisfied: *i)* $P_l^{m,min} \le P_l^m \le P_l^{m,max}, \forall m$ and *ii)* no other cognitive nodes in $\mathcal{N}_l^I$ start their transmissions.

Owing to channel contention, in the kth link's viewpoint, node n with at least one outgoing link which causes interference to it forms an edge $e \in \mathcal{E}$ in a weighted node-to-link conflict graph $G^c(\mathcal{V}, \mathcal{E}, \mathcal{W})$, where the vertex set $\mathcal{V}$ is the set of the links and nodes and $\mathcal{W}$ is the set of edge weights. It is noteworthy that in this paper the weight of the node n-to-link k edge is the sum of persistence probabilities of outgoing links from node n which cause interference to link k whereas, in [15], [16], the edge weight is the sum of persistence probabilities of all outgoing links from node n. In addition, in this paper, our conflict graph changes with the link transmit powers so as to obtain the balance of interference and contention among links on the basis of offered load regulated by sources. Fig. 2 illustrates an example under this investigation that $I_l^{th} = 0.1 \times 3\delta/2$(W), where δ is the distance between any two neighboring nodes. To reduce collisions, the number of edges and the edge weight that strongly depend on the power allocation policy on each link at each node should be designed appropriately.

Our key objective is to achieve the best net revenue that jointly considers maximizing social welfare and minimizing the total energy consumption for OSA-based CRAHNs. However, to stabilize the system while assuring the LUs' QoS, the following constraints should be held.



(a) Physical topology.

(b) Node-to-link conflict graph with weights.

Fig. 2. An example of interference-dependent contention model for wireless ad-hoc networks.

A. Protocol Capacity Constraints

We recall that there is only one transmission in one clique of conflict graph G^c at each time slot. From the cognitive link l's viewpoint, the whole channel is idle if all other contending cognitive nodes and all pairs of LUs whose transmissions make an adverse interference to the lth link's reception are silent. This channel state occurs with probability:

$$q_l^{\text{idle}} = \prod_{n' \in \mathcal{N}_l^I} (1 - \sum_{k \in L_{out}^l(n')} q_k) \times \prod_{h \in \mathcal{N}_{l,0}^I} \pi^h, \forall l \in L_{out}(n) \quad (1)$$

where $\mathcal{N}_{l,0}^I = \{\text{Tx}_h : \sum_{m_h}^{m_{h+1}-1} I_{lh,0}^m \geq I_{lh}^{th}\}$ is the set of licensed transmitters which cause harmful interference at the lth link's receiver and $L_{out}^l(n')$ is the set of outgoing links from the other node n' which cause a considerable interference at the lth link's receiver. Note that we use the special symbol "0" to denote what is related to LUs. Then, the link l successfully transmits its data at q_l if the whole channel is idle. The probability of this event can be calculated as

$$\begin{aligned} q_l^{\text{suc}} &= q_l^{\text{idle}} \times q_l \\ &= \prod_{n' \in \mathcal{N}_l^I} (1 - \sum_{k \in L_{out}^l(n')} q_k) \times \prod_{h \in \mathcal{N}_{l,0}^I} \pi^h \times q_l, \forall l \in L_{out}(n). \end{aligned} \quad (2)$$

For its stability at the output buffer, the lth link's offered load avoids overwhelming its capacity, which can be calculated based on Shannon capacity $c_l(\boldsymbol{q}, \boldsymbol{P}_l)$ over the additive white Gaussian noise channels. We can express these constraints as follows:

$$\sum_{s \in \mathcal{S}_l} x_s \leq \underbrace{\sum_{m}^{M} \log(1 + G_{MC} \gamma_l^m P_l^m) \times q_l^{\text{suc}}}_{c_l(\boldsymbol{q}, \boldsymbol{P}_l)}, \forall l \quad (3)$$

where $\mathcal{S}_l = \{s : l \in \mathcal{L}_s\}$ is the set of sources s using link l and $\gamma_l^m = \frac{G_{ll}^m}{N_0^m}$ is channel gain-to-interference ratio (CIR) of link l on sub-band m [21]. N_0^m denotes the additive white noise power at the lth link's receiver on sub-band m. The constant G_{MC} denotes the processing gain; for ease of presentation we absorb it into γ_l^m, henceforth.

B. Licensed User Packet Collision Constraints

At the beginning of each time slot, from the hth LU's viewpoint, the hth band is assumed to be busy if at least one cognitive node starts data transmission with an adverse interference to the hth LU's receiver. This probability of band state is calculated as

$$\zeta_h^{\text{busy}} = 1 - \prod_{n \in \mathcal{N}_{h,0}^I} (1 - \sum_{l \in L_{out}^h(n)} q_l), \forall h \quad (4)$$

where $\mathcal{N}_{h,0}^I = \cup_l \{\text{CU} - \text{Tx}_l : \sum_{m=m_h}^{m_{h+1}-1} I_{hl,0}^m \geq I_{h,0}^{th}\}$ and $L_{out}^h(n)$ is the set of outgoing links from node n which cause a considerable interference to the hth licensed link's receiver. The collision to the LU h occurs only when it starts data transmission while its band is being interfered by CUs. To guarantee the LUs' QoS, the maximum packet collision rate caused by the CUs' transmissions must be below the tolerable and preset threshold $\mu^h \leq 1 - \pi^h$:

$$\begin{aligned} \zeta_h^{\text{col}} &= \zeta_h^{\text{busy}} \times (1 - \pi^h) \\ &= \Big(1 - \prod_{n \in \mathcal{N}_{h,0}^I} (1 - \sum_{l \in L_{out}^h(n)} q_l)\Big) \times (1 - \pi^h) \leq \mu^h, \forall h. \end{aligned} \quad (5)$$

It is clear that ζ_h^{col} only depends on the persistence probabilities of those cognitive links whose total interference level to the hth LU's receiver exceeds a certain limit $I_{h,0}^{th}$. Hence, given stochastic spectrum opportunities $(\boldsymbol{\pi}, \boldsymbol{\mu})$ with an acceptable interference threshold $I_{h,0}^{th}$, how to balance interference and contention among CUs motivates an optimization control framework in the next section.

III. Optimal Cross-Layer Cognitive MAC Design

A. Optimization Formulation of Cross-Layer Cognitive MAC

We augment the NUM (Network Utility Maximization) framework [22] to include the MAC layer, the physical layer, and the transport layer which leads to a novel cross-layer design. Our cross-layer optimization framework is to globally maximize the total net revenue subject to the link capacity conservation (3) and the LUs' QoS (5) as follows:

$$\max_{\boldsymbol{x} \in \mathcal{X}, \mathbf{P} \in \mathcal{P}, \mathbf{q} \in \mathcal{Q}} \sum_{s \in \mathcal{S}} U_s(x_s) - \text{CPP} \sum_{l=1}^{L} \sum_{m=1}^{M} P_l^m \quad (6)$$
$$\textit{s.t.} \quad (3), \quad (5),$$

where $\mathcal{X} = \{x_s; s \in \mathcal{S} | x_s^{min} \leq x_s \leq x_s^{max}\}$, $\mathcal{P} = \{P_l^m; \forall l, m | P_l^{m,min} \leq P_l^m \leq P_l^{m,max}\}$, $\mathcal{Q} = \{\mathcal{Q}_l, l \in \mathcal{L}\}$. $U_s(x_s)$ is assumed to be twice continuously differentiable, non-decreasing and strictly concave in its domain. CPP is the cost per unit of consumed power. The fairness under our resource allocation can be characterized by a general α-fair utility function [12]. It is noteworthy that congestion in wireless networks is defined by a bottleneck link where traffic load (i.e., $\sum_{s \in \mathcal{S}_l} x_s$) is larger than link capacity, in terms of the attainable data rate under the given conditions of power vector $\mathbf{P}$, persistence probability q_l, and spectrum opportunities $\boldsymbol{\pi}$, which is a global function of all the persistence probabilities of the interfering links $\Upsilon_{n'}^{l}, \forall n' \in \mathcal{N}_l^I$. As a result, our proposed cross-layer framework tightly couples the optimization variables $(\boldsymbol{x}, \mathbf{P}, \boldsymbol{q})$ which correspond to the wireless protocol layers in constraints (3) and (5).

It is very important to note that the optimization problem (6) is non-convex and inseparable. Now, let us denote $\chi^h = 1 - \frac{\mu^h}{1-\pi^h}$. By using the log change of rate variables ($\hat{\boldsymbol{x}} = \log \boldsymbol{x}$), taking logarithm of both sides of (3) and (5), the problem (6) can be equivalently transformed as

$$\max_{\hat{\boldsymbol{x}} \in \hat{\mathcal{X}}, \boldsymbol{P} \in \mathcal{P}, \mathbf{q} \in \mathcal{Q}} \sum_{s \in \mathcal{S}} U_s(e^{\hat{x}_s}) - \text{CPP} \sum_{l=1}^{L} \sum_{m=1}^{M} P_l^m \quad s.t. \tag{7}$$

$$\log \sum_{s \in \mathcal{S}_l} e^{\hat{x}_s} \leq \log \left(\sum_{m=1}^{M} \log\left(1 + \gamma_l^m P_l^m\right) \times \prod_{h \in \mathcal{N}_{l,0}^I} \pi^h \right) + \log \left(q_l \prod_{n' \in \mathcal{N}_l^I} (1 - \Upsilon_{n'}^l) \right), \forall l, \tag{8}$$

$$\log \chi^h \leq \sum_{n \in \mathcal{N}_{h,0}^I} \log(1 - \Upsilon_n^h), \forall h, \tag{9}$$

$$\sum_{k \in L_{out}^l(n)} q_k = \Upsilon_n^l, \sum_{l \in L_{out}^h(n)} q_l = \Upsilon_n^h, \sum_{l \in L_{out}(n)} q_l = \Upsilon_n \leq 1, \forall n \tag{10}$$

where Υ_n^l and Υ_n^h are interpreted as the interference weights in which cognitive node n interferes link l and LU h, respectively.

