Cross-Layer Optimization for Congestion and Power Control in OFDM-Based Multi-Hop Cognitive Radio Networks

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***Abstract*—Efficient and fair power allocation associated with congestion control in orthogonal frequency division multiplexing (OFDM)-based multi-hop cognitive radio networks (CRNs) is a challenging and complicated problem. In this paper, we consider their mutual relationship through a cross-layer optimization design that addresses both aggregate utility maximization and energy consumption minimization. By introducing the unique outage constraint of primary user (PU) protection, the joint congestion control and power control (JCPC) formulation is shown to be a nonlinear non-convex optimization problem. Using dual decomposition approach, we first propose a distributed algorithm that can attain the optimal solution via message passing while maintaining the architectural modularity between the layers. Next, we develop a suboptimal algorithm using a new heuristic method to alleviate the overhead burden of the first solution. Finally, the numerical results confirm that the OFDM-based multi-hop CRNs can optimally exploit the spectrum opportunity if the PU outage probability is kept below the target.**

***Index Terms*—Cross-layer optimization, congestion control, power control, outage probability, message passing, CRNs.**

I. Introduction

COGNITIVE Radio, a new communication paradigm for more efficient utilization of radio spectrum, has been attracting substantial attention from the wireless communication community. In fact, CRNs are based on the principles of spectrum sensing and dynamic spectrum access. However, as recently proposed by many researchers, the secondary users (SUs) may simultaneously perform their transmission over the licensed bands providing that the harmful interference introduced by them to the PU receiver (Rx) is below an acceptable threshold, known as spectrum underlay [1]. In a spectrum underlay environment, one of major challenges of a multi-hop CRN with the interference-limited link capacities are how to optimally resolve the mutual relationship between congestion control and power control in order to improve the overall network utility while guaranteeing the protection of the PUs from the excessive interference introduced by the SUs. More importantly, the network control algorithm design must maintain its modular layer architecture. However, harmful interference and congestion due to the concurrent use of common-use radio bands and links become obstacles to overcoming those challenges. As a consequence, the overall performance of a multi-hop CRN calls for a cross-layer design and an optimization approach. The major motivation of this paper is the provision of high total utility for the SUs and the preservation of architectural modularity of Internet Protocol (IP) via a cross-layer methodology without degrading the performance of the PUs.

The basic idea behind spectrum pooling [2] is merging dynamic spectral ranges from different spectrum owners into a common pool for secondary use during PU's under-utilized and/or idle periods. OFDM has already received tremendous recognition as a promising candidate for the SUs' transmission in cognitive radio systems, mainly due to its robustness against multipath fading and subcarrier structure allowing the adaption of transmission parameters. In OFDM-based CRNs, the unused subcarriers left by the PUs can be exploited by the SUs in an *interweave* fashion [2]–[5]. However, the SUs can adapt their transmission parameters into any subcarriers in an *underlay* fashion if the aggregate interference inflicted on the PU receivers remains below the acceptable threshold [6]–[11].

Resource allocation in OFDM-based CRNs has been extensively studied. Most studies [3]–[5], [7] consider maximizing the SUs' throughput with an adjacent channel interference (ACI) constraint to the PUs due to their coexistence in side-by-side bands. However, our scope focuses on the second approach [8], in which both ACI constraint and common channel interference (CCI) constraint due to coexistence of both PUs and SUs in the same band are taken into consideration. Some recent works ([3], [9]–[11]) addressed the problem of total capacity optimization under OFDM-based CRNs and proposed different ways to protect the PUs. In [9], Chen *et al.* investigated the joint subcarrier and power allocation for uplink OFDM-based CRNs with a peak power constraint in order to protect the PUs. By introducing an interference temperature constraint for guaranteeing PUs' quality of service (QoS), the authors [6] proposed an optimal subcarrier and power allocation algorithm to maximize the overall utility for SUs. For situations in which the PU's channel state information (CSI) is unavailable at the SU's transmitter, a

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hybrid of the new rate loss constraint and the conventional interference power constraint was proposed for primary transmission protection [11]. Through an optimal power allocation strategy, Kang *et al.* in [11] showed that a significant rate gain under the hybrid protection constraint can be achieved as compared to that obtained under the conventional interference power constraint.

It is implicitly understood that most constraints of interference temperature, interference power, or rate loss are linked directly with a channel's deterministic state (i.e., both slow-fading and fast-fading channel gains are fixed). Hence, for each secondary-to-primary channel state, power allocation algorithms must be performed twice in order to seek a new optimal solution satisfying these constraints. This makes the message passing-based distributed algorithms infeasible and unscalable with fast-fading. In [10], Son *et al.* addressed this problem by proposing the optimal and suboptimal algorithms to maximize the capacity of a single pair of SUs in a simple OFDM-based cognitive system under the interference-power outage constraint to protect the PUs. However, even in the absence of the SUs, the PUs's transmission may be failed by severe fading channels. To cope with this effect, the outage probability which is defined as the fraction of time a transmitter/receiver pair experiences an outage over fading blocks [12] and [13], is introduced in this paper as the target constraint to protect the PU's transmission using spectrum underlay approach. As a result, it is not necessary to update the optimum whenever the primary-related fading channels change their states. In particular, in this work, we present a cross-layer optimization framework for multi-hop CRNs by investigating the joint congestion and power control problem via Network Utility Maximization (NUM) [14]. Our contributions are summarized as follows:

- A cross-layer framework is developed to address both congestion control and power control in OFDM-based multi-hop CRNs as a nonlinear non-convex optimization problem. Unlike previous works, the use of outage constraint to protect the PUs allows the SUs to adaptively adjust their transmission parameters (i.e., transmit power, spreading gain, and transmission rate) without depending on the dynamic fading channel of the PU-related links.
- We propose two distributed algorithms for the problem of JCPC to maximize the SUs' net revenue. The first algorithm can obtain the global optimum with message passing while the second solution is sub-optimal without explicit message passing.
- In order to maintain the architectural modularity between layers, we show that our proposed algorithms can couple with the congestion control mechanism of the existing transmission control protocol (TCP) to obtain the spectrum utilization and increase the secondary system's aggregate throughput.

It is important to emphasize that the adaptation of dual decomposition methodology having been used in [15] and in the vast, existing literature to disjoint the global dependence of primal variables for our NUM-based optimization problem of JCPC is not a *trivial* application due to essentially different scenarios of CRNs in terms of dynamic spectrum, multi-carriers, spectrum incumbent protection, and network model. In addition, the unique outage constraint of PU protection in this paper makes our optimization problem very challenging and therefore demands new solutions.

The rest of paper is organized as follows. Section II presents the system model and problem formulation. Section III introduces the optimally distributed JCPC algorithm based on the primal-dual method. In Section IV, we present a new heuristic method to develop a sub-optimal JCPC algorithm in order to obtain scalability as well as preservation of the TCP's congestion mechanism with no explicit message passing. We present numerical results to illustrate the performance of our proposed algorithms in Section V, and finally conclude the paper in Section VI.

II. System Model and Problem Formulation

We consider a multi-hop CRN in which the combination of code division multiple access (CDMA) and multicarrier transmission technology OFDM, so-called MC-CDMA [16], is adopted at the physical layer over each hop, such that multiple secondary transmissions can concurrently occur in a common frequency band. The entire spectrum that can potentially be used by SUs is divided into M narrow-band flat-fading subcarriers, and each has a bandwidth of W_m. These subcarriers are grouped into K subchannels that are correspondingly licensed to K primary links (not necessarily OFDM) as shown in Fig.1. Note that we also use m_k to denote the index of the first subcarrier in the k^{th} subchannel. All SUs, equipped with cognitive radios, can perform simultaneous two-way information transfer over a common wide-band channel known as division free duplex (DFD) [17]. A set of links $\mathcal{L} = \{1, ..., L\}$ logically formed among SUs are shared by a set of sources $\mathcal{S} = \{1, ..., S\}$. Suppose that source $s \in \mathcal{S}$ traverses multiple hops on its path to reach the target destination through the pre-established set of links, $L(s) \subseteq \mathcal{L}$. We also assume that the SUs' buffers are infinite and sources always have data to send such that delay constraints are ignored. In addition, the source s regulates its transmission rates in response to congestion levels within the network. This is typically done by using a utility $U_s(x_s) : \mathbb{R}_+ \rightarrow \mathbb{R}$, which can be interpreted as the level of satisfaction attained by a source at the allocated rate $x_s \in [x_s^{min}, x_s^{max}]$. This utility function is assumed to be twice continuously differentiable, non-decreasing, and strictly concave.

