Data Offloading in Heterogeneous Cellular Networks: Stackelberg Game Based Approach

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***Abstract*—In heterogeneous networks (HetNets), low power smallcells, i.e., Wifi, can be offered an economic incentive in order to offload traffic from high-power macrocell, which is usually overloaded. This becomes important in order to maintain efficient operation of the network and generate benefit of tradeoff between macrocell and smallcells. The benefit to smallcells comes from the economic incentive offered by macrocell and the benefit to macrocell is achieved by reducing the load and saving spectrum. However, two important challenges are faced in this cooperation: 1) How much economic incentive can be offered by macrocell, and 2) How much offloading traffic volumes can be admitted by the smallcells. In this paper, we propose a novel game based approach for data offloading scheme to determine the amount of economic incentive a macrocell should offer to smallcells and to determine how much traffic each smallcell should admit from the macrocell. In our proposal, a two-stage non-cooperative Stackelberg game theory is applied to optimize the strategies of both macrocell and smallcells in order to maximize their utilities.**

Index Terms*—Heterogeneous Networks, Data Offloading, Stackelberg Game, Game Theory.

I. Introduction

Wireless data traffic has seen significant growth in volume in recent years due to development of the infrastructure of mobile market (e.g., new generation of wireless network and mobile devices in 4G and 5G) [1]. Moreover, there is a fundamental trend in traffic pattern shifting from data-centric to video-centric. This considerable increase leads to new serious challenges for the mobile network operators (MNOs) who have to enhance and maintain their network infrastructure accordingly. However these operations are often costly and time-consuming [1]. MNOs must find alternative methods to address this problem. One of the approach used by the MNOs is to install small cells to enhance the capacity of cellular networks which are termed as heterogeneous networks. Installation of small cells under the coverage of macrocell causes interference due to use of same frequency bands. Mobile data offloading has been considered as an effective approach to cater with the bursty traffic of cellular networks.

In traditional heterogeneous networks, data traffic of macrocells is deliberately routed to the complementary networks, namely smallcells such as WiFi, picocells and femtocells networks [1][2][3] in order to offload its bursty traffic to small cells. Nevertheless, mobile data offloading may reduce the heavy load of macrocell but it also leads to congestion in small cells. Consequently, it is necessary to have an efficient economic incentive mechanism in order to achieve optimal data offloading. We assume that smallcells belong to third party operators and operate on different frequency bands hence there is no interference between macrocell and smallcells. This paper focuses on data offloading between macrocell and Wifi in heterogeneous cellular network. There are two important issues that need to be answered:

1) How much economic incentive can be offered by macrocell?
2) How much traffic volumes can be admitted by the Wifi access points (APs)?

The *mobile data offloading* problem in heterogeneous wireless networks has recently been widely studied [1][2][3][8]. The authors in [1][2][3] investigated the economics aspect of mobile data offloading. Gao *et al.* [1] defined a multi-leader multi-follower data offloading game, where MBSs (leaders) propose market prices, and accordingly APs (followers) determine the traffic volume they are willing to offload. Using different approach Gao *et al.* [2] model and analyze the interaction between one MNO and multiple AP owners (APOs) for the amount of MNO's offloading data and the respective APOs compensations by using the Nash bargaining theory. Iosifidis *et al.* [3] designed an iterative double auction mechanism that ensures the market where mobile network operators maximize their offloading benefits and APs minimize their offloading costs. Ho *et al.* [8] investigate load coupling problem of interfered regular-cellular base stations in heterogeneous wireless network. They optimize the demand to be served in the regular-cellular or the complementary networks, so as to maximize a utility function.

The key results and contributions of this paper are: This paper proposes a *network economic incentive non-cooperative approach* for the data offloading problem in two-tier heterogeneous networks. We use game-theoretic model to design an economic incentive scheme that encourages each individual AP to admit offloaded traffic for MBS in a non-cooperative fashion. We show that the optimal solution for the proposed scheme can be attained by a two-stage Stackelberg game and the optimal solution represents a unique Nash equilibrium.

The rest of the paper is organized as follows: Section II introduces the network model and defines the problem. Section III analyzes game theoretic framework applied in non-cooperative data offloading scheme based on the Stackelberg Game theory. Numerical results are illustrated in section IV.

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Finally, we conclude our work in section V.

II. System Model

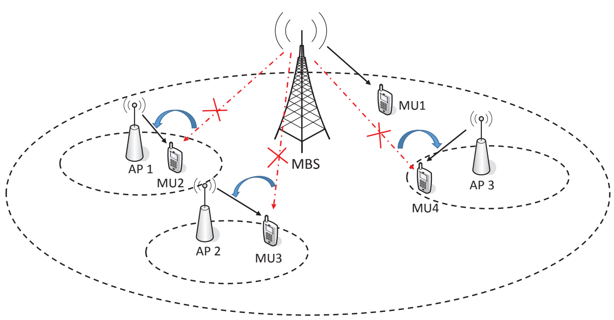

Fig. 1: System model: Two-tier heterogeneous networks consisting of macrocell and multi APs. Mobile user can be offloaded from macrocell to APs

We consider a downlink two-tier heterogeneous network with a MBS, a set of APs, and M is the size of set. MBS and APs use the different orthogonal frequency bands to transmit data, hence there is no cross-tier interference. MBS serves its own group of mobile users (MUs) which are randomly distributed within the MBS and APs' coverages with infinite traffic sources. We study for one time period. The MUs' location and traffic may change over time but for simplicity they are considered fixed within each period.

Let l_m denote the traffic volume that AP m can admit from MBS. Let l_0 be the total MBS's traffic that cannot be offloaded to any AP, i.e., those generated by all MUs of MBS not in coverage of any AP (e.g., MU1 in Fig. 1). The offloaded traffic profile of MBS is denoted by $\boldsymbol{l} \triangleq (l_1, ..