Theorem 1. *The equivalent optimization problem* (7) *is separable and convex in* $(\hat{\boldsymbol{x}}, \boldsymbol{P}, \boldsymbol{q})$*-space.*

Proof: First of all, we observe that the objective of (7) is a concave and separable function due to the strict concavity assumption of $U_s(.)$ and that the optimization variables in all constraints are decoupled after taking logarithm. Moreover, $\log \sum_{s \in \mathcal{S}_l} e^{\hat{x}_s}$ is convex because the *log-sum-exp* function is convex [23]. Next, we need to prove $f(\boldsymbol{P}) = \log\left(\sum_{m=1}^{M} \log\left(1 + \gamma_l^m P_l^m\right)\right)$ is concave. We consider $f(\boldsymbol{P}) = \log\left(g(\boldsymbol{P})\right)$ as a composition function, where $g(\boldsymbol{P}) = \sum_{m=1}^{M} \log(1 + \gamma_l^m P_l^m) \geq 0$. We have $\frac{\partial g(\boldsymbol{P})}{\partial P_l^m} = \frac{\gamma_l^m}{1+\gamma_l^m P_l^m} \geq 0$, $\frac{\partial^2 g(\boldsymbol{P})}{\partial (P_l^m)^2} = -\left(\frac{\gamma_l^m}{1+\gamma_l^m P_l^m}\right)^2$, and $\frac{\partial^2 g(\boldsymbol{P})}{\partial P_l^m \partial P_l^i} = 0, \forall m \neq i$. Denoting $z = \frac{\gamma_l^m}{1+\gamma_l^m P_l^m}$, for all $\mathbf{v} \in \mathbb{R}^M$ we always have $\mathbf{v}^T \nabla^2 g(\boldsymbol{P}) \mathbf{v} = -\sum_i z_i^2 v_i^2 \leq 0$, which shows that $g(\boldsymbol{P})$ is concave and nondecreasing. Therefore, $f(\boldsymbol{P})$ is concave by the composition rules [23]. The convexity of remaining constraints finalizes this proof. ■

By augmenting the objective in (7) with a weighted sum of the constraints (8) and (9), we have the partial Lagrangian:

$$L(\hat{\boldsymbol{x}}, \boldsymbol{P}, \boldsymbol{q}, \boldsymbol{\lambda}, \boldsymbol{\nu}) = L_x(\hat{\boldsymbol{x}}, \boldsymbol{\lambda}) + L_P(\boldsymbol{P}, \boldsymbol{\lambda}) + L_q(\boldsymbol{q}, \boldsymbol{\lambda}, \boldsymbol{\nu}) \tag{11}$$

where $\boldsymbol{\lambda} = [\lambda_1, ..., \lambda_L]$ and $\boldsymbol{\nu} = [\nu^1, ..., \nu^H]$ are the Lagrange nonnegative multipliers which are interpreted as congestion prices and spectrum prices, respectively. $L_x(\hat{\boldsymbol{x}}, \boldsymbol{\lambda})$, $L_P(\mathbf{P}, \boldsymbol{\lambda})$, and $L_q(\boldsymbol{q}, \boldsymbol{\lambda}, \boldsymbol{\nu})$ are defined as

$$L_x(\hat{\boldsymbol{x}}, \boldsymbol{\lambda}) \triangleq \sum_s U_s(e^{\hat{x}_s}) - \sum_{l \in \mathcal{L}} \lambda_l \log \sum_{s \in \mathcal{S}_l} e^{\hat{x}_s}. \tag{12}$$

We refer to the problem (7) as the *primal* problem, then its *duality* can be described as

$$(\mathbf{D}) \qquad \min_{\boldsymbol{\lambda} \succeq 0, \boldsymbol{\nu} \succeq 0} \quad g(\boldsymbol{\lambda}, \boldsymbol{\nu}) \tag{15}$$

$$\text{where} \quad g(\boldsymbol{\lambda}, \boldsymbol{\nu}) = \max_{\hat{\boldsymbol{x}}, \mathbf{P}, \mathbf{q}} L(\hat{\boldsymbol{x}}, \boldsymbol{P}, \boldsymbol{q}, \boldsymbol{\lambda}, \boldsymbol{\nu}) \quad s.t. \ (10). \tag{16}$$

B. Dual Decomposition and Optimal Solution

Since the primal problem (7) is convex [Theorem 1] and it satisfies the Slaters conditions, the strong duality holds. This means duality gap is zero and (7) can hence be optimally solved via the dual problem (15). By decomposing (16) into three subproblems which correspond to the maximization of (12), (13), and (14), we then obtain the primal solution as follows:

1) Rate Control Subproblem : Given the multipliers $\boldsymbol{\lambda}$, each s regulates its data rate x_s such that $L_x(\hat{\boldsymbol{x}}, \boldsymbol{\lambda})$ is maximized. Let $\lambda_s \doteq \sum_{l \in \mathcal{L}_s} \frac{\lambda_l}{\sum_{\acute{s} \in \mathcal{S}_l} x_{\acute{s}}}$, the optimal rate can be found by using a gradient-ascent method [24] with a sufficiently small step size $\kappa_t \geq 0$:

$$x_s^{(t+1)} = \left[x_s^{(t)} + \kappa_t \left(U_s'(x_s^{(t)}) - \lambda_s^{(t)} \right) \right]^{\mathcal{X}} \tag{17}$$

where $U_s'(.)$ is the first derivative of utility and $[x]^{\mathcal{X}}$ is the projection of x onto the set $\mathcal{X}$.

2) Power Control Subproblem: Given λ_l, the link l alternatively adjusts its transmit power P_l^m on each sub-band to maximize $L_P(\boldsymbol{P}, \boldsymbol{\lambda})$ via the following iterative algorithm.

$$P_l^{m,(t+1)} = \left[P_l^{m,(t)} + \kappa_t \left(\lambda_l^{(t)} \Theta_l^{m,(t)} - \text{CPP} \right) \right]^{\mathcal{P}} \tag{18}$$

where $\Theta_l^{m,(t)} = \frac{\gamma_l^m}{(1+\gamma_l^m P_l^{m,(t)}) \sum_{m=1}^{M} \log(1+\gamma_l^m P_l^{m,(t)})}$.

Proof: Since $L_P(\boldsymbol{P}, \boldsymbol{\lambda})$ is strictly concave in $\boldsymbol{P}$, its first-order derivative with respect to P_l^m is $\frac{\partial L_P(\boldsymbol{P}, \boldsymbol{\lambda})}{\partial P_l^m} = \frac{\lambda_l \gamma_l^m}{1+\gamma_l^m P_l^m} \frac{1}{\sum_{m=1}^{M} \log(1+\gamma_l^m P_l^m)} - \text{CPP}$. Then, we adopt the projected gradient-ascent method [24] with a step size $\kappa_t \geq 0$ for the link power updates on each sub-band. ■

3) Medium Access Control Subproblem: We change the indices in the sums by using the following facts:

1. $\sum_l \lambda_l = \sum_n \sum_{l \in L_{out}(n)} \lambda_l$.
2. $\sum_l \lambda_l \sum_{n' \in \mathcal{N}_l^I} \log\left(1 - \sum_{k \in L_{out}^l(n')} q_k\right)$
   $= \sum_n \sum_{k \in L_{out^-}(n)} \lambda_k \log\left(1 - \sum_{l \in L_{out}^k(n)} q_l\right)$,

where $L_{out^-}(n) = \{k \neq l : \sum_m I_{kl}^m \geq I_k^{th}, \forall l \in L_{out}^n\}$ is the set of links whose receptions are affected by the interference from the transmission of node n, excluding the outgoing links

$$L_P(\boldsymbol{P},\boldsymbol{\lambda}) \triangleq \sum_{l=1}^{L}\lambda_l \log\left(\sum_{m=1}^{M}\log\left(1+\gamma_l^m P_l^m\right)\times\prod_{h\in\mathcal{N}_{l,0}^I}\pi^h\right) - \text{CPP}\sum_{l=1}^{L}\sum_{m=1}^{M}P_l^m. \quad (13)$$

$$L_q(\boldsymbol{q},\boldsymbol{\lambda},\boldsymbol{\nu}) \triangleq \sum_{l=1}^{L}\lambda_l\log\Big(q_l\prod_{n'\in\mathcal{N}_l^I}(1-\Upsilon_{n'}^l)\Big)+\sum_{h=1}^{H}\nu^h\Big(\sum_{n\in\mathcal{N}_{h,0}^I}\log(1-\Upsilon_n^h)-\log\chi^h\Big). \quad (14)$$

from node n. Then, we rewrite $L_q(\boldsymbol{q},\boldsymbol{\lambda},\boldsymbol{\nu})$ in (14) as

$$\begin{aligned}L_q(\boldsymbol{q},\boldsymbol{\lambda},\boldsymbol{\nu}) &= \sum_{n=1}^{N}\sum_{l\in L_{out}(n)}\lambda_l\log q_l \\ &+\sum_{n\in\mathcal{N}_{h,0}^I}\sum_{h=1}^{H}\nu^h\log(1-\Upsilon_n^h)-\sum_{h=1}^{H}\nu^h\log\chi^m \\ &+\sum_{n=1}^{N}\sum_{k\in L_{out^-}(n)}\lambda_k\log\left(1-\Upsilon_n^k\right).\end{aligned} \quad (19)$$

Given $(\boldsymbol{\lambda},\boldsymbol{\nu})$, the medium access control subproblem in (16),

$$\max_{\mathbf{q}\in\mathcal{Q}} \quad L_q(\boldsymbol{q},\boldsymbol{\lambda},\boldsymbol{\nu}) \qquad \textit{s.t. } (10), \quad (20)$$

can obtain its optimal solution thanks to the Karush-Kuhn-Tucker (KKT) conditions [24]:

$$q_l^{(t+1)} = \left[\frac{\lambda_l^{(t)}}{\sum_{k\in L_{out^-}(n)}\frac{\lambda_k^{(t)}}{1-\Upsilon_n^{k,(t)}}+\sum_h^H\frac{a_n^h\nu^{h,(t)}}{1-\Upsilon_n^{h,(t)}}}\right]^{\mathcal{Q}}, \quad \forall l\in L_{out}(n) \quad (21)$$

where

$$a_n^h = \begin{cases}1, & \text{if } n\in\mathcal{N}_{h,0}^I,\\ 0, & \text{otherwise.}\end{cases}$$

is a binary function which indicates whether cognitive node n interferes with LU h or not.