A. Multiuser Frequency-Selective Fading Model and Link Capacity Constraint

We consider additive Gaussian noise with power η_l as background noise at the receiver of each link l. The instantaneous signal-to-interference ratio (SIR), $\gamma_l^m(\mathbf{P}^m)$, at link l on subcarrier m is expressed as

$$\gamma_l^m(\mathbf{P}^m) = \frac{S_{ll}^m P_l^m}{\eta_l + \sum_{h \neq l} S_{lh}^m P_h^m + I_{l0}^m}, \quad (1)$$

where $\mathbf{P}^m = [P_1^m, ..., P_L^m]$ is a vector of secondary link powers on subcarrier m and $I_{l0}^m = \sum_{k=1}^K I_{lk}^m$ is the total interference introduced by PU-transmitters into the mth subcarrier on link l. We use the special index of $l = 0$ to

TABLE I
IMPORTANT NOTATION

Symbol	Definition
L	Number of seconday links
$\mathcal{L}$	Set of secondary links, $L =\mid \mathcal{L} \mid$
K	Number of subchannels or primary links
$\mathcal{K}$	Set of primary links, $K =\mid \mathcal{K} \mid$
M	Number of subcarriers
$\mathcal{M}$	Set of subcarriers, $M =\mid \mathcal{M} \mid$
W_m	Subcarrier bandwidth
m_k	Index of the first subcarrier in the k^{th} subchannel
S	Number of data sources
$\mathcal{S}$	Set of data sources , $S =\mid \mathcal{S} \mid$
$L(s)$	Set of links on the path of source s
$S(l)$	Set of sources using link l
x_s	Transmssion rate of source s
P_l^m	Power of link l on subcarrier m
γ_l^m	SIR at SU-Rx of link l on subcarrier m
γ_0^k	SIR at the kth PU-Rx
$\beta_l^{m,k}$	Interference factor of link l to PU-Rx k on subcarrier m
ζ_{th}^k	The k^{th} PU-Rx's outage probability threshold
γ_{th}^k	The k^{th} PU-Rx's SIR threshold
γ_0^k	The k^{th} PU-Rx's outage probability in SUs' absence
CPP	Cost Per unit of Consumed Power

denote the primary link. We assume that the interference I_{lk}^m, introduced by the kth PU into the mth subcarrier on link l, can be modeled as white noise, which can be estimated by integrating the power spectral density (PSD) of the PU signal across the mth subcarrier [3]. $S_{lk}^m = G_{lk}^m F_{lk}^m$, where G_{lk}^m and F_{lk}^m represent the large-scale channel gain and random small-scale channel fading gain between the kth link's transmitter and the lth link's receiver on subcarrier m, respectively. In this paper, we assume that G_{lk}^m only depends on the physical link distance d_{lk} with the path loss exponent n, i.e., $G_{lk}^m = d_{lk}^{-n}$. All channel fading gains F_{lk}^m are assumed to be independent and identically distributed (i.i.d) random variables (RVs). We further assume that F_{lk}^m is not dependent on all statistical variations of both signal and noise power in each power adaption interval. Hence, it remains constant during the power adaption interval but may vary over the time scale of interest.

The instantaneous capacity of link l modeled on the Shannon capacity is a global and nonlinear function of the transmit power vector $\mathbf{P} = (\mathbf{P}^m, m \in \mathcal{M})$.

$$C_l(\mathbf{P}) = W_m \sum_{m}^{M} \ln\left(1 + G_{MC}\gamma_l^m(\mathbf{P}^m)\right). \tag{2}$$

Here, constant $G_{MC} = -\phi_1/\log(\phi_2 \text{ BER})$ denotes the processing gain, where ϕ_1 and ϕ_2 are constants depending on the modulation method, coding scheme, and bit-error rate (BER) [18]. In the MC-CDMA systems, it is assumed that G_{MC} and the number of subcarriers M are equal [16]. In this case, $G_{MC}\gamma_l^m(\mathbf{P}^m)$ is much greater than one. Therefore, the link capacity can be approximated as $\sum_m^M \ln(\gamma_l^m(\mathbf{P}^m))$, where henceforth G_{MC} is absorbed into G_{ll} and W_m is assumed to be unit without loss of generality. At link l, the ingress rate should not exceed its link capacity, i.e.,

$$\sum_{s \in S(l)} x_s \leq C_l(\mathbf{P}), \forall l, \tag{3}$$

where $S(l) = \{s : l \in L(s)\}$ is the set of sources using link l.

B. PU Outage Constraint

We assume that the radio transmission environment between SU-transmitter (Tx) and the PU-Rx is non-line-of-sight. In this case, we can employ a Rayleigh fading channel model, where the small-scale channel fading gains $F_{0l}^{m_k}$ among the SU transmitters and the PU receivers follow an independent exponential distribution with unit mean. We further assume that the PSD of the mth subcarrier on link l can be modeled as an ideal Nyquist pulse [3]:

$$\phi_l^m(f) = P_l^m T_m \left(\frac{\sin \pi f T_m}{\pi f T_m}\right)^2, \tag{4}$$

where $T_m = 1/W_m$ is the OFDM symbol duration. Let d_m^k represents the spectral distance between the mth subcarrier and the center frequency of PU k. The interference power, introduced by the mth subcarrier of link l to PU-Rx on the kth subchannel, is given by

$$J_l^{m,k}(d_m^k, P_l^m) = P_l^m \beta_l^{m,k}, \tag{5}$$

where $\beta_l^{m,k} = T_m \int_{d_m^k - W_k/2}^{d_m^k + W_k/2} \left(\frac{\sin \pi f T_m}{\pi f T_m}\right)^2 df$ denotes the interference factor of the mth subcarrier.

Let η_0^k be the thermal noise power at the kth PU-Rx, the instantaneous SIR at the kth PU-Rx is expressed as

$$\gamma_0^k(\mathbf{P}^k) = \frac{G_{00}^k F_{00}^k P_0^k}{\eta_0^k + \sum_{l=1}^{L} \sum_{m=m_k}^{m_{k+1}-1} G_{0l}^m F_{0l}^m J_l^{m,k}}, \forall k \in \mathcal{K}, \tag{6}$$

where $\mathbf{P}^k = [P_l^i], \forall i \in [m_k, m_{k+1} - 1], \forall l \in \mathcal{L}$ is the power vector of all links over those subcarriers that are inside the kth subchannel. We also note that the large-scale fading gains $G_{0l}^m, \forall m \in [m_k, m_{k+1} - 1]$ on the same link l to the PU-Rx are the same. However, the small-scale fading gains $F_{0l}^m, \forall m \in [m_k, m_{k+1} - 1]$ on the same link l may be different.

As can be observed from (6), the random variable $\gamma_0^k(\mathbf{P}^k)$ has a complex distribution, i.e, we can not employ its approximation with using either a Rayleigh distribution or any other common distribution. Even in the absence of SUs, a fast Rayleigh fading may also make PU-Rx unable to decode the receiving signal from its PU-Tx's transmission. In such a case, the outage probability of PU should be taken into account in order to accurately evaluate the PU's QoS. To allow the SUs' channel access while maintaining its QoS, the kth PU-Rx requires its outage probability to stay below a certain threshold ζ_{th}^k. The constraint is set as follows:

$$\Pr[\gamma_0^k(\mathbf{P}^k) \leq \gamma_{th}^k] \leq \zeta_{th}^k, \forall k \in \mathcal{K}, \tag{7}$$

where γ_{th}^k is the SIR threshold at the k^{th} PU-Rx. In other words, the outage probability at PU-Rx k for a given SU transmit power vector $\mathbf{P}^k$ is [13]:

$$\Pr[\gamma_0^k(\mathbf{P}^k) \leq \gamma_{th}^k] = 1 - (1 - \zeta_0^k) \prod_{l=1}^{L} \left(\prod_{m=m_k}^{m_{k+1}-1} \left(1 + \frac{G_{0l}^m P_l^m \beta_l^{m,k} \gamma_{th}^k}{G_{00}^k P_0^k}\right)^{-1} \right), \forall k, \tag{8}$$

where $\zeta_0^k = 1 - \exp\left(\frac{-\eta_0^k \gamma_{th}^k}{P_0^k G_{00}^k}\right)$ is the outage probability of the kth PU-Rx in the absence of SUs. Substituting (8) into

(7), rewriting the resulting inequality as a lower bound on a posynomial function with respect to $\mathbf{P}^k$, then taking the natural logarithm on both sides, we have

$$\sum_{l=1}^{L}\sum_{m=m_k}^{m_{k+1}-1}\ln(1+\rho_l^{m,k}\beta_l^{m,k}P_l^m)\le \ln\mu^k, \forall k\in\mathcal{K}, \quad (9)$$

where $\mu^k = (1-\zeta_0^k)/(1-\zeta_{th}^k)$ and $\rho_l^{m,k} = \frac{G_{0l}^m\gamma_{th}^k}{G_{00}^k P_0^k}$. We assume that the primary requirements including the transmit power P_0^k, ζ_0^k, and ζ_{th}^k must be declared *a priori* to all SUs.

C. Problem Formulation: JCPC with PU Outage Constraint

As mentioned in Section I, it is noteworthy that the hard interference constraint may be inappropriate for PUs' QoS adaptation due to the fast-fading channel under distributed *spectrum underlay* systems. For instance, when the fading speed is dramatically increased, the rate of power update for SUs must also increase in order to keep the harmful interference to PU receivers below a certain threshold. This produces either so much message passing overhead leading to a collapse or a shortage of control information making the message passing based distributed algorithms impossible to converge. In order to cope with those difficulties while maximizing the net revenue for secondary system, we propose the following optimization framework in which the PUs are supposed to suffer their certain outage probabilities so that the SUs' resource allocation is adapted on a much slower time scale than the fluctuation of fading.

$$(\mathbf{P1}) \quad \max_{\boldsymbol{x}\in\mathcal{X},\mathbf{P}\in\mathcal{P}} \sum_{s\in\mathcal{S}}U_s(x_s)-\text{CPP}\sum_{l\in\mathcal{L}}\sum_{m\in\mathcal{M}}P_l^m \quad (10)$$

$$\text{subject to} \quad (3) \quad \text{and} \quad (7),$$

where $\mathcal{X} = \{x_s; s\in\mathcal{S}|x_s^{min}\le x_s\le x_s^{max}\}$ and $\mathcal{P} = \{P_l^m; l\in\mathcal{L}, m\in\mathcal{M}|P_l^{min}\le P_l^m\le P_l^{max}\}$ indicate QoS constraints for each source and the power restrictions for each link, respectively. CPP is the cost per unit of consumed power. Hereafter, we assume CPP to be unit without loss of optimality.

As can clearly be observed from (2), the capacity of each link is a nonlinear and neither convex nor concave function with respect to the nonnegative optimization variables, i.e., the transmit power vector $\mathbf{P}$. Moreover, the PU outage probabilities in (8) are non-convex on $\mathbf{P}$. Therefore, $\mathbf{P1}$ is generally a non-linear non-convex optimization problem.