Proof: Since $L_q(\mathbf{q},\boldsymbol{\lambda},\boldsymbol{\nu})$ is strictly concave in $\mathbf{q}$, its first-order derivative with respect to q_l is: $\frac{\partial L_q(\mathbf{q},\boldsymbol{\lambda},\boldsymbol{\nu})}{\partial q_l} = \frac{\lambda_l}{q_l}-\sum_{h=1}^{H}\frac{a_n^h\nu^h}{1-\Upsilon_n^h}-\sum_{k\in L_{out^-}(n)}\frac{\lambda_k}{1-\Upsilon_n^k}$. By letting the above derivative equals zero, solving the resultant equation, we obtain (21). ■

It can be observed from (21) that congestion control, power control, and medium access control have a mutual relationship, where congestion control (17) regulates source rates to avoid overwhelming the attainable link capacity which depends on the success probability of channel access q_l^{suc} and allocated power as shown in Fig. 3. In fact, a change in source rates will in turn affect the spending decisions of power and persistence probability at cognitive links through congestion prices λ_l. Furthermore, any increase/decrease in the set of interfering nodes $\mathcal{N}_l^I$ or the interference weights (i.e., Υ_n^k and Υ_n^h) due to the increasing/decreasing power level of outgoing links from node n will force their channel access probabilities to decrease/increase accordingly. And any increase/decrease in one cognitive node's persistence probability also changes the other contending nodes' behavior. Hence, the source rate regulation (17) associated with the sufficient power policy (18) and the optimal channel access strategy (21) is essentially driven by CUs to yield a good balance of interference, contention and congestion.

C. Optimal Cross-Layer Cognitive MAC Protocol (OCC-MAC)

In this section, we describe the OCC-MAC protocol (see Algorithm 1) in which the congestion subproblem is handled by cognitive sources via *Cognitive Source Algorithm* that can realize the TCP congestion control mechanism in a fully distributed manner. Specifically, each cognitive source s locally regulates its rate (17) based on the feedback of the aggregate congestion price $\lambda_s \doteq \sum_{l\in\mathcal{L}_s}\frac{\lambda_l}{\sum_{\acute{s}\in\mathcal{S}_l}x_{\acute{s}}}$ from the intermediate cognitive nodes along its route from destination while cognitive link adjusts its transmission power and probability on the basis of control information exchange. We further assume that one of cognitive radios at each CU is set to the common control channel (CCC) for signaling.

Note that the updates $\lambda_l^{(t+1)}$ and $\nu^{h,(t+1)}$ are obtained by the dual solution using the projected gradient-descent method [24]. The algorithm will stop whenever the convergence criterion $\max\|\mathbf{q}^{*(t)}-\mathbf{q}^{*(t-1)}\|\le\varepsilon$, where ε is the error tolerance, is reached. For the sake of convenience, we use the same step-size k_t for all updates without loss of generality, henceforth.

Theorem 2. *For any initial source rate $\boldsymbol{x}^{(0)}\in\mathcal{X}$, link power $\boldsymbol{P}^{(0)}\in\mathcal{P}$, persistence probability $\boldsymbol{q}^{(0)}\in\mathcal{Q}$, and shadow prices $(\boldsymbol{\lambda}^{(0)},\nu^{(0)})\succeq 0$, the sequence of primal-dual variables generated by **Algorithm 1** converges to the global optimum of the original problem* (6) *provided that the step sizes κ_t satisfy*

$$\kappa_t\ge 0,\quad \sum_{t=0}^{\infty}\kappa_t=\infty,\quad \sum_{t=0}^{\infty}(\kappa_t)^2<\infty. \quad (24)$$

Proof: It is straightforward that the equivalent problem (7) is convex [Theorem 1]. Hence, with any initial values of primal and dual variables in their feasible domain, Algorithm 1 converges to the global optimum with sufficiently small step sizes κ_t satisfying (24) [24]. ■

Remarks: It is important to note that OCC-MAC requires only the explicit message passing of $(q_l^{(t)};\lambda_l^{(t)};\Upsilon_n^{h,(t)},\forall h)$ between the cognitive links which make an considerable interference to each other and LUs for the power and persistence probability allocations on the CCC. Hence, the number of explicit message passing required in OCC-MAC depends on both network topology and transmit power at each cognitive link. However, we assume that each cognitive node will broadcast only one control message containing the fields of $(q_l^{(t)};\lambda_l^{(t)};\Upsilon_n^{h,(t)},\forall h)$ for all outgoing cognitive links from it on the CCC at each iteration. As a result, the maximum number of control overheads at each iteration is N.

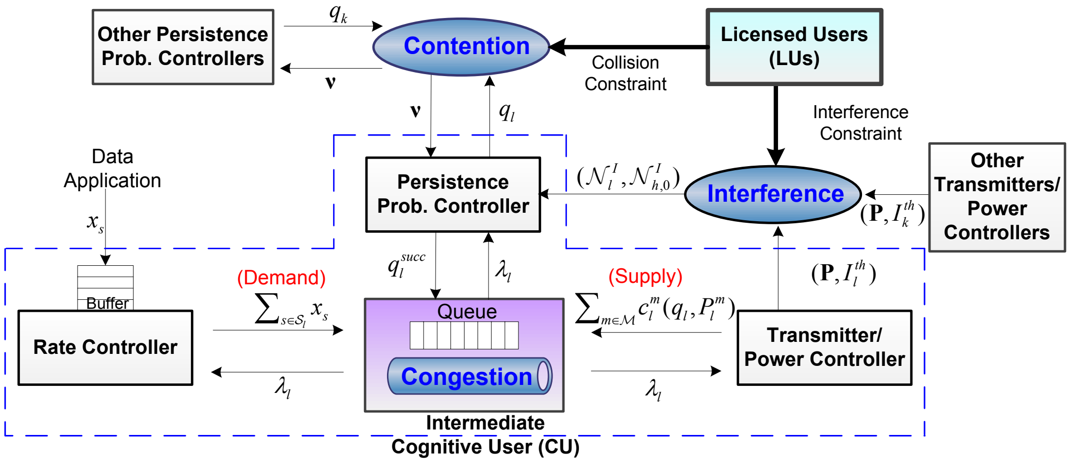


Fig. 3. Cross-layer design for congestion control, contention control, and power control in CRAHNs.

Algorithm 1: Optimal Cross-Layer Cognitive MAC Protocol (OCC-MAC)

Sources and links initialize $\boldsymbol{x}^{(0)}, \boldsymbol{P}^{(0)}, \boldsymbol{q}^{(0)}, \boldsymbol{\lambda}^{(0)}, \boldsymbol{\nu}^{(0)}$. At time t:

Cognitive Source Algorithm: For each source $s \in \mathcal{S}$:

1: Receive the total price that accumulates the intermediate links' congestion prices per sum ingress rate $\lambda_l^{(t)} / \sum_{\acute{s} \in \mathcal{S}_l} x_{\acute{s}}^{(t)}$ along its path through a feedback message from its destination.

2: Update rate $x_s^{(t+1)}$ using (17) with $\lambda_s^{(t)}$.

Cognitive Link Algorithm: For each link $l \in L_{out}(n); n \in \mathcal{N}$:

1: Broadcast a message with $(q_l^{(t)}; \lambda_l^{(t)}; \Upsilon_n^{h,(t)}, \forall h)$ on CCC.

2: Calculate its powers $P_l^{m,(t+1)}, \forall m \in \mathcal{M}$ using (18).

3: Update $L_{out^-}(n)$, a_n^h, and $\mathcal{N}_{h,0}^I$, $\forall h$ by a private graph G_n^c.

4: Calculate its interference weights $\Upsilon_n^{k,(t)}, \forall k \in L_{out^-}(n)$ and $\Upsilon_n^{h,(t)}, \forall h$.

5: Receive congestion prices $\lambda_k^{(t)}, k \in L_{out^-}(n)$ and interference weights $\Upsilon_{n'}^{h,(t)}, \forall h$. Update its persistence probability $q_l^{(t+1)}$ using (21).

6: Receive $q_k^{(t)}, k \in L_{out}^l(n'), \forall n' \in \mathcal{N}_l^I$, calculate $c_l(\mathbf{q}^{(t)}, \mathbf{P}^{(t)})$ using (3).

7: Get ingress rate $\sum_{s \in \mathcal{S}_l} x_s^{(t)}$ from input queue, then update its congestion price:

$$\lambda_l^{(t+1)} = \left[\lambda_l^{(t)} + \kappa_t \log \left(\sum_{s \in \mathcal{S}_l} x_s^{(t)} / c_l(\mathbf{q}^{(t)}, \mathbf{P}^{(t)}) \right) \right]^{\mathbb{R}^+}. \quad (22)$$

8: Update its spectrum price:

$$\nu^{h,(t+1)} = \left[\nu^{h,(t)} + \kappa_t \left(\log \chi^h - \sum_{n \in \mathcal{N}_{h,0}^I} \log(1 - \Upsilon_n^{h,(t)}) \right) \right]^{\mathbb{R}^+}. \quad (23)$$

IV. Heuristic Cross-Layer Cognitive MAC Scheme (HCC-MAC)

In OCC-MAC, the link's transmit power and persistence probability are jointly adjusted on the basis of ingress rate regulated by sources to maximize the total net revenue of the secondary system while keeping the LU collision probability below a tolerable threshold. Despite its optimality's attraction, the overhead congestion and unsuccessful reception of control messages on the CCC may be a key bottleneck to deploy OCC-MAC in practice. Using a heuristic solution [19] to cope with these difficulties might seem desirable. In this section, we propose another distributed scheme, HCC-MAC which would perform local measurements for estimating the success probability q_l^{suc} and the collision probability ζ_h^{col} of the LU h to omit message passing in an implementation of OCC-MAC at the expense of optimality.

It is different from OCC-MAC that, in HCC-MAC, since CUs do not use their cognitive radio to exchange control information, one of their cognitive radios is dedicated to overhear ongoing communications (idle, collision, or success) within each band. Here, we reuse the optimal solution of rate control and power control subproblems as discussed in Section III to solve (6). However, the medium access control subproblem is solved in the following heuristic fashion.

A. Heuristic Medium Access Control Solution

We assume that at the beginning of each time slot the lth link's ingress rate $\sum_{s \in \mathcal{S}_l} x_s$ and power $P_l^m, \forall m$ given by (17) and (18) are fixed. Then, q_l must be controlled in such a way that its capacity can meet the ingress rate demand. Accordingly, the constraints in (3) are re-expressed:

$$q_l \times \prod_{n' \in \mathcal{N}_l^I} (1 - \Upsilon_{n'}^l) \geq \frac{\sum_{s \in \mathcal{S}_l} x_s}{\sum_m^M \log(1 + \gamma_l^m P_l^m) \times \prod_{h \in N_{l,0}^I} \pi^h}, \quad \forall l \in L_{out}(n). \quad (25)$$

In this regard, the right-hand side of (25) is considered constant. Therefore, the medium access control subproblem given by (16),

$$\max_{\mathbf{q} \in \mathcal{Q}} \left\{ \sum_{l=1}^{L} \lambda_l \log \left(q_l \times \prod_{n' \in \mathcal{N}_l^I} (1 - \Upsilon_{n'}^l) \right) + \sum_{h=1}^{H} \nu^h \left(\sum_{n \in \mathcal{N}_{h,0}^I} \log(1 - \Upsilon_n^h) - \log \chi^h \right) \right\} \quad s.t.\ (10),$$

can be approximately rewritten as follows:

$$\max_{\mathbf{q}\in\mathcal{Q}} \sum_{h=1}^{H} \nu^h \sum_{n\in\mathcal{N}_{h,0}^I} \log(1-\Upsilon_n^h) \quad s.t. \quad (10),(25). \tag{26}$$

Then we can reform (26) as:

$$\max_{\mathbf{q}\in\mathcal{Q}} \sum_{h=1}^{H} \nu^h \sum_{n\in\mathcal{N}_{h,0}^I} \log(1-\sum_{l\in L_{out}^h(n)} q_l) \quad s.t. \quad (25). \tag{27}$$

Under observing from (25), the problem (27) states that how to seek $\mathbf{q}$ for minimizing the LUs' collision probability while balancing the bandwidth supply and rate demand at each cognitive link. Now let us take the log change of persistence variables (i.e., $\hat{\mathbf{q}} = \log \mathbf{q} \preceq 0$) and the logarithm both sides of (25), we then obtain an equivalent problem of (27):

$$\max_{\hat{\mathbf{q}}\in\hat{\mathcal{Q}}} \Psi(\hat{\mathbf{q}}) \triangleq \sum_{h}^{H} \nu^h \sum_{n\in\mathcal{N}_{h,0}^I} \log\left(1-\sum_{l\in L_{out}^h(n)} e^{\hat{q}_l}\right) \tag{28}$$

$$s.t. \quad \hat{q}_l \geq \Phi_l(\hat{\mathbf{q}}), \quad \forall l, \tag{29}$$

where $\Phi_l(\hat{\mathbf{q}}) \triangleq \log\left(\frac{\sum_{s\in\mathcal{S}_l} x_s}{\sum_{m}^{M} \log(1+\gamma_l^m P_l^m)\times\prod_{h\in\mathcal{N}_{l,0}^I}\pi^h}\right) - \sum_{n'\in\mathcal{N}_l^I} \log\left(1-\sum_{k\in L_{out}^l(n')} e^{\hat{q}_k}\right)$.

Proposition 1. *Given $(\boldsymbol{x}, \boldsymbol{P}, \boldsymbol{\nu})$, the approximate MAC subproblem* (28) *is convex.*

Proof: See Appendix A. ∎

Since (28) is convex, its optimal solution can be found via the following iterative algorithm modifying gradient-descent method [24] after transforming back to $\mathbf{q}$-space as follows:

$$q_l^{(t+1)} = \left[q_l^{(t)} + \kappa_t\left(\frac{\partial\Psi(\mathbf{q}^{(t)})}{\partial q_l}\frac{1}{q_l^{(t)}} + \Phi_l(\mathbf{q}^{(t)})\right)\right]^{\mathcal{Q}_l}, \forall l\in\mathcal{L}. \tag{30}$$

Proposition 2. *Given a triple of fixed variables $(\boldsymbol{x}, \boldsymbol{P}, \boldsymbol{\nu})$, the medium access control algorithm* (30) *solving the subproblem* (28) *converges to the unique optimum.*

Proof: We use two following facts:

1) $\frac{\partial\Psi(\hat{\mathbf{q}})}{\partial\hat{q}_l} = -\sum_h^H \frac{a_n^h \nu^h e^{\hat{q}_l}}{1-\sum_{l\in L_{out}^h(n)} e^{\hat{q}_l}} \leq 0.$

2) $\Phi_l(\hat{\mathbf{q}})$ satisfies:
   a) Negativity: $\Phi_l(\hat{\mathbf{q}}) < 0$.
   b) Scalability: $\delta\Phi_l(\hat{\mathbf{q}}) < \Phi_l(\delta\hat{\mathbf{q}}), \forall\delta > 1$.
   c) Monotonicity: $\Phi_l(\hat{\mathbf{q}}) < \Phi_l(\hat{\mathbf{q}}'), \forall\hat{\mathbf{q}} > \hat{\mathbf{q}}'$.

to approve the following inequality:

$$\frac{\partial\Psi(\hat{\mathbf{q}}^{(t)})}{\partial\hat{q}_l} + \Phi_l(\hat{\mathbf{q}}^{(t)}) \leq 0, \forall l\in\mathcal{L}. \tag{31}$$

Also, using the facts that $\frac{\partial\Psi(\hat{\mathbf{q}})}{\partial\hat{q}_l} = \frac{1}{q_l}\frac{\partial\Psi(\mathbf{q})}{\partial q_l}$ and $q_l = e^{\hat{q}_l}$, we then equivalently transform (31) into

$$\frac{\partial\Psi(\mathbf{q}^{(t)})}{\partial q_l}\frac{1}{q_l^{(t)}} + \Phi_l(\mathbf{q}^{(t)}) \leq 0, \forall l\in\mathcal{L}. \tag{32}$$

We assume that the step size κ_t satisfies (24). As a result, the sequence of $q_l^{(t)}$ in (30) is monotonically decreasing. On the other hand, this sequence is lower-bounded by q_l^{min} the medium access control algorithm (30) always converges to the unique optimum $\mathbf{q}^*$. ∎

B. Estimation of LU Collision Probability and Idle Channel Probability

Since link l knows the idle probabilities of LUs, it can locally estimate its idle channel probability q_l^{idle} and/or success probability q_l^{suc} through the statistics of channel contention observation. It is straightforward that the number of time slots in which link l experiences to obtain a successful transmission is an independent, identically distributed (i.i.d.) geometric random variable, denoted by $\xi_l \sim Geo(q_l^{\mathsf{suc}})$. Link l can estimate q_l^{suc} by observing its success events over a time window T (time slots). We assume that there exist K bursts of successive failures during T. Let b_k denote the number of successive failures of the k^{th} burst, link l then can calculate the mean number of successive failures preceding a success during T. In this regard, the noisy estimation of q_l^{suc} is given by using maximum log-likelihood (ML) estimation:

$$\check{q}_l^{\mathsf{suc}} = \frac{\sum_{k=1}^K b_k}{K+\sum_{k=1}^K b_k}. \tag{33}$$

Proof: The likelihood and log-likelihood of the geometric distribution can be written as $\Lambda(\check{q}_l^{\mathsf{suc}}) = \prod_{k=1}^K Pr(\xi_l = b_k|\check{q}_l^{\mathsf{suc}}) = \prod_{k=1}^K (1-\check{q}_l^{\mathsf{suc}})(\check{q}_l^{\mathsf{suc}})^{b_k} = (1-\check{q}_l^{\mathsf{suc}})^K(\check{q}_l^{\mathsf{suc}})^{\sum_{k=1}^K b_k}$ and $\ln\Lambda(\check{q}_l^{\mathsf{suc}}) = K\ln(1-\check{q}_l^{\mathsf{suc}}) + \sum_{k=1}^K b_k \ln\check{q}_l^{\mathsf{suc}}$. By setting the first derivative of log-likelihood with respect to $\check{q}_l^{\mathsf{suc}}$ to zero and solving the resultant equation, we obtain (33). ∎

Similarly, CUs can also estimate the collision probability of each pair of LUs by overhearing RTS collisions taken place over each band. Let N_h^T denote the number of collisions of the hth LU observed at one CU during T and R_h denote the hth LU's packet rate during T. Then, the noisy estimation of the hth LU collision probability can be calculated as [19, Eq.27]:

$$\check{\zeta}_h^{\mathsf{col}} = \begin{cases} 1/(R_h\times T), & \text{if } N_h^T = 0, \\ N_h^T/(R_h\times T), & \text{otherwise.}\end{cases} \tag{34}$$

C. Heuristic Cross-Layer Cognitive MAC Protocol (HCC-MAC)

Our aforementioned novel solution of the medium access control subproblem motivates a heuristic power control MAC protocol which can be deployed in a decentralized fashion as described in Algorithm 2. In HCC-MAC, the cognitive links perform local measurements for estimating their success probabilities $\check{q}_l^{\mathsf{suc},(t)}$ and the LU collision probabilities $\check{\zeta}_h^{\mathsf{col},(t)}$ to update the transmission parameters without message passing.

Theorem 3. *For κ_t satisfying* (24)*, $\boldsymbol{x}^{(0)} \in \mathcal{X}$, $\boldsymbol{P}^{(0)} \in \mathcal{P}$, $\boldsymbol{q}^{(0)} \in \mathcal{Q}$, and $(\boldsymbol{\lambda}^{(0)}, \boldsymbol{\nu}^{(0)}) \succeq 0$, **Algorithm 2** converges to an equilibrium which has a negligible gap compared to the global optimum of the original problem* (6).

Proof: See Appendix B. ∎

Remarks: Our cross-layer cognitive MAC protocols operate in a distributed fashion and adopt a slotted p-persistent

Algorithm 2: Heuristic Cross-Layer Cognitive MAC Protocol (HCC-MAC)

Sources and links initialize $\boldsymbol{x}^{(0)}, \boldsymbol{P}^{(0)}, \boldsymbol{q}^{(0)}, \boldsymbol{\lambda}^{(0)}, \boldsymbol{\nu}^{(0)}$. At time t:
Cognitive Source Algorithm: Source rate updates as (17) in OCC-MAC.
Cognitive Link Algorithm: For each link $l \in L_{out}(n); n \in \mathcal{N}$.