III. Dual Decomposition and Optimal Solution

A. Equivalent Convex Problem

By substituting (2) into (3), replacing (7) with (9), and performing the optimization variable transformation, $\hat{P}_l^m = \ln P_l^m$, the problem $\mathbf{P1}$ is equivalently rewritten as

$$(\mathbf{P2}) \quad \max_{\boldsymbol{x}\in\mathcal{X},\hat{\mathbf{P}}\in\hat{\mathcal{P}}} \sum_{s\in\mathcal{S}}U_s(x_s)-\sum_{l\in\mathcal{L}}\sum_{m\in\mathcal{M}}e^{\hat{P}_l^m} \quad (11)$$

$$\text{subject to} \quad \sum_{s\in S(l)}x_s\le\sum_{m\in\mathcal{M}}\ln\left(\gamma_l^m(e^{\hat{\mathbf{P}}^m})\right), \quad \forall l, \quad (12)$$

$$\sum_{l=1}^{L}\sum_{m=m_k}^{m_{k+1}-1}\ln(1+\rho_l^{m,k}\beta_l^{m,k}e^{\hat{P}_l^m})\le\ln\mu^k, \quad \forall k, \quad (13)$$

where $\hat{\mathcal{P}} = \{\hat{P}_l^m; \forall l, m|\ln P_l^{min}\le\hat{P}_l^m\le\ln P_l^{max}\}$.

Theorem 1. *The transformed problem* $\mathbf{P2}$ *is an convex optimization problem.*

Proof: All the constraints (12) and (13) are convex in $(\boldsymbol{x},\hat{\mathbf{P}})$ since the log-sum-exp is convex in its domain [19]. Moreover, the utilities in (11) are assumed to be strictly concave. Therefore, $\mathbf{P2}$ is convex in $(\boldsymbol{x},\hat{\mathbf{P}})$. ■

B. Dual Decomposition and Optimal Solution

By augmenting the objective function (11) with a weighted sum of the constraints (12) and (13), we obtain the Lagrangian function of $\mathbf{P2}$:

$$\begin{aligned} L(\boldsymbol{x},\hat{\mathbf{P}},\boldsymbol{\lambda},\boldsymbol{\nu}) &= \sum_{s\in\mathcal{S}}U_s(x_s)-\sum_{l\in\mathcal{L}}\sum_{m\in\mathcal{M}}e^{\hat{P}_l^m} \\ &-\sum_{l\in\mathcal{L}}\lambda_l\Bigg(\sum_{s\in S(l)}x_s-\sum_{m\in\mathcal{M}}\ln\left(\gamma_l^m(e^{\hat{\mathbf{P}}^m})\right)\Bigg) \\ &-\sum_{k\in\mathcal{K}}\nu_k\Bigg(\sum_{l=1}^{L}\sum_{m=m_k}^{m_{k+1}-1}\ln\left(1+\rho_l^{m,k}\beta_l^{m,k}e^{\hat{P}_l^m}\right)-\ln\mu^k\Bigg), \end{aligned} \quad (14)$$

where $\boldsymbol{\lambda} = [\lambda_1, ..., \lambda_L]$ and $\boldsymbol{\nu} = [\nu_1, ..., \nu_K]$ are the Lagrangian nonnegative multipliers that are interpreted as congestion prices and PU outage prices to efficiently balance the conflict resource among the SUs, respectively. The former reflects the degree of congestion on a link while the latter reflects the outage status of each pair of PUs. The dual problem of $\mathbf{P2}$ can be described as an unconstrained max-min problem:

$$\min_{\boldsymbol{\lambda},\boldsymbol{\nu}}\max_{\boldsymbol{x}\in\mathcal{X},\hat{\mathbf{P}}\in\hat{\mathcal{P}}} L(\boldsymbol{x},\hat{\mathbf{P}},\boldsymbol{\lambda},\boldsymbol{\nu}). \quad (15)$$

Thanks to the separable nature with respect to $\mathbf{x}$ and $\hat{\mathbf{P}}$ of (14), the objective function of the minimization problem in (15) can be decomposed into the two subproblems as follows:

$$\max_{\boldsymbol{x}}\left\{L_x(\mathbf{x},\boldsymbol{\lambda})\doteq\sum_{s\in\mathcal{S}}U_s(x_s)-\sum_{s\in\mathcal{S}}\sum_{l\in L(s)}\lambda_l x_s\right\}; \quad (16)$$

$$\begin{aligned} \max_{\hat{\mathbf{P}}}\Bigg\{ L_{\hat{P}}(\hat{\mathbf{P}},\boldsymbol{\lambda},\boldsymbol{\nu}) &\doteq \sum_{l\in\mathcal{L}}\sum_{m\in\mathcal{M}}\lambda_l\ln\left(\gamma_l^m(e^{\hat{\mathbf{P}}^m})\right)-e^{\hat{P}_l^m} \\ &-\sum_{k=1}^{K}\sum_{l=1}^{L}\sum_{m=m_k}^{m_{k+1}-1}\nu_k\left(\ln(1+\rho_l^{m,k}\beta_l^{m,k}e^{\hat{P}_l^m})-\ln\mu^k\right)\Bigg\}. \end{aligned} \quad (17)$$

The first subproblem (16) is the canonical congestion control problem already solved implicitly or explicitly in TCP variants (e.g., Tahoe, Reno, Vegas and FAST), where each source adjusts its transmission rate via the feedback of the loss-based or delay-based congestion signal. The second subproblem (17) is the resource allocation problem that exactly allocates the power per subcarrier to each link.

Since $\mathbf{P2}$ is a convex optimization problem [**Theorem 1**], a feasible point satisfying the Slater constraint qualification in its domain exists [20]. According to the Strong Duality Theorem [21], there is no duality gap. Hence, the optimal solution of the maximization problem in (15) can then be obtained by solving the dual problem (15) via the iterative distributed algorithm

JCPC-OP. For simplicity of presentation, we use the same step size κ_t for all updates without loss of generality. In practice, the step sizes may be different provided that the updates jointly achieve the best convergence.

We also note that JCPC-OP is implemented in a distributed manner as follows:

1) The source algorithm can preserve the existing TCP congestion mechanism, where source s gets the aggregate congestion price $\lambda_s = \sum_{l \in L(s)} \lambda_l$ accumulated through a feedback message from its destination in the path of s. Then, source s adjusts its rate using (18).
2) In the link algorithm, the SU-Rx of link h locally measures $\gamma_h^m(\mathbf{P}^m)$ on each subcarrier and broadcasts its message $RxCtrlMsg$ containing M real-value fields reserved for $\chi_h^{m,(t)}, \forall m \in \mathcal{M}$. The SU-Tx of link l receives $RxCtrlMsg$ with $\chi_h^{m,(t)}$ and $TxCtrlMsg$ with $P_h^{m,(t)}, \forall h \neq l$, estimates S_{hl}^m through training sequence, and updates its transmit power per subcarrier by (19) via congestion price (20) and PU outage price (21). Then, the SU-Tx of link l broadcasts $TxCtrlMsg$ containing M real-value fields reserved for $P_l^{m,(t)}, \forall m \in \mathcal{M}$.
3) At each SU-Tx, the congestion price update (20) requires only links local information for ingress rate and SIR.
4) At each SU-Tx, the PU outage price update (21) needs $P_h^{m,(t)}, \forall m \in \mathcal{M}$ through $TxCtrlMsg$ received from the other SU-Tx and $\rho_l^{m,k} = \frac{G_{0l}^m \gamma_{th}^k}{G_{00}^k P_0^k}$ declared by the PU system.

Algorithm 1. *Optimal JCPC Algorithm (JCPC-OP)*

Primal/dual variables are updated iteratively until convergence.

- Congestion Control: *The source rate updates*

$$x_s^{(t+1)}(\lambda_s) = \left[U_s'^{-1}\left(\lambda_s^{(t)}\right)\right]^{\mathcal{X}_s}, \tag{18}$$

where $\lambda_s^{(t)} = \sum_{l \in L(s)} \lambda_l^{(t)}$.

- Power Control: *The link power updates per subcarrier*

$$P_l^{m,(t+1)} = \left[P_l^{m,(t)} + \kappa_t\left(\frac{\lambda_l^{(t)}}{P_l^{m,(t)}} - \sum_{h \neq l} \chi_h^{m,(t)} S_{hl}^m - \nu_k^{(t)} \frac{\rho_l^{m,k}\beta_l^{m,k}}{1+\rho_l^{m,k}\beta_l^{m,k}P_l^{m,(t)}} - 1\right)\right]^{\mathcal{P}_l}, \tag{19}$$

where $\chi_h^{m,(t)} = \frac{\lambda_h \gamma_h^{m,(t)}}{S_{hh}^m P_h^{m,(t)}}$.

- Congestion price updates

$$\lambda_l^{(t+1)} = \left[\lambda_l^{(t)} + \kappa_t\left(\sum_{s \in S(l)} x_s^{(t)} - C_l(\boldsymbol{P}^{(t)})\right)\right]^{\mathbf{R}_+}. \tag{20}$$

- PU outage price updates

$$\nu_k^{(t+1)} = \left[\nu_k^{(t)} + \kappa_t\left(\sum_{l=1}^{L}\sum_{m=m_k}^{m_{k+1}-1} \ln\left(\frac{1+\rho_l^{m,k}\beta_l^{m,k}P_l^{m,(t)}}{\mu^k}\right)\right)\right]^{\mathbf{R}_+} \tag{21}$$

where $[x]^{\mathcal{A}}$ *is the projection of* x *onto the feasible set* $\mathcal{A}$*,* κ_t *is the positive scalar step-size and* $U_s'^{-1}(.)$ *is the inverse of the first derivative of utility.*

Proposition 1. *Source rate update* (18) *solves the congestion control subproblem* (16) *for the fixed primal variables* $\boldsymbol{\lambda}$*.*

Proof: Since $L_x(\boldsymbol{x}, \boldsymbol{\lambda})$ is strictly concave and separable in $\boldsymbol{x}$, maximizer

$$x_s(\lambda_s) = arg \max_{x_s \in \mathcal{X}} \sum_{s \in \mathcal{S}} U_s(x_s) - \sum_{s \in \mathcal{S}} \lambda_s x_s.$$

can be found by the Karush-Kuhn-Tucker (KKT) necessary conditions [20, Proposition 3.3.1]. In fact, we take the first-order derivative of $L_x(\mathbf{x}, \boldsymbol{\lambda})$ with respect to x_s. Then we have (18) by letting the resulting quantity equal zero. ■

Proposition 2. *Link power update* (19) *solves the power allocation subproblem* (17) *for a pair of fixed primal variables* $(\boldsymbol{\lambda}, \boldsymbol{\nu})$.