1: Update power $P_l^{(t+1)}$ using (18).
2: Calculate $\tilde{c}_l(\mathbf{q}^{(t)}, P_l^{(t)})$ using (3) with ML estimation $\check{q}_l^{\mathsf{suc},(t)}$:

$$\tilde{c}_l(\mathbf{q}^{(t)}, P_l^{(t)}) = \sum_m^M \log\left(1 + \gamma_l^m P_l^{m,(t)}\right) \times \check{q}_l^{\mathsf{suc},(t)}. \qquad (35)$$

3: Get $\sum_{s\in S_l} x_s^{(t)}$ from input queue. Update the interference weights to LUs: $\Upsilon_n^{h,(t)} = \sum_{l\in L^h_{out}(n)} q_l^{(t)}, \forall h$.
4: Update persistence probability $q_l^{(t+1)}$ using our proposed algorithm (30) as follows:

$$q_l^{(t+1)} = \left[q_l^{(t)} + \kappa_t\left(\log\left(\frac{\sum_{s\in\mathcal{S}_l} x_s^{(t)} q_l^{(t)}}{\tilde{c}_l(\mathbf{q}^{(t)}, P_l^{(t)})}\right) - \sum_h^H \frac{a_n^{h,(t)}\nu^{h,(t)}}{1-\Upsilon_n^{h,(t)}}\right)\right]^{\mathcal{Q}_l}. \qquad (36)$$

5: Update congestion prices using (22) with $\tilde{c}_l(\mathbf{q}^{(t)}, P_l^{(t)})$ given in (35).

$$\lambda_l^{(t+1)} = \left[\lambda_l^{(t)} + \kappa_t \log\left(\sum_{s\in\mathcal{S}_l} x_s^{(t)}/\tilde{c}_l(\mathbf{q}^{(t)}, P_l^{(t)})\right)\right]^{\mathbb{R}^+}. \qquad (37)$$

6: Update spectrum price using (23) with $\zeta_m^{\mathsf{col},(t)}$ as follows:

$$\nu^{h,(t+1)} = \left[\nu^{h,(t)} + \kappa_t\left(\log\chi^h - \log\left(1-\zeta_h^{\mathsf{col},(t)}/(1-\pi^h)\right)\right)\right]^{\mathbb{R}^+}. \qquad (38)$$

CSMA (Carrier Sensing Multiple Access) algorithm where all control messages are exchanged through the CCC. Though our proposed protocols only need loose synchronization and the communications among neighboring nodes on the CCC can facilitate this, we assume a perfect synchronization. Consideration of imperfect synchronization is left for future works. Furthermore, almost current contention-based MAC protocols in traditional wireless networks work well with bi-directional links. However, to achieve more efficient channel utilization we investigate the *interference-based* contention resolution where we have explicitly captured wireless interference among transmitting links in making scheduling decisions. Hence, simultaneous transmissions on scheduled *unidirectional* links would be successful. This would essentially remove the need of having a feedback link for reporting an ACK (Acknowledgment) message.

V. Performance Evaluations

To evaluate OCC-MAC and HCC-MAC, we have developed two separate Monte Carlo simulators which model the interaction of three protocol layers as described in this paper through a simplified CRAHN as Fig. 1. Without loss of generality, we assume the system has 3 frequency bands which are licensed to 3 corresponding pairs of LUs, each band with bandwidth of 20MHz comprises 16 orthogonal sub-bands. Each CU with $P_l^m \in [-10, 20]$dBm and $q_l \in [0.01, 0.9]$ opportunistically accesses all 3 licensed bands characterized by an ON/OFF Markov Chain with the idle probabilities as $\pi^1 = 60\%$, $\pi^2 = 80\%$, and $\pi^3 = 55\%$. The minimum data rate for each source is assumed to be 200bps while the maximum rate is dynamically updated to the achievable link capacity at each time slot. The tolerable collision probabilities of licensed bands are $\mu^1 = 15\%$, $\mu^2 = 8\%$, and $\mu^3 = 6\%$. We choose $G_{MC} = -1.5/\log(5\mathrm{BER})$, for all l where target bit error rate BER$= 10^{-5}$. We further assume that the power spectral density of white noise is -174dBm/Hz at both CU and LU receivers, the path loss exponent is 4, and $\delta = 1m$. For a proportional fair allocation, we choose $U_s(x_s) = \log x_s$ as source's utility function (i.e., $\alpha = 1$) for all CUs. The same step-size $k_t = 10^{-3}/t$ is chosen for both proposed protocols and the error tolerance $\varepsilon = 10^{-5}$. In all experiments, we assume sources and links are time-synchronized and always have up-to-date information to perform the deterministic computations on the optimization parameters.

A. *Efficiency and Fairness of Proposed Algorithms*

Here, we validate the efficiency and optimality of our proposed algorithms (i.e., OCC-MAC and HCC-MAC) against the Fixed-Power Cognitive MAC (FPC-MAC) scheme where we integrate the MAC protocols [15], [16] into our CRAHN scenario with fixed power on each link per sub-band at $P_l^{m,max}$. Then, we then vary the value of $P_l^{m,max}$ to show the negative effect of no power control in FPC-MAC. In this regard, we show the FPC-MAC's performance at the different fixed powers (i.e., $P_l^m \leq P_l^{m,max}$) to compare with our proposed MAC protocols. It is noteworthy that FPC-MAC is the optimal solution of the cross-layer cognitive MAC problem (6) where no power control is considered and the contention relation is location-dependent. Specifically, in FPC-MAC, the collision caused by CUs to the LU's receiver only depends on the predefined distance between the former's transmitter to the latter's receivers. This threshold is derived from the interference threshold setting for LUs in both OCC-MAC protocol and HCC-MAC protocol. Furthermore, in this experiment, for ease of exposition, we also assume that the whole channel experiences the Rayleigh flat fading where the channel gains on all subbands of any band are the same. Therefore, the powers at each link per sub-band within a band are allocated at the same level, however, they may be different over the different bands.

Fig. 4 shows the comparison of efficiency-fairness tradeoff across OCC-MAC, HCC-MAC, and FPC-MAC versus the minimum tolerable interference power, I_l^{th}, at $P_l^{m,max} = 75mW$. In fact, OCC-MAC and HCC-MAC's social welfares globally converge to the fixed optimal points with a negligible gap and they outperform FPC-MAC for all values of I_l^{th}. This is achieved because our proposed protocols balance the interference level and the contention level among cognitive links to best-utilize spectrum opportunities in a unified optimization framework whereas FPC-MAC's fixed interference levels at each links make its location-dependent contention resolution passive. We also observe that the smaller the links' interference power budgets are, the bigger the optimality

TABLE I
PERFORMANCE COMPARISON OF PROTOCOLS VERSUS **CPP**† AT $P_l^{m,max} = 50mW$

Protocols	**SocialWelfare** (CPP)			**TEC*** (CPP)			**NetRevenue** (CPP)		
	CPP = 2	CPP = 3	CPP = 4	CPP = 2	CPP = 3	CPP = 4	CPP = 2	CPP = 3	CPP = 4
OCC-MAC	68.95	68.89	68.87	$8.17W$	$6.80W$	$5.59W$	52.61	48.49	46.51
HCC-MAC	68.89	68.85	68.78	$8.32W$	$6.97W$	$5.75W$	52.25	47.94	45.78
FPC-MAC	68.22	68.22	68.22	$9.60W$	$9.60W$	$9.60W$	49.02	39.42	29.82

* Total Energy Consumption (W); † Cost Per unit of consumed Power (1/W)

TABLE II
PERFORMANCE COMPARISON OF PROTOCOLS VERSUS $P_l^{m,max}$ AT **CPP**† = **5**

Protocols	**SocialWelfare** ($P_l^{m,max}$)			**TEC*** ($P_l^{m,max}$)			**NetRevenue** ($P_l^{m,max}$)		
	$7mW$	$40mW$	$100mW$	$7mW$	$40mW$	$100mW$	$7mW$	$40mW$	$100mW$
OCC-MAC	68.71	68.84	68.91	$1.12W$	$5.00W$	$5.98W$	63.11	43.84	39.01
HCC-MAC	68.67	68.75	68.83	$1.15W$	$5.30W$	$6.32W$	62.92	42.25	37.23
FPC-MAC	68.56	68.20	67.03	$1.20W$	$7.20W$	$21.60W$	62.56	32.20	-40.97

* Total Energy Consumption (W); † Cost Per unit of consumed Power (1/W)

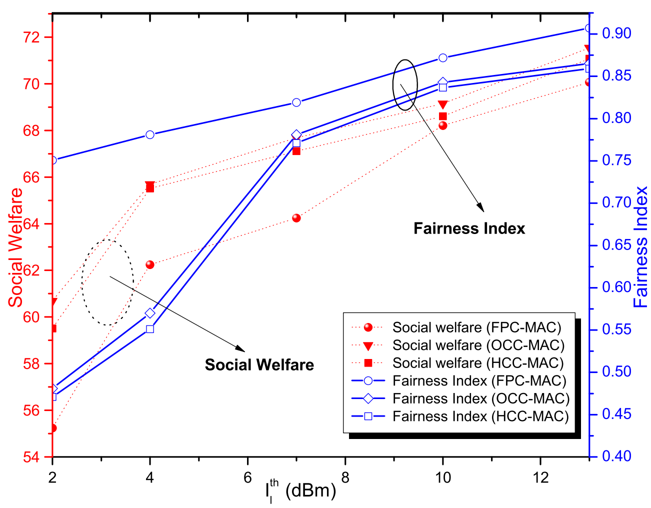


Fig. 4. Efficiency-fairness tradeoff versus I_l^{th}.

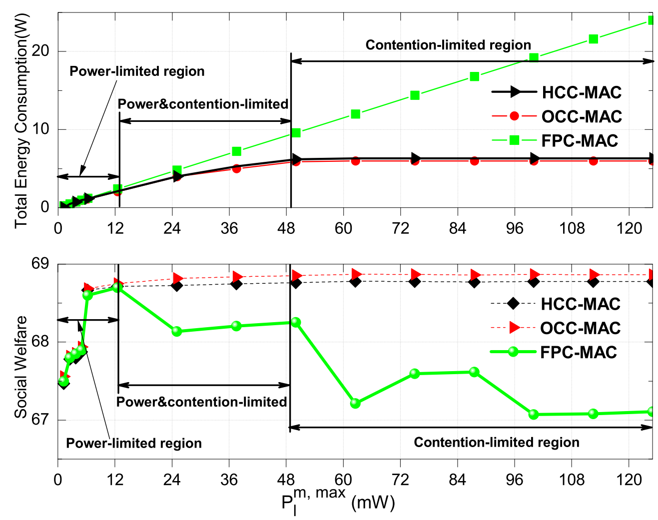


Fig. 5. Saturation effect of social welfare and total energy consumption vs. $P_l^{m,max}$.