Proof: Since $L_{\hat{P}}(\hat{\mathbf{P}}, \boldsymbol{\lambda}, \boldsymbol{\nu})$ is strictly concave in $\hat{\mathbf{P}}$, its first-order derivative with respect to $\hat{P}_l^m$ is given by

$$\frac{\partial L_{\hat{P}}(\hat{\mathbf{P}}, \boldsymbol{\lambda}, \boldsymbol{\nu})}{\partial \hat{P}_l^m} = \lambda_l - e^{\hat{P}_l^m} - \sum_{h \neq l} \lambda_h \frac{S_{hl}^m e^{\hat{P}_l^m}}{\eta_h + \sum_{j \neq h} S_{hj}^m e^{\hat{P}_j^m} + I_{h0}^m} - \nu_k \frac{\rho_l^{m,k}\beta_l^{m,k}e^{\hat{P}_l^m}}{1+\rho_l^{m,k}\beta_l^{m,k}e^{\hat{P}_l^m}}.$$

Using the facts that $\nabla_l L_P(\mathbf{P}, \boldsymbol{\lambda}, \boldsymbol{\nu}) = \frac{1}{P_l^m} \nabla_l L_{\hat{P}}(\hat{\mathbf{P}}, \boldsymbol{\lambda}, \boldsymbol{\nu})$ and $P_l^m = e^{\hat{P}_l^m}$, we then adopt the projected gradient-ascent method [21]:

$$P_l^{m,(t+1)} = \left[P_l^{m,(t)} + \kappa_t \frac{\partial L_P(\mathbf{P}^{(t)}, \boldsymbol{\lambda}^{(t)}, \boldsymbol{\nu}^{(t)})}{\partial P_l^m}\right]^{\mathcal{P}_l}$$

with a step size $\kappa_t \geq 0$ for link power updates as (19). ■

Coming back to the dual problem (15), the objective is differentiable for all $\boldsymbol{\lambda}$ and $\boldsymbol{\nu}$. Therefore, we can also apply the projected gradient-descent method [21] to solve the dual problem (15) via link congestion price updates (20) and primary outage price updates (21).

Theorem 2. *For any initial source rates* $\boldsymbol{x}^{(0)} \in \mathcal{X}$*, link powers* $\boldsymbol{P}^{(0)} \in \mathcal{P}$ *and shadow prices* $(\boldsymbol{\lambda}^{(0)}, \boldsymbol{\nu}^{(0)}) \geq 0$*, the sequence of primal-dual variables generated by JCPC-OP converges to the global optimum of the original problem* $\mathbf{P1}$ *provided that the step sizes satisfy:*

$$\kappa_t \geq 0, \ \lim_{t \to \infty} \kappa_t = 0, \ \sum_{t=0}^{\infty} \kappa_t = \infty, \ \sum_{t=0}^{\infty} (\kappa_t)^2 < \infty. \tag{22}$$

Proof: From Propositions 3 and 4, we conclude that JCPC-OP solves $\mathbf{P2}$. For any initial values of the primal and dual variables, and the step sizes satisfying (22), JCPC-OP always converges to a unique point [21]. Since $\mathbf{P2}$ is a convex optimization problem [Theorem 1], any locally optimal point obtained from JCPC-OP is also the global optimum [20]. ■

IV. New Heuristic-based Suboptimal Solution

In the previous section, we proposed the JCPC-OP algorithm in which the optimal congestion control and power control are jointly designed for OFDM-based multi-hop CRNs. The aim of this scheme is to maximize the net revenue of secondary system while keeping the PU outage probabilities below the predefined thresholds. However, the highly induced computational complexity, large signaling overhead, and the PU-related links' CSI may make the system infeasible and unscalable in practice. Using a suboptimal solution to address these shortcomings with a low computational complexity is desired. Deploying the noisy estimation of the PU outage probabilities in conjunction with local measurements, the SUs can alternatively adjust their rates and powers via the PU outage prices and congestion prices with neither explicit message passing nor the PU-related links' CSI.

Here, we also use the same approach of dual decomposition technique as discussed in Section III to solve $\mathbf{P1}$. However, the Lagrangian is taken on the basis of the original problem $\mathbf{P1}$ instead of the equivalent convex problem $\mathbf{P2}$ as follows:

$$L(\boldsymbol{x},\mathbf{P},\boldsymbol{\lambda},\boldsymbol{\nu}) = \sum_{s\in\mathcal{S}} U_s(x_s) - \sum_{l\in\mathcal{L}}\sum_{m\in\mathcal{M}} P_l^m - \sum_{l\in\mathcal{L}}\lambda_l \sum_{s\in S(l)} x_s + \sum_{l\in\mathcal{L}}\lambda_l C_l(\mathbf{P}) - \sum_{k\in\mathcal{K}}\nu_k\left(\Pr[\gamma_0^k(\mathbf{P}^k)\le\gamma_{th}^k] - \zeta_{th}^k\right). \quad (23)$$

The dual problem of $\mathbf{P1}$ can be expressed as an unconstrained max-min problem:

$$\min_{\boldsymbol{\lambda},\boldsymbol{\nu}}\ \max_{\boldsymbol{x},\mathbf{P}} L(\boldsymbol{x},\mathbf{P},\boldsymbol{\lambda},\boldsymbol{\nu}). \quad (24)$$

Our major task is to solve the primal problem of $\mathbf{P1}$ (i.e., the maximization problem in (24)) via the dual variables λ_l and ν_k. In fact, the objective in (24) can be decomposed into two separate subproblems with respect to primal variables $\boldsymbol{x}$ and $\mathbf{P}$ as follows:

$$\max_{\boldsymbol{x}\in\mathcal{X}}\left\{L_x(\boldsymbol{x},\boldsymbol{\lambda}) \doteq \sum_{s\in\mathcal{S}} U_s(x_s) - \sum_{s\in\mathcal{S}}\sum_{l\in L(s)}\lambda_l x_s\right\}; \quad (25)$$

$$\max_{\mathbf{P}\in\mathcal{P}}\left\{L_P(\mathbf{P},\boldsymbol{\lambda},\boldsymbol{\nu}) \doteq \sum_{l\in\mathcal{L}}\lambda_l C_l(\mathbf{P}) - \sum_{l\in\mathcal{L}}\sum_{m\in\mathcal{M}} P_l^m - \sum_{k\in\mathcal{K}}\nu_k\left(\Pr[\gamma_0^k(\mathbf{P}^k)\le\gamma_{th}^k] - \zeta_{th}^k\right)\right\}. \quad (26)$$

The optimal solution of the unchanged subproblem (25) is still the same as (18) in JCPC-OP. Our most important task is now moving forward to the power allocation subproblem (26).

A. Estimation of PU Outage Probability and Dual Solution

We assume that the number of outage events of the kth PU-Rx observed during the power update interval T is N_k^T. Then, the noisy estimation of the kth PU-Rx outage probability $\Pr[\gamma_0^k(\mathbf{P}^k)\le\gamma_{th}^k]$ is [22]:

$$\hat{\zeta}_k^{(T)} = \begin{cases} 1/(R_k\times T), & \text{if } N_k^T = 0,\\ N_k^T/(R_k\times T), & \text{otherwise}\end{cases} \quad (27)$$

where R_k and $R_k\times T$ represent the packet rate and the number of packets emitted by the k-th PU-Tx during power update interval T, respectively. It is clear that $\hat{\zeta}_k^{(T)}$ becomes more accurate as T is large enough. However, it takes a longer time for the noisy estimation based solutions to converge.

Since $L(\boldsymbol{x},\mathbf{P},\boldsymbol{\lambda},\boldsymbol{\nu})$ given by (19) is affine in $(\boldsymbol{\nu},\ \boldsymbol{\lambda})$, its sub-gradients with respect to $\boldsymbol{\lambda}$ and $\boldsymbol{\nu}$ yield:

$$\frac{\partial L(\boldsymbol{x},\mathbf{P},\boldsymbol{\lambda},\boldsymbol{\nu})}{\partial\lambda_l} = \sum_{s\in S(l)} x_s - C_l(\mathbf{P}); \quad (28)$$

$$\frac{\partial L(\boldsymbol{x},\mathbf{P},\boldsymbol{\lambda},\boldsymbol{\nu})}{\partial\nu_k} = \Pr[\gamma_0^k(\mathbf{P}^k)\le\gamma_{th}^k] - \zeta_{th}^k = \hat{\zeta}_k - \zeta_{th}^k. \quad (29)$$

The dual problem (24) can be solved using the sub-gradient projection method [21], where the congestion prices $\lambda_l(l\in\mathcal{L})$ and the primary outage prices $\nu_k(k\in\mathcal{K})$ are adjusted in the descending direction of sub-gradients $\nabla_{\boldsymbol{\lambda}}L(\boldsymbol{x},\mathbf{P},\boldsymbol{\lambda},\boldsymbol{\nu})$, and $\boldsymbol{\nabla}_{\boldsymbol{\nu}}L(\boldsymbol{x},\mathbf{P},\boldsymbol{\lambda},\boldsymbol{\nu})$, respectively.