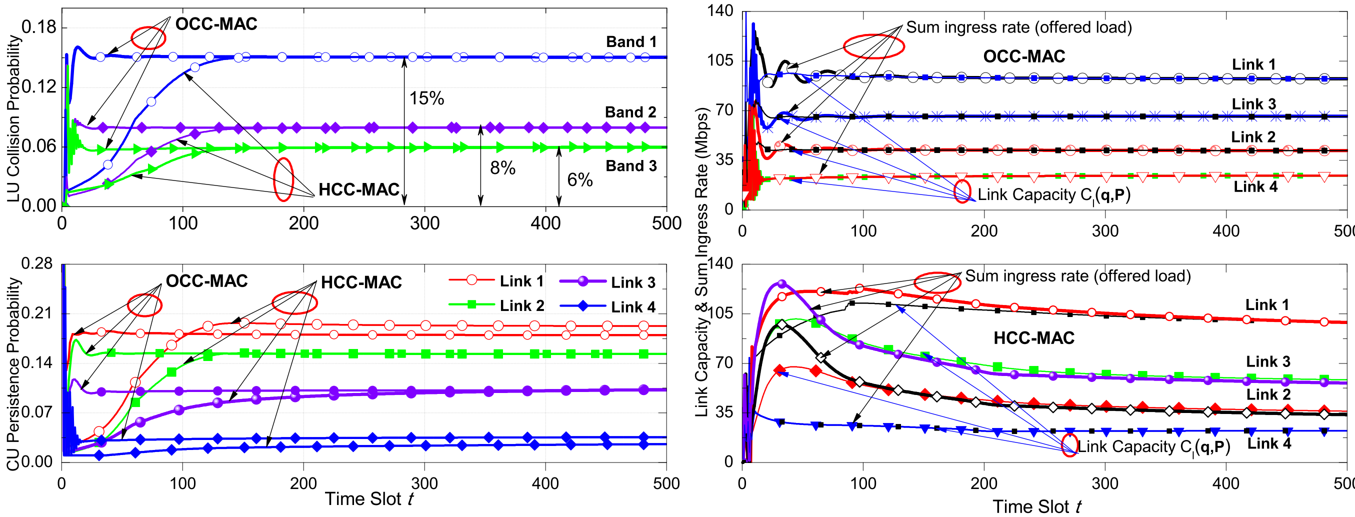
gap between our proposed protocols' and FPC-MAC's social welfare. This is because, in the small interference power budget region, in FPC-MAC, almost cognitive nodes make an adverse interference to each other and hence its performance is significantly worse.

In Table I, we compare the performance of our proposed protocols with FPC-MAC in terms of net revenue versus the different values of CPP. First of all, we can see that the achievable social welfare of FPC-MAC is the lowest but the total energy consumption is the highest in almost all cases. As a consequence, FPC-MAC's net revenue is becoming much worse when CPP inflicted on each unit of consumed power is increasing. In economic aspects, the power resource wastage leads an inefficiency of FPC-MAC. Next, it is straightforward that the higher CPP will force the link power per sub-band to decrease proportionally as pointed out in (18). As a result, in both OCC-MAC and HCC-MAC, the link's reduced total energy consumption due to the increase in CPP forces its congestion price to become higher such that sources must proportionally decrease their rates according to (17). However, the increase in CPP slightly decrease the net revenues of both OCC-MAC and HCC-MAC whereas the FPC-MAC's net revenue is decreased dramatically.

To evaluate fairness, we use Jain's fairness index $f(\boldsymbol{c}) = (\sum_{l=1}^{L} c_l)^2/(L \times \sum_{l=1}^{L} c_l^2)$ [25] where c_l is the lth link's attainable data rate. Again, it is observed from Fig. 4 that all interference-dependent contention-based MAC protocols consistently achieve higher fairness as I_l^{th} increases (i.e., interference constraints are less stringent). Moreover, results from Fig. 4 and 5 also show the efficiency-fairness tradeoff that the resource allocation among links in FPC-MAC is fairer than that in both OCC-MAC and HCC-MAC at much lower efficiency.

B. *Effect of Power Control on Contention-based MAC Protocols' Performance*

In this section, we observe the effect of power control on the contention-based MAC protocols' performance under OSA approach. By tracking the set of other cognitive nodes whose transmissions cause a considerable interference to one cognitive link l and the PU receiver h (i.e., $\mathcal{N}_l^I$ and $\mathcal{N}_{h,0}^I$)



(a) (b)

Fig. 6. (a) Trajectories of CU persistence probabilities and LU tolerable collision probabilities at $I_l^{th} = 10dBm$. (b) Measured link capacity with slowly converging offered load of OCC-MAC and HCC-MAC at $I_l^{th} = 10dBm$.

while increasing $P_l^{m,max}$ at CPP $= 4$ and $I_l^{th} = 10dBm$ for our proposed MAC protocols, we identify the different regions of $P_l^{m,max}$ as follows: *i) power-limited* region in which these sets are almost unchanged for a very low range of $P_l^{m,max}$ and hence social welfare depends only on the optimal power allocation at each cognitive link, *ii) power&contention-limited* region in which these sets become larger for a successive range of $P_l^{m,max}$ and the social welfare depends not only on the optimal transmit power allocation at each cognitive link but also on the optimal contention resolution among cognitive links, and *iii) contention-limited* region in which these sets are unchanged at some cognitive links for a very high range of $P_l^{m,max}$ because the power is reallocated on the basis of increasing power budget at some cognitive link but the total power consumption is maintained thanks to the balancing strategy between capacity supply and rate demand. Hence the social welfare depends only on the optimal contention resolution among cognitive links.

Fig. 5 depicts the comparative performance in terms of social welfare and total energy consumption as $P_l^{m,max}$ increases. The social welfare of both OCC-MAC and HCC-MAC is significantly improved with a higher power budget and getting saturated as $P_l^{m,max}$ is greater than a certain threshold, 48mW. We also observe that for a very low $P_l^{m,max}$ (i.e., power-limited regime), the FPC-MAC's social welfare is slightly worse, but an further increase in $P_l^{m,max}$ (i.e., power & contention-limited regime) provides a great opportunity for our proposed protocols to dramatically outperform FPC-MAC. The key reason is that cognitive links in OCC-MAC and HCC-MAC jointly adjust their powers and probabilities to balance between interference and contention. More specifically, for a very high $P_l^{m,max}$ (i.e., contention-limited regime), the FPC-MAC's social welfare seriously degrades whereas our proposed protocols are moving towards stable state of lower potential energy. This is because, in both OCC-MAC and HCC-MAC, any further increases in power which force the interference weights and the number of interference edges for a cognitive link to increase, the balance policy between capacity supply and rate demand guides the link powers to stability region while FPC-MAC with no power control can cause an unchangeable and considerable harmful interference among links, then the channel contention becomes heavily dense. Consequently, the capacities of interfered links are dramatically reduced because of much more inflicted contention as shown in (3). We can clearly see this from observing Fig. 5 that FPC-MAC consumes a huge amount of energy and degrades as the link powers are in contention-limited regime.

To further observe the benefits of OCC-MAC and HCC-MAC over FPC-MAC, let us consider the net revenue at a fixed CPP. Table II shows the comparative performance of protocols for three different values of $P_l^{m,max}$ at CPP$= 5$. At $P_l^{m,max} = 7mW$, their performance gap is almost negligible. However, their performance gap gradually becomes bigger at $P_l^{m,max} = 40mW$. When $P_l^{m,max}$ falls into contention-limited regime (i.e., $P_l^{m,max} = 100mW$), the FPC-MAC's net revenue becomes negative. However, the equilibrium solution HCC-MAC still achieves a good performance compared with OCC-MAC.

C. Cross-Layer Adaptive Control for Stochastic Spectrum Opportunities

In this section, let us turn our attention on how CUs can opportunistically exploit spectrum holes for stochastic configuration settings through our proposed approach. It is observed from Fig. 6a that the cognitive links in our proposals (i.e., OCC-MAC and HCC-MAC) can optimally adjust their persistence probabilities in such a way that their achievable capacities can meet the total ingress rate. The convergence curves of persistence probabilities in Fig. 6a also show that in HCC-MAC the links' behavior for the persistence probability

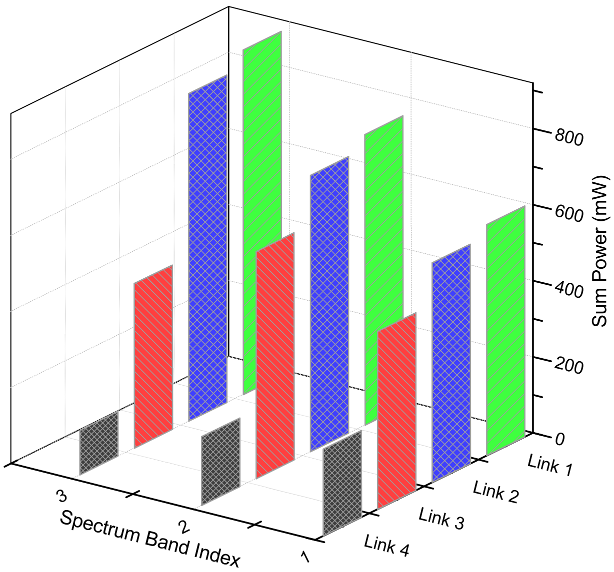


Fig. 7. Optimal powers at $I_l^{th} = 10dBm$.