B. Power Allocation Algorithm

Next, we substitute the constraint on the PU outage probability (9) into (26) (since (9) is equivalent to (7)), the power allocation subproblem $\max_{\mathbf{P}\in\mathcal{P}} L_P(\mathbf{P},\boldsymbol{\lambda},\boldsymbol{\nu})$ is rewritten as follows:

$$\max_{\mathbf{P}\in\mathcal{P}}\left\{L_P(\mathbf{P},\boldsymbol{\lambda},\boldsymbol{\nu}) = \sum_{l\in\mathcal{L}}\lambda_l C_l(\mathbf{P}) - \sum_{l\in\mathcal{L}}\sum_{m\in\mathcal{M}} P_l^m - \sum_{k\in\mathcal{K}}\sum_{l\in\mathcal{L}}\sum_{m=m_k}^{m_{k+1}-1}\nu_k\left(\ln(1+\rho_l^{m,k}\beta_l^{m,k}P_l^m) - \ln\mu^k\right)\right\}. \quad (30)$$

As can be observed from (30), $L_P(\mathbf{P},\boldsymbol{\lambda},\boldsymbol{\nu})$ are generally not concave in $\mathbf{P}$. To solve (30), we equivalently transform this non-convex subproblem (30) into an approximate convex optimization problem using a fixed rate assignment approach. Then, we propose an iterative power allocation algorithm and prove that it converges to the unique fixed point.

We assume that prior to each power update iteration, the link's ingress rate allocated by the congestion control policy (25) is fixed. Hence, the link powers P_l^m must be controlled in such a way that its capacity can meet bandwidth demand, i.e.,

$$\sum_{s\in S(l)} x_s \le C_l(\mathbf{P}) = \sum_{m=1}^{M}\ln\left(\gamma_l^m(\mathbf{P}^m)\right), \forall l\in\mathcal{L}. \quad (31)$$

Consequently, the SIR requirement of each link at its ingress rate $\sum_{s\in S(l)} x_s$ is given by

$$\prod_{m=1}^{M}\gamma_l^m(\mathbf{P}^m) \ge \exp\left(\sum_{s\in S(l)} x_s\right) \doteq \mathrm{SIR}_l^{th}, \quad \forall l\in\mathcal{L}. \quad (32)$$

Under this fixed rate assignment scheme, the power allocation subproblem (30) is equivalent to the problem seeking a feasible power vector $\mathbf{P}$ to minimize the total interference impact on PUs while satisfying the SIR constraints (32) as follows:

$$(\mathbf{P3})\ \min_{\mathbf{P}\in\mathcal{P}} \sum_{k\in\mathcal{K}}\sum_{l\in\mathcal{L}}\sum_{m=m_k}^{m_{k+1}-1}\nu_k\ln\left(1+\rho_l^{m,k}\beta_l^{m,k}P_l^m\right) + \sum_{l\in\mathcal{L}}\sum_{m\in\mathcal{M}} P_l^m \quad (33)$$

$$s.t. \qquad \prod_{m=1}^{M}\gamma_l^m(\mathbf{P}^m) \ge \mathrm{SIR}_l^{th}, \quad \forall l\in\mathcal{L}.$$

The set of constraints on SIR in (33) can be rewritten as

$$P_l^m \geq \frac{\mathrm{SIR}_l^{th}}{\Omega_l(\mathbf{P}^m)\prod_{n\neq m}\gamma_l^n(\mathbf{P}^m)} \doteq \Psi_l^m(\mathbf{P}^m), \forall l \in \mathcal{L}, \quad (34)$$

where

$$\Omega_l(\mathbf{P}^m) = \frac{S_{ll}^m}{\eta_l + \sum_{h\neq l} S_{lh}P_h^m + I_{l0}^m}, \quad (35)$$

and $\Psi_l^m(\mathbf{P})$ is the effective interference impact on the link l on subcarrier m. Since the objective of (33) is not convex in **P**, the link powers are transformed logarithmically to ensure that it is convex. The problem (33) is equivalent to

$$(\mathbf{P4}) \ \min_{\hat{\mathbf{P}}\in\hat{\mathcal{P}}} \ h(\hat{\mathbf{P}}) \doteq \sum_{k\in\mathcal{K}}\sum_{l\in\mathcal{L}}\sum_{m=m_k}^{m_{k+1}-1} \nu_k \ln\left(1+\rho_l^{m,k}\beta_l^{m,k}e^{\hat{P}_l^m}\right) + \sum_{l\in\mathcal{L}}\sum_{m\in\mathcal{M}} e^{\hat{P}_l^m} \quad (36)$$

$$s.t. \qquad \ln e^{\hat{P}_l^m} \geq \ln \Psi_l^m(\hat{\mathbf{P}}), \quad \forall l \in \mathcal{L}. \quad (37)$$

Proposition 3. *For a given pair of* $(\boldsymbol{x}, \boldsymbol{\nu})$*,* $\mathbf{P4}$ *is convex in* $\hat{\mathcal{P}}$.

Proof: We note that ν_k and x_s respectively represent the PU outage status and the ingress rate before performing a power update. In this regard, they can be assumed to be fixed. Therefore, the objective of (36) is clearly convex. Additionally, the set of constraints (37) can be rewritten as

$$\ln \mathrm{SIR}_l^{th} + \sum_{i=1}^{M} \ln\left(\eta_l + \sum_{h\neq l} S_{lh}^i e^{\hat{P}_h^i} + I_{l0}^i\right) - \sum_{i=1}^{M}\left(\ln S_{ll}^i + \hat{P}_l^i\right) \leq 0, \ \forall l \in \mathcal{L}$$

which are convex in $\hat{\mathbf{P}}$ because the log-sum-exp is convex [19]. Hence, $\mathbf{P4}$ is convex. ■

Then, the optimal solution of (36) can be found via an iterative power control algorithm modifying gradient-descent method [21] after transforming back to **P**-space as follows:

$$P_l^{m,(t+1)} = \left[P_l^{m,(t)} - \kappa_t\left(\frac{\partial h(\mathbf{P}^{(t)})}{\partial P_l^m}P_l^{m,(t)} - \Psi_l^m(\mathbf{P}^{(t)})\right)\right]^{\mathcal{P}_l}. \quad (38)$$

Proposition 4. *The power control algorithm* (38) *solving the subproblem* (36) *for a pair of fixed variables* $(\boldsymbol{x}, \boldsymbol{\nu})$ *converges to a unique fixed point.*

Proof: First, it is apparent from (34) that $\Psi_l^m(\mathbf{P})$ is a function satisfying the triple properties of positivity, monotonicity, and scalability for all $\mathbf{P} \in \mathcal{P}$. Since $\nabla h(\mathbf{P}) \succeq 1$, we have the following inequalities:

$$\Psi_l^m(\mathbf{P}^{(t)}) \leq P_l^{m,(t)} \leq P_l^{m,(t)}\frac{\partial h(\mathbf{P}^{(t)})}{\partial P_l^m}, \forall l \in \mathcal{L}. \quad (39)$$

Hence,

$$\frac{\partial h(\mathbf{P}^{(t)})}{\partial P_l^m}P_l^{m,(t)} - \Psi_l^m(\mathbf{P}^{(t)}) \geq 0, \forall l \in \mathcal{L}. \quad (40)$$

With the step size κ_t satisfied (22), $P_l^{m,(t)}$ is a monotonically decreasing sequence. Since this sequence has a lower-bound of P_l^{min}, the power control algorithm (38) always converges to the optimum $\mathbf{P}^*$. In other words, since $\mathbf{P4}$ is convex [Proposition 3], $\mathbf{P}^*$ is the unique optimal point [19]. ■

C. Sub-Optimal JCPC Algorithm

Finally, the optimal solution of $\mathbf{P1}$ can now be readily found via the following iterative algorithm.

Algorithm 2. Sub-Optimal JCPC Algorithm (JCPC-SOP)

Primal/dual variables are updated iteratively until convergence.

- Congestion Control*: The source rate updates using* (18)*.*
- Power Control*: The link power updates per subcarrier*

$$P_l^{m,(t+1)} = \left[P_l^{m,(t)} + \kappa_t\left(\frac{P_l^{m,(t)}\exp\left(\sum_{s\in S(l)} x_s^{(t)}\right)}{\prod_{m=1}^{M}\gamma_l^m(\boldsymbol{P}^{m,(t)})} - \frac{\nu_k^{(t)}\rho_l^{m,k}\beta_l^{m,k}P_l^{m,(t)}}{1+\rho_l^{m,k}\beta_l^{m,k}P_l^{m,(t)}} - P_l^{m,(t)}\right)\right]^{\mathcal{P}_l}. \quad (41)$$

- Congestion price updates*:*

$$\lambda_l^{(t+1)} = \left[\lambda_l^{(t)} + \kappa_t\left(\sum_{s\in S(l)} x_s^{(t)} - C_l(\boldsymbol{P}^{(t)})\right)\right]^{\mathbf{R}_+}. \quad (42)$$

- PU outage price updates*:*

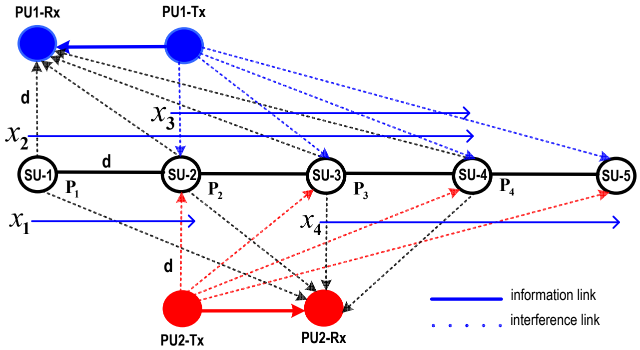
$$\nu_k^{(t+1)} = \left[\nu_k^{(t)} + \kappa_t\left(\hat{\zeta}_k - \zeta_{th}^k\right)\right]^{\mathbf{R}_+}. \quad (43)$$

Theorem 3. *For any initial source rate vector* $\boldsymbol{x}^{(0)} \in \mathcal{X}$*, link power vector* $\boldsymbol{P}^{(0)} \in \mathcal{P}$*, shadow prices* $(\boldsymbol{\lambda}^{(0)}, \boldsymbol{\nu}^{(0)}) \geq 0$*, and* κ_t *satisfying* (22) *the sequence of primal-dual variables generated by JCPC-SOP converges to the point near the global optimum of the original problem* $\mathbf{P1}$.

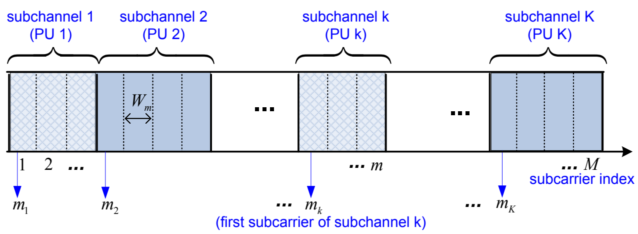
Proof: In the iterative procedure of JCPC-SOP, the congestion control portion is referred to in the current step, whereas the power control portion is the result from the solution of the previous step. In each step, only the congestion control subproblem (25) is globally maximized and the obtained solution is used for forming a new convex approximation (36) of the non-convex power allocation subproblem in (26). The results achieved from this new convex problem are used for the next iteration of the algorithm. Hence, from the point view of network, the optimal powers obtained by JCPC-SOP is local. By Proposition 1 & 4, JCPC-SOP converges to the local fixed optimum. However, from the point view of link, the value of achievable optimal powers per band are global since $\mathbf{P}4$ is convex [Proposition 3]. In association of the following experimental results in Section V, we strongly conclude that the solution of JCPC-SOP reaches the global optimum of the original problem $\mathbf{P1}$ with a negligible gap. ■

Our JCPC-SOP algorithm can be implemented in a distributed manner without control information exchange:

1) The congestion control mechanism is still the same as described in JCPC-OP. In fact, it can preserve TCP stack by embedding congestion prices in a header field of control packets traversing the source's reverse path.



(a) Physical/logical topologies for a coexistence of PUs/SUs.



(b) Spectrum allocation in OFDM-based CRN systems.

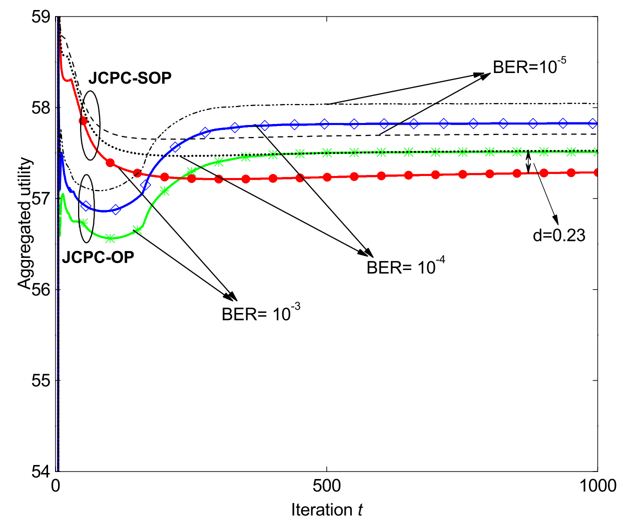
Fig. 1: System model of OFDM-based multi-hop CRNs.

2) The power control strategy requires only the link's local SIR measurement per subcarrier and the allocated ingress rate $\sum_{s \in S(l)} x_s^{(t)}$ without message passing.
3) The congestion price updates use only the link's local information such as ingress rate and SIR.
4) The PU outage price updates are based on the number of outage feedbacks from primary links that can be locally measured by SUs.

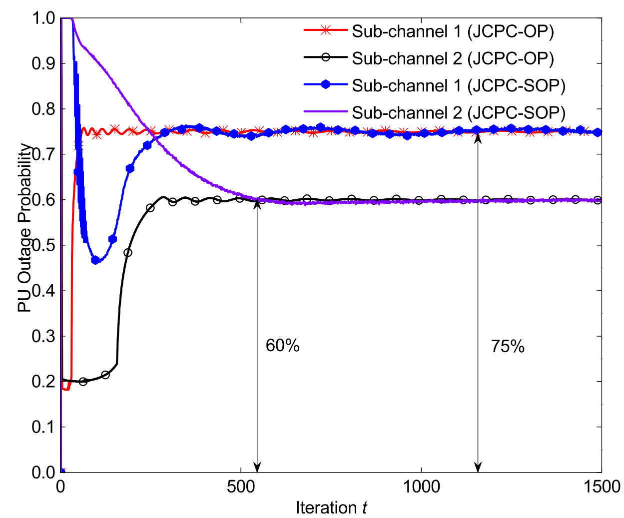
V. Performance Evaluation

A. Simulation Settings

We consider a multi-hop CRN system with N = 5 secondary nodes, K = 2 pairs of PUs and 4 flows as illustrated in Fig. 1. We assume that the system has two sub-channels. The first sub-channel consists of the subcarriers 1-3 while the second sub-channel consists of the subcarriers 4-8. They are licensed to two primary links, respectively. We choose path loss exponent $n = 4$ and the Rayleigh fading channel with mean $E(F_{lk}) = 1$. Each secondary link with a transmit power constraint of $P_l^{min} = 1.5mW$ and $P_l^{max} = 400mW$ per subcarrier can access all the licensed subcarriers, bandwidth of each subcarrier is $W_m = 0.125MHz$. The minimum data rate for each elastic flow is $x_{min} = 100bps$, while x_{max} is adjusted dynamically with respect to link capacities. The target BER $= 10^{-4}$ corresponding to MQAM modulation is the same for all secondary nodes with $G_{MC} = -1.5/\log(5\text{BER})$ [18]. For PUs, we set the outage probability thresholds for the licensed subchannels 1 and 2 as 75% and 60%, respectively. The SIR thresholds for PU receivers 1 and 2 are $\gamma_{th}^1 = 4.71dB$ and $\gamma_{th}^2 = 6.02dB$ at transmit powers $20dBm$ and $23dBm$, respectively. The PSD of white noise is assumed to be $-174dBm/Hz$ at the PU and SU receivers. We choose $U_s(x_s) = \log x_s$ as the source's utility function for all secondary nodes. The criterion used to evaluate the convergence of algorithms is $\max \|\mathbf{P}^{*(t)} - \mathbf{P}^{*(t-1)}\| \le \varepsilon$, where $\varepsilon = 10^{-5}$ is the error tolerance. The step sizes of both algorithms are chosen as follows: the power updates $(6e^{-3}/t)$, the congestion price updates (e^{-13}/t), and the PU outage price updates $(125/\sqrt{t})$



(a)



(b)

Fig. 2: Convergence of aggregated utility which is averaged over 20 random realizations of the logical topologies versus BER (a) and PU outage probability (b) at $\varepsilon = 10^{-5}$.

B. Performance of the Proposed Algorithms

In this experiment, we investigate the evolution of the two proposed algorithms and confirm the practical optimality achieved by them. Fig. 2a shows that the aggregate utilities of our proposed algorithms (i.e., JCPC-OP and JCPC-SOP) converge to the optimum within a reasonable time at some various values of BER. Their trajectories also show that the gap between JCPC-OP and JCPC-SOP is negligible. This is a remarkable point for the close-to-optimal solution with no message passing, JCPC-SOP. Fig. 2b shows that the PU outage probabilities due to secondary transmissions converge to their

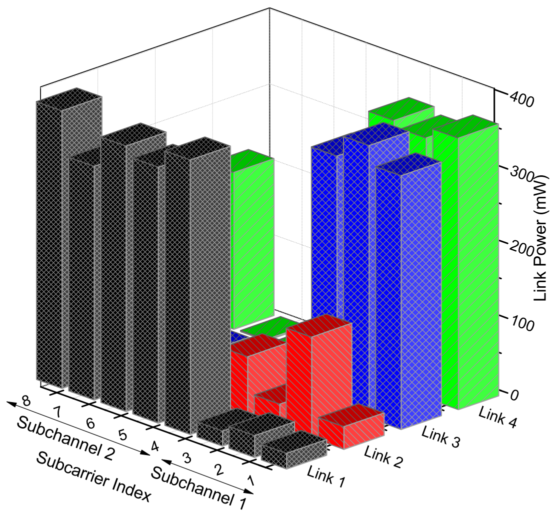


Fig. 3: The optimal allocated power at the average small-scale fading gains per subcarrier corresponding to $[1.7, 0.31, 1.6, 0.3, 1.2, 0.7, 2.0, 0.15]$ and $\varepsilon = 10^{-5}$.

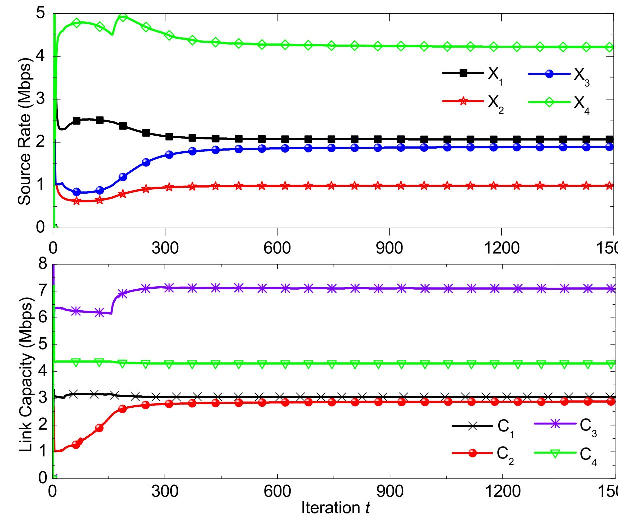


Fig. 4: Convergence of source rate and link capacity using JCPC-OP at $\varepsilon = 10^{-5}$.

pre-specified thresholds for both JCPC-OP and JCPC-SOP. As can be observed from Fig. 3, the transmit power depends not only on the mutual interference among SUs, but also on the physical distance between SUs and PUs. Moreover, they are also affected by fading channel conditions. Also, the optimal rate in Fig. 4 shows that the greedy sources try to get a larger rate to obtain a higher utility. However, sources must follow the diminishing marginal return due to the concave property of the utility function. They must just their rate via both the congestion prices and the PU outage prices, which reflect the total cost to pay.