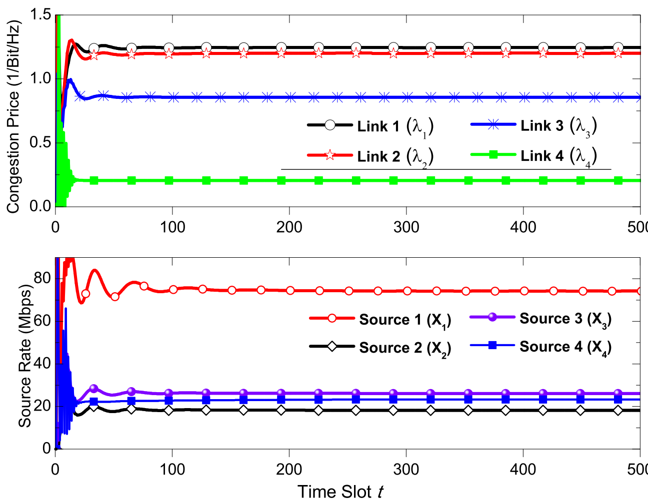


Fig. 8. Convergence of congestion price and source rate at $I_l^{th} = 10dBm$.

adjustment is totally different from these in OCC-MAC. This is because the links in HCC-MAC control their access channel probabilities on the *direct* balance basis of total ingress rate and instant protocol capacity provided that they must satisfy the LU collision constraints. Hence, the convergence curves of persistence probabilities have the same tendency to increase. In contrary, in OCC-MAC, the links must regulate their persistence probabilities on the basis of the law of diminishing returns via pricing strategy. As a result, in OCC-MAC, the behavior of one link in the persistence probability adjustment may be different from those of the other links.

To visualize the optimal behaviors of both OCC-MAC and HCC-MAC, in Fig. 6b we illustrate the balance process between demand (i.e., sum ingress rate) and supply (i.e., attainable link capacity) for congestion control at each cognitive

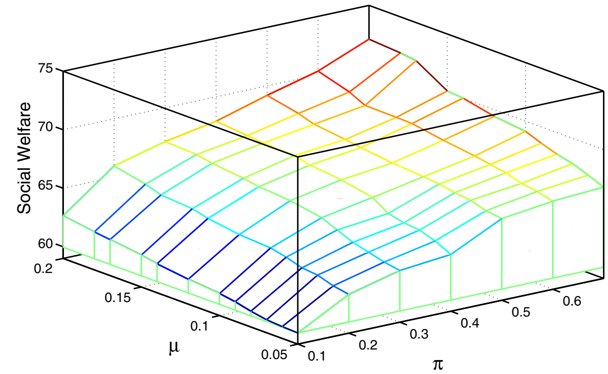


Fig. 9. Social welfare of OCC-MAC versus the LU1's idle probability π^1 versus the LU1's tolerable collision probability μ^1 while fixing the other LUs' spectrum opportunities.

link. In OCC-MAC, at each link, the convergence curves of sum ingress rate and link capacity are totally different. This is because the sources and links alternatively control their parameters in an independent and distributed manner via the feedback of prices, leading a globally optimal point. However, in HCC-MAC, the link capacity follows asymptotically the measured offered load. Such a way of adaptive control may take HCC-MAC a longer time to converge to the equilibrium point with no control overhead.

As can clearly seen from Fig. 7, for an interference balance among links and spectrum best-utilization, the 1st link's optimal powers are the highest whereas the 4th link's powers are the lowest. This is because the spectrum opportunities at the 1st link are the most potential and its interference effect to the other links is the lowest. In addition, the 1st link needs to carry a large amount of data from sources 1 and 2. Similarly, for congestion balance in a proportional-fair fashion, source 1 (i.e., x_1) can potentially achieve the highest data rate while source 2's rate (i.e., x_2) is the lowest. This is because source 2 traverses the most number of hops (i.e., links 1, 2, and 3) in which the bottleneck links 2 and 3 have the smallest spectrum opportunities, the largest interference effect on the other links. As can be shown in Fig. 8, thanks to the link's pricing strategy through the balance between sum ingress rate and attainable link capacity which totally depends on the balance of interference and contention among links (see Fig. 6a for the contention balance and Fig. 7 for the interference balance), sources regulates their rates on the basis of congestion prices and converge to the optimal values.

Fig. 6a also shows that the LUs' collision probabilities of both OCC-MAC and HCC-MAC converge to the corresponding target thresholds $\mu^1 = 15\%$, $\mu^2 = 8\%$, and $\mu^3 = 6\%$. In this observation, we confirm that OCC-MAC and HCC-MAC can optimally utilize the spectrum holes left by LUs even in LUs' presence without degrading primary system's QoS if LUs can slightly tolerate their specific levels of collision and suffer from a certain interference. In fact, it is interesting to find from Fig. 9 that the cognitive sources and cognitive links in our cross-layer MAC protocols can adaptively control their parameters to get more social welfare when the stochastic spectrum opportunities $(\boldsymbol{\pi}, \boldsymbol{\mu})$ are further relaxed.

VI. Conclusion

In this paper, a cross-layer MAC framework for contention-based multi-hop CRAHNs is proposed. We specifically consider the co-existence of both licensed and unlicensed users under the collision-rate-constrained stochastic spectrum opportunities. We show that our cross-layer MAC protocols which perform the *interference-dependent* contention resolution significantly outperform those existing MAC protocols. Moreover, our proposed MAC protocols can achieve the best energy efficiency and the optimal social welfare by balancing three barriers of congestion, interference, and contention among links and sources over all dynamic spectrum bands. They treat all CUs in an α-fairness manner via a pricing strategy. Importantly, we provide the trade-off between efficiency in net revenue and scalability in implementation through OCC-MAC and HCC-MAC.

Appendix A
Proof of Proposition 1

We first prove that $h(\hat{\boldsymbol{q}}) = \log\big(1 - \sum_{l\in L_{out}(n)} e^{\hat{q}_l}\big)$ is a concave function. In this regard, we need to prove that the first derivative $\frac{\partial h(\mathbf{x})}{\partial x_i} \geq 0, \forall i$ for the decreasing property and the Hessian matrix $\nabla^2 h(\mathbf{x})$ is a negative semi-definite matrix for concavity. We have the following derivatives $\forall l \neq k$: $\frac{\partial h(\hat{\boldsymbol{q}})}{\partial \hat{q}_l} = \frac{-e^{\hat{q}_l}}{1-\sum_l e^{\hat{q}_l}}$, $\frac{\partial^2 h(\hat{\boldsymbol{q}})}{\partial \hat{q}_l^2} = -\frac{e^{2\hat{q}_l}}{\left(1-\sum_l e^{\hat{q}_l}\right)^2} - \frac{e^{\hat{q}_l}}{1-\sum_l e^{\hat{q}_l}}$, and $\frac{\partial^2 h(\hat{\boldsymbol{q}})}{\partial \hat{q}_l \partial \hat{q}_k} = -\frac{e^{\hat{q}_l+\hat{q}_k}}{\left(1-\sum_l e^{\hat{q}_l}\right)^2}$. Let us denote $z_i = \frac{-\partial h(\hat{\boldsymbol{q}})}{\partial \hat{q}_l} = \frac{e^{\hat{q}_l}}{1-\sum_l e^{\hat{q}_l}} \geq 0, \forall i \in [1,2,...,|L^n_{out}|]$, the Hessian matrix $\nabla^2 h(\hat{\boldsymbol{q}})$ can be shown as follows:

$$\nabla^2 h(\hat{\boldsymbol{q}}) = \begin{pmatrix} -z_1^2 - z_1 & -z_1 z_2 & \cdots \\ -z_2 z_1 & -z_2^2 - z_2 & \cdots \\ \vdots & \vdots & \ddots \end{pmatrix}.$$

For all $\mathbf{v}$ that has the same dimension as $\hat{\boldsymbol{q}}$, we have

$$\mathbf{v}^T \nabla^2 h(\hat{\boldsymbol{q}}) \mathbf{v} = -\sum_i z_i v_i^2 - \left(\sum_i z_i v_i\right)^2 \leq 0.$$

Hence, $\nabla^2 h(\hat{\boldsymbol{q}})$ is a negative semi-definite matrix. Similarly, the constraints in (29) are convex in $\hat{\boldsymbol{q}}$-space. This completes the proof.

Appendix B
Proof of Theorem 3

Proof: As proved in Algorithm 1, the rate updates (17), the power updates (18) optimally solve the rate control and power control subproblems in (16). Moreover, the MAC algorithm (30) solving the approximate MAC subproblem (28) always converges to the unique optimum [Proposition 2]. Hence, Algorithm 2 totally solves the original problem (6) and converge to the fixed point. Now, we use the stochastic Lyapunov function method to prove the optimality property of HCC-MAC. For the dual problem $D(\boldsymbol{\lambda}, \boldsymbol{\nu})$ in (15), we establish the Lyapunov function $V(\boldsymbol{\lambda}, \boldsymbol{\nu}) = \|\boldsymbol{\lambda} - \boldsymbol{\lambda}^*\|_2^2 + \|\boldsymbol{\nu} - \boldsymbol{\nu}^*\|_2^2$, where $(\boldsymbol{\lambda}^*, \boldsymbol{\nu}^*)$ is the dual optimum. By using congestion prices and spectrum prices in both OCC-MAC and HCC-MAC, we have

$$\|\boldsymbol{\lambda}^{(t+1)} - \boldsymbol{\lambda}^*\|_2^2 = \|\boldsymbol{\lambda}^{(t)} - \kappa_t \boldsymbol{Z}^{(t)} - \boldsymbol{\lambda}^*\|_2^2 \leq \|\boldsymbol{\lambda}^{(t)} - \boldsymbol{\lambda}^*\|_2^2 - 2\kappa_t[\boldsymbol{\lambda}^{(t)} - \boldsymbol{\lambda}^*]^T \boldsymbol{Z}^{(t)} + \kappa_t^2 \|\boldsymbol{Z}^{(t)}\|_2^2;$$

$$\|\boldsymbol{\nu}^{(t+1)} - \boldsymbol{\nu}^*\|_2^2 = \|\boldsymbol{\nu}^{(t)} - \kappa_t \boldsymbol{Q}^{(t)} - \boldsymbol{\nu}^*\|_2^2 \leq \|\boldsymbol{\nu}^{(t)} - \boldsymbol{\nu}^*\|_2^2 - 2\kappa_t[\boldsymbol{\nu}^{(t)} - \boldsymbol{\nu}^*]^T \boldsymbol{Q}^{(t)} + \kappa_t^2 \|\boldsymbol{Q}^{(t)}\|_2^2$$

where

1) OCC-MAC: $Z^{(t)}_{\mathsf{OCC}} = \log(c_l(\mathbf{q}^{(t)}, P_l^{(t)}) / \sum_{s\in\mathcal{S}_l} x_s^{(t)})$; $Q^{(t)}_{\mathsf{OCC}} = \sum_{n\in\mathcal{N}^I_{0,m}} \log(1 - \Upsilon_n^{(t)}) - \log \chi^m$.
2) HCC-MAC: $Z^{(t)}_{\mathsf{HCC}} = \log(\tilde{c}_l(\mathbf{q}^{(t)}, P_l^{(t)}) / \sum_{s\in\mathcal{S}_l} x_s^{(t)})$; $Q^{(t)}_{\mathsf{HCC}} = \log\big(1 - \tilde{\zeta}_m^{col,(t)}/(1-\pi^m)\big) - \log \chi^m$.

We obtain the following bound for Lyapunov drift of HCC-MAC:

$$V(\boldsymbol{\lambda}^{(t+1)}, \boldsymbol{\nu}^{(t+1)}) - V(\boldsymbol{\lambda}^{(t)}, \boldsymbol{\nu}^{(t)}) \leq -2\kappa_t\big([\boldsymbol{\lambda}^{(t)} - \boldsymbol{\lambda}^*]^T \boldsymbol{Z}^{(t)}_{\mathsf{HCC}} + [\boldsymbol{\nu}^{(t)} - \boldsymbol{\nu}^*]^T \boldsymbol{Q}^{(t)}_{\mathsf{HCC}}\big) + \kappa_t^2(\|\boldsymbol{Z}^{(t)}_{\mathsf{HCC}}\|_2^2 + \|\boldsymbol{Q}^{(t)}_{\mathsf{HCC}}\|_2^2). \tag{39}$$

In HCC-MAC, since $\sum_{s\in\mathcal{S}_l} x_s^{(t)} = \tilde{c}_l(\mathbf{q}^{(t+1)}, P_l^{(t)})$ and $\mathbf{q}^{(t+1)}$ is a monotonic decreasing sequence, we have

$$\sum_{s\in\mathcal{S}_l} x_s^{(t)} = \tilde{c}_l(\mathbf{q}^{(t+1)}, P_l^{(t)}) \leq \tilde{c}_l(\mathbf{q}^{(t)}, P_l^{(t)}) \leq c_l(\mathbf{q}^{(t)}, P_l^{(t)}).$$

The last inequality follows from the facts that $\mathbf{q}^{(t)}$ in $\tilde{c}_l(\mathbf{q}^{(t)}, P_l^{(t)})$ of HCC-MAC is obtained from an approximate MAC subproblem (28). Then, we have $\boldsymbol{Z}^{(t)}_{\mathsf{HCC}} = \boldsymbol{Z}^{(t)}_{\mathsf{OCC}} - \boldsymbol{\Theta}^{(t)}$, where $\Theta^{(t)} \succeq 0$. In this regard, we further assume that noisy estimation in HCC-MAC does not affect its optimality, i.e., $\boldsymbol{Q}^{(t)}_{\mathsf{OCC}} \approx \boldsymbol{Q}^{(t)}_{\mathsf{HCC}}$. Using these facts for (39), we obtain

$$V(\boldsymbol{\lambda}^{(t+1)}, \boldsymbol{\nu}^{(t+1)}) - V(\boldsymbol{\lambda}^{(t)}, \boldsymbol{\nu}^{(t)}) \leq -2\kappa_t\big([\boldsymbol{\lambda}^{(t)} - \boldsymbol{\lambda}^*]^T \boldsymbol{Z}^{(t)}_{\mathsf{OCC}} + [\boldsymbol{\nu}^{(t)} - \boldsymbol{\nu}^*]^T \boldsymbol{Q}^{(t)}_{\mathsf{OCC}}\big) + 2\kappa_t\big([\boldsymbol{\lambda}^{(t)} - \boldsymbol{\lambda}^*]^T \boldsymbol{\Theta}^{(t)}\big) + \kappa_t^2(G^2_{\mathsf{HCC}} + G'^2_{\mathsf{HCC}}), \tag{40}$$

where we assume $\|\boldsymbol{Z}^{(t)}_{\mathsf{HCC}}\|_2^2 \leq G^2_{\mathsf{HCC}}$ and $\|\boldsymbol{Q}^{(t)}_{\mathsf{HCC}}\|_2^2 \leq G'^2_{\mathsf{HCC}}$. Since $g(\boldsymbol{\lambda}, \boldsymbol{\nu})$ is convex in $(\boldsymbol{\lambda}, \boldsymbol{\nu})$ space; and both $\boldsymbol{Z}^{(t)}_{\mathsf{OCC}}$ and $\boldsymbol{Q}^{(t)}_{\mathsf{OCC}}$ are the partial gradients of $g(\boldsymbol{\lambda}, \boldsymbol{\nu})$ at $(\boldsymbol{\lambda}^{(t)}, \boldsymbol{\nu}^{(t)})$, we have:

$$V(\boldsymbol{\lambda}^{(t+1)}, \boldsymbol{\nu}^{(t+1)}) - V(\boldsymbol{\lambda}^{(t)}, \boldsymbol{\nu}^{(t)}) \leq -2\kappa_t\big(g(\boldsymbol{\lambda}, \boldsymbol{\nu}) - g(\boldsymbol{\lambda}^*, \boldsymbol{\nu}^*)\big) + \kappa_t^2(G^2_{\mathsf{HCC}} + G'^2_{\mathsf{HCC}}) + 2\kappa_t\big([\boldsymbol{\lambda}^{(t)} - \boldsymbol{\lambda}^*]^T \boldsymbol{\Theta}^{(t)}\big). \tag{41}$$

By taking expectation of (41) over $(\boldsymbol{\lambda}, \boldsymbol{\nu})$, we get

$$E[V(\boldsymbol{\lambda}^{(t+1)}, \boldsymbol{\nu}^{(t+1)}) - V(\boldsymbol{\lambda}^{(t)}, \boldsymbol{\nu}^{(t)})] \leq -2\kappa_t\big(E[g(\boldsymbol{\lambda}, \boldsymbol{\nu})] - g(\boldsymbol{\lambda}^*, \boldsymbol{\nu}^*)\big) + \kappa_t^2(G^2_{\mathsf{HCC}} + G'^2_{\mathsf{HCC}}) + 2\kappa_t E[\big([\boldsymbol{\lambda}^{(t)} - \boldsymbol{\lambda}^*]^T \boldsymbol{\Theta}^{(t)}\big)]. \tag{42}$$

Summing (42) from $\tau = 0$ to $\tau = t$, we obtain

$$\frac{1}{t}\sum_{\tau=0}^{t} E[g(\boldsymbol{\lambda}^{(\tau)},\boldsymbol{\nu}^{(\tau)})] - g(\boldsymbol{\lambda}^*,\boldsymbol{\nu}^*) \le \frac{E[V(\boldsymbol{\lambda}^{(1)},\boldsymbol{\nu}^{(1)})]}{2t\sum_{\tau=0}^{t}\kappa_\tau} + \frac{\sum_{\tau=0}^{t}\kappa_\tau(G^2_{\mathsf{HCC}} + G'^2_{\mathsf{HCC}})}{2t} + \frac{\sum_{\tau=0}^{t} E[([\boldsymbol{\lambda}^{(\tau)} - \boldsymbol{\lambda}^*]^T\boldsymbol{\Theta}^{(\tau)})]}{t}. \quad (43)$$

where the inequality follows the fact that $E[V(\boldsymbol{\lambda}^{(t+1)},\boldsymbol{\nu}^{(t+1)})] \ge 0$. For some sufficiently large t, we have

$$\lim_{t\to\infty}\frac{1}{t}\sum_{\tau=0}^{t} E[g(\boldsymbol{\lambda}^{(\tau)},\boldsymbol{\nu}^{(\tau)})] - g(\boldsymbol{\lambda}^*,\boldsymbol{\nu}^*) \le \frac{\kappa_\infty}{2}(G^2_{\mathsf{HCC}} + G'^2_{\mathsf{HCC}}) + E[([\boldsymbol{\lambda}^{(\infty)} - \boldsymbol{\lambda}^*]^T\boldsymbol{\Theta}^{(\infty)})].$$

$$\Leftrightarrow E[g(\boldsymbol{\lambda}^{(\infty)},\boldsymbol{\nu}^{(\infty)})] - g(\boldsymbol{\lambda}^*,\boldsymbol{\nu}^*) \le \frac{\kappa_\infty}{2}(G^2_{\mathsf{HCC}} + G'^2_{\mathsf{HCC}}) + E[([\boldsymbol{\lambda}^{(\infty)} - \boldsymbol{\lambda}^*]^T\boldsymbol{\Theta}^{(\infty)})]. \quad (44)$$

Since $g(\boldsymbol{\lambda},\boldsymbol{\nu})$ is a convex function, by Jensens inequality, we can rewrite (44) as:

$$g(E[\boldsymbol{\lambda}^{(\infty)},\boldsymbol{\nu}^{(\infty)}]) - g(\boldsymbol{\lambda}^*,\boldsymbol{\nu}^*) \le \frac{\kappa_\infty}{2}(G^2_{\mathsf{HCC}} + G'^2_{\mathsf{HCC}}) + \rho, \quad (45)$$

where $E[([\boldsymbol{\lambda}^{(\infty)} - \boldsymbol{\lambda}^*]^T\boldsymbol{\Theta}^{(\infty)})] = \rho$. Similarly, in OCC-MAC, we can also obtain:

$$g(E[\boldsymbol{\lambda}^{(\infty)},\boldsymbol{\nu}^{(\infty)}]) - g(\boldsymbol{\lambda}^*,\boldsymbol{\nu}^*) \le \frac{\kappa_\infty}{2}(G^2_{\mathsf{OCC}} + G'^2_{\mathsf{OCC}}). \quad (46)$$

Hence, the algorithms OCC-MAC and HCC-MAC converge statistically to within $\kappa_\infty(G^2_{\mathsf{OCC}} + G'^2_{\mathsf{OCC}})/2$ and $\kappa_\infty(G^2_{\mathsf{HCC}} + G'^2_{\mathsf{HCC}})/2 + \rho$ of the optimal value $g(\boldsymbol{\lambda}^*,\boldsymbol{\nu}^*)$, respectively. Then, their optimal gap is $(\kappa_\infty\|\boldsymbol{\Theta}\|_2^2/2+\rho)$ which is negligible because ρ and $\|\boldsymbol{\Theta}\|_2^2$ are very small. ∎

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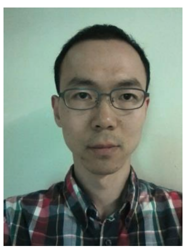

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