Next, we compare the performance of our algorithms with some TCP Vegas's congestion control schemes without coupling our proposals, i.e.,

- Scheme 1 allocates the fixed maximum power $400mW$ to all links per subcarrier.
- Scheme 2 allocates the fixed powers as $P_1^{1-3} = 2mW; P_1^{4-8} = 400mW; P_2^{1-3} = 50mW; P_2^{4-8} = 1mW; P_3^{1-3} = 400mW; P_3^{4-8} = 1mW; P_4^{1-3} = 400mW; P_4^{4-8} = 120mW$.
- Scheme 3 allocates the fixed powers as $P_1^{1-3} = 100mW; P_1^{4-8} = 200mW; P_2^{1-3} = 25mW; P_2^{4-8} = 10mW; P_3^{1-3} = 300mW; P_3^{4-8} = 2mW; P_4^{1-3} = 400mW; P_4^{4-8} = 340mW$.

Fig. 5 shows that the TCP Vegas's schemes cannot achieve as high net revenue as JCPC-OP and JCPC-SOP, even in some cases in which their spectrum utilizations exceed the pre-specified PU outage thresholds of PU1 (75%) and PU2 (60%) as in schemes 1 and 3. This is because the power randomly allocated to each link per subcarrier does not address the spectrum sharing problem and their mutual interference. For example, at TCP Vegas's scheme 3, compared with the JCPC-OP's optimal powers as in Fig. 6, it is not sufficient to allocate more power to subcarriers 4-8 on the link 4 and less power to subcarriers 4-8 on the link 1. This wrong power allocation causes not only the excessive harmful interference to PUs but also serious degradation of the secondary system's performance (e.g., scheme 1's performance is the worst although it spends the more spectrum and power than the others).

C. Effect of Link Behavior on Objective Function

As can be observed from Fig. 7, when the link 2's congestion price is increasing, the link 2 must raise its power (as shown in Fig. 6) and the sources 2 and 3 traversing the link 2 must simultaneously reduce their rates (as illustrated in Fig. 4) because the price decrease of the other links (i.e., links 1 and 3) on their path can not compensate this link 2's inflicted price increase. When the link 2's capacity is accelerated by the increase of power and becomes greater than the total ingress rate, the link 2's congestion price starts to decrease so that the sources 2 and 3 can pay the lower cost, then raise its rates, and the total ingress rate may catch up to its increasing capacity. At this time, the link 2's powers on all subcarriers are still increasing until convergence (as shown in Fig. 6). This is because the link 2 has not used up its potential spectrum opportunity as shown via the PU outage probability in Fig. 2b.

In contrary, the links 1, 3, and 4 reduce their powers to raise the values of objective function (i.e., net revenue) when both their capacities are larger than the total ingress rates and their powers are still larger than the minimum powers, as shown in Figs. 4, 6, and 7. Note that the powers of links 1, 3, and 4 increase on some of their subcarriers but decrease on some of their other subcarriers due to the adjustment of their mutual interference and spectrum fairness allocation strategy. However, their average powers are decreasing so as to minimize energy consumption and maximize the net revenue.

It is noteworthy that the total rate of the traffic through one link equals its capacity at the optimality. This may be obtained only if the link capacity is larger than the total rate of the traffic through this link and none of its decreasing powers reaches the minimum value before convergence. In this regard, we can clearly see from all links' behavior in JCPC-OP's. And, we can conclude that the link 3's capacity exactly equals the sum of the link 2's and the link 4's capacities as in Fig. 4.

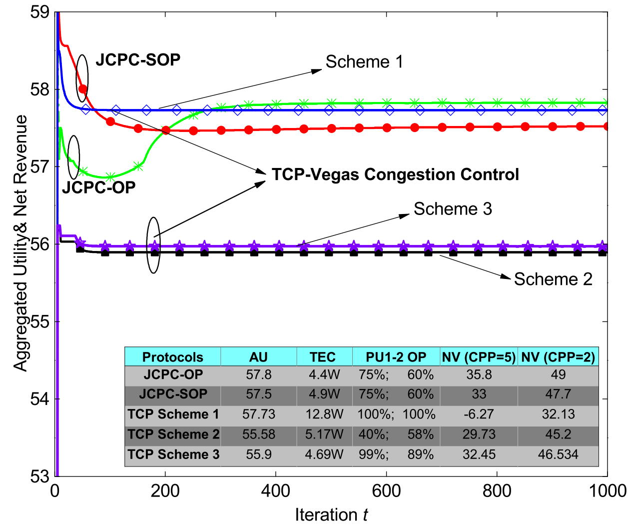


Fig. 5: Evolution of proposals and TCP-Vegas Schemes at BER $= 10^{-4}$; where Schemes 1, 2, and 3 randomly allocate the fixed powers to all links and AU=Aggregated Utility, TEC=Total Energy Consumption, PU OP= Primary User Outage Probability, NV= Net Revenue, CPP= Cost Per unit of consumed Power.

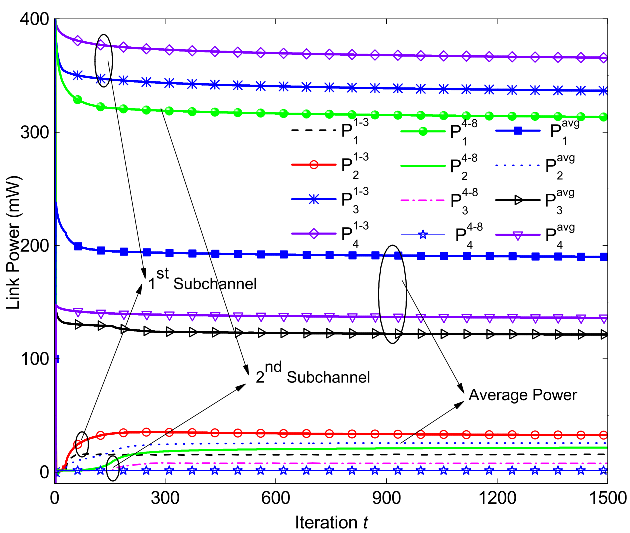


Fig. 6: Trajectory of link powers using JCPC-OP.

D. Aggregate Utility under the OFDM-based Spectrum Underlay and PU Outage Approach

In this section, we examine the aggregate utility under the OFDM-based spectrum underlay and PU outage approach. First, we investigate the effect of the number of subcarriers and the PU outage probability thresholds on the aggregate utility. In Fig. 8, we observe that for a fixed number of subcarriers, the aggregate utility increases as we continue relaxing the PU outage probability thresholds. This is because the more PU outage probabilities are relaxed or the more interference the PUs tolerate, the more additional spectrum opportunity is definitely brought to the SUs. Similarly, increasing the number of subcarriers on each subchannel raises the aggregate utility

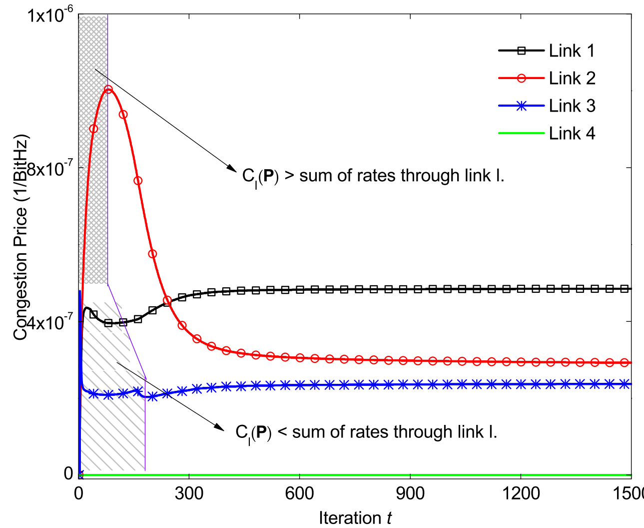


Fig. 7: Trajectory of congestion price on links using JCPC-OP.

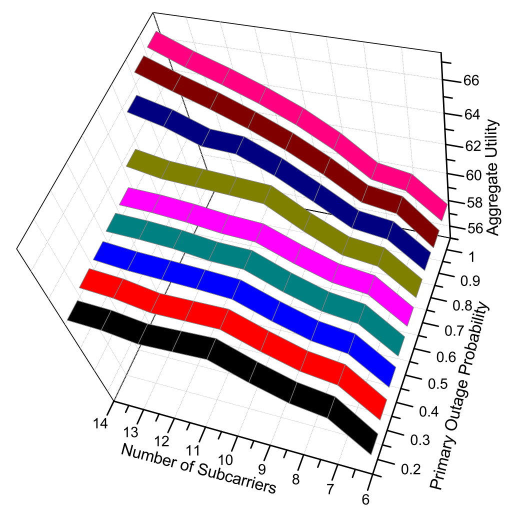


Fig. 8: Aggregated utility vs. number of subcarriers vs. PU1 outage probability using JCPC-OP with $\zeta_{th}^2 = 60\%$.

for the fixed PU outage probability thresholds.

Next, we vary the distance parameter d_0 and note the PU1's position differentiation when it moves horizontally in order to show the relationship between the spectrum opportunity and the distance between the SUs and the PUs. In this regard, we fix the PU2's position and the PU outage probability thresholds. It can be seen from Fig. 9 that the secondary link capacities quickly increases as d_0/d increases and vice versa. The reason is that the harmful interference, introduced by the SUs, is inversely proportional to the distance between them and the PUs. Hence, the SUs can adjust their transmit power to achieve optimal capacity while keeping the PU outage probability at the target. As a result, in the low d_0 regime, the link 1's capacity increases when the PU1 moves far away from it while the capacities of links 2, 3, and 4 gradually decrease because the PU1 comes closer to them. We can see a similar relationship in the high d_0 regime. Finally, Fig. 10 shows the saturation effect of aggregate utility and PU outage probabilities to the maximal transmit power P_l^{max}. In the low

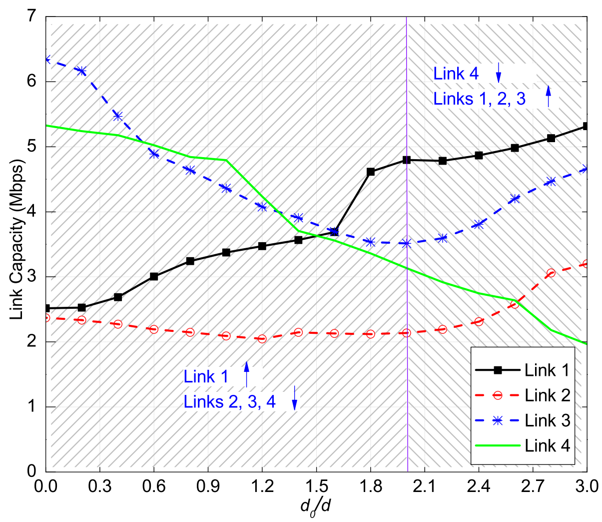


Fig. 9: Effect of PU1's position while fixing PU2's position.

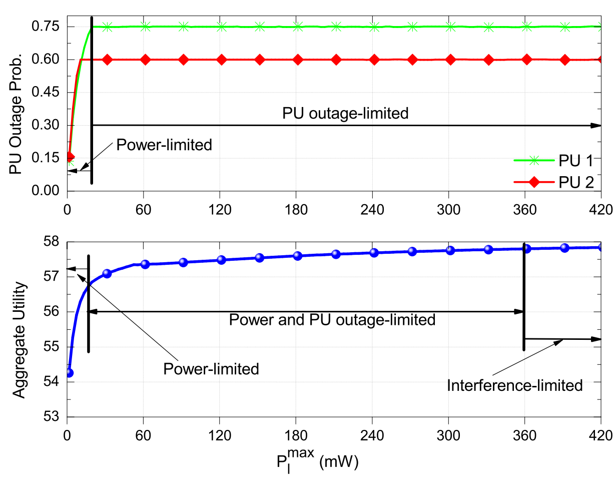


Fig. 10: Saturation effect of aggregate utility and PU outage probability as P_l^{max} increases.

and middle P_l^{max} regime, the aggregate utility increases as the available power increases. However, when P_l^{max} is greater than a certain turning point, the aggregate utility does not increase further because the PU outage probability constraints and the SU mutual interference constraints that are implicitly represented by the link capacity constraints become dominant. Also, in the power-limited regime, the PU outage probabilities are much lower than their corresponding outage thresholds $\zeta_{th}^1 = 75\%$ and $\zeta_{th}^2 = 60\%$. If we continue increasing P_l^{max} in the PU outage-limited regime, then the PU outage probabilities are saturated to their corresponding outage thresholds.

E. Fairness in Resource Allocation

Fig. 11 shows the fairness index versus the number of active sessions for our considered algorithms. We use Jain's fairness index $(\sum_{s=1}^{S} x_s)^2/(S * \sum_{s=1}^{S} x_s^2)$ as a standard fairness measurement. Here, we can see that the overall fairness among competing sources achieved by JCPC-OP is slightly better than that achieved by JCPC-SOP. This observation

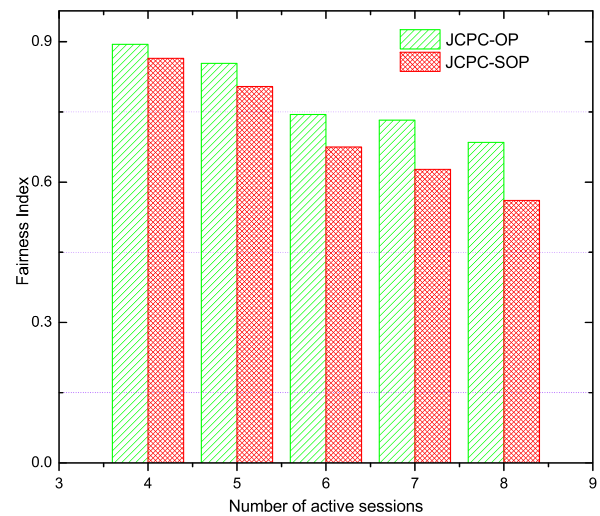


Fig. 11: Fairness index versus the number of active sessions.

verifies the feasibility of the heuristic-based algorithm, JCPC-SOP. Moreover, our proposed algorithms are stable when the sessions are dynamic because the algorithms adaptively adjust source rates and link powers according to the current transmissions.

VI. Conclusion

In this paper, the joint congestion control and power control scheme in OFDM-based multi-hop CRNs is studied. We specifically consider the co-existence of licensed and unlicensed users and the spectrum opportunity under the unique outage constraints in a spectrum underlay fashion. A distributed algorithm, JCPC-OP, is proposed to achieve the optimal power allocation vector for each link per subcarrier as well as rate for each source. Although the JCPC-OP algorithm's attraction is the global optimality, its message passing brings an overhead burden. A suboptimal algorithm without message passing, JCPC-SOP, is also proposed to alleviate this issue. Fairness in resource allocation and energy efficiency are kindly investigated to maximize the social welfare, as well.

References

[1] S. Srinivasa and S. Jafar, "The throughput potential of cognitive radio: a theoretical perspective," *IEEE Commun. Mag.*, vol. 45, no. 5, pp. 73–79, May 2007.

[2] T. A. Weiss and F. K. Jondral, "Spectrum pooling: an innovative strategy for the enhancement of spectrum efficiency," *IEEE Commun. Mag.*, vol. 42, no. 3, pp. S8–14, Aug. 2004.

[3] G. Bansal, M. J. Hossain, and V. K. Bhargava, "Optimal and suboptimal power allocation schemes for OFDM-based cognitive radio systems," *IEEE Trans. Wireless Commun.*, vol. 7, no. 11, pp. 4710–4718, Nov. 2008.

[4] D. T. Ngo, C. Tellambura, and H. H. Nguyen, "Resource allocation for OFDM-based cognitive radio multicast networks," in *Proc. 2009 IEEE Wireless Commun. Netw. Conf.*, pp. 1–6.

[5] Y. Zhang and C. Leung, "Resource allocation in an OFDM-based cognitive radio system," *IEEE Trans. Commun.*, vol. 57, no. 7, pp. 1928–1931, July 2009.

[6] Q. Lu, T. Peng, W. Wang, and W. Wang, "Optimal subcarrier and power allocation under interference temperature constraints," in *Proc. 2009 IEEE Wireless Commun. and Networking Conf.*, pp. 1–5.

[7] Z. Hasan, G. Bansal, E. Hossain, and V. Bhargava, "Energy-efficient power allocation in OFDM-based cognitive radio systems: a risk-return model," *IEEE Trans. Wireless Commun.*, vol. 8, no. 12, pp. 6078–6088, Dec. 2009.

[8] P. Wang, X. Zhong, L. Xiao, S. Zhou, and J. Wang, "A general power allocation algorithm for OFDM-based cognitive radio systems," in *Proc. 2009 IEEE Int. Conf. Commun. Workshops*, pp. 1–5.

[9] C.-H. Chen and C.-L. Wang, "Joint subcarrier and power allocation in multiuser OFDM-based cognitive radio systems," in *Proc. 2010 IEEE Int. Commun. Conf.*, pp. 1–5.

[10] K. Son, B. C. Jung, S. Chong, and D. K. Sung, "Power allocation for OFDM-based cognitive radio systems under outage constraints," in *Proc. 2010 IEEE Int. Commun. Conf.*, pp. 1–5.

[11] X. Kang, H. Garg, Y.-C. Liang, and R. Zhang, "Optimal power allocation for OFDM-based cognitive radio with new primary transmission protection criteria," *IEEE Trans. Wireless Commun.*, vol. 9, no. 6, pp. 2066–2075, June 2010.

[12] C. Caini, G. Immovilli, and M. L. Merani, "Outage probability for cellular mobile radio systems: simplified analytical evaluation and simulation results," *IET Electron. Lett.*, vol. 28, no. 7, pp. 669–671, Mar. 1992.

[13] S. Kandukuri and S. Boyd, "Optimal power control in interference-limited fading wireless channels with outage-probability specifications," *IEEE Trans. Wireless Commun.*, vol. 1, no. 1, pp. 46–55, Jan. 2002.

[14] F. P. Kelly, "Charging and rate control for elastic traffic," *European Trans. Tel.*, vol. 8, no. 1, pp. 33–37, Jan. 1997.

[15] M. Chiang, "Balancing transport and physical layers in wireless multihop networks: jointly optimal congestion control and power control," *IEEE J. Sel. Areas Commun.*, vol. 23, no. 1, pp. 104–116, Jan. 2005.

[16] S. Hara and R. Prasad, "Overview of multicarrier CDMA," *IEEE Commun. Mag.*, vol. 35, no. 12, pp. 126–133, Dec. 1997.

[17] S. Chen, M. A. Beach, and J. P. McGeehan, "Division-free duplex for wireless applications," *IEEE Electron. Lett.*, vol. 34, no. 2, pp. 147–148, Jan. 1998.

[18] A. Goldsmith, *Wireless Communications*. Cambridge University Press, 2004.

[19] S. Boyd and L. Vandenberghe, *Convex Optimization*. Cambridge University Press, 2004.

[20] D.Bertsekas, *Convex Analysis and Optimization*. Athena Scientific, 2003.

[21] ——, *Nonelinear Programming*. Athena Scientific, 2003.

[22] S. Huang, X. Liu, and Z. Ding, "Distributed power control for cognitive user access based on primary link control feedback," in *Proc. 2010 IEEE INFOCOM*, pp. 1–9.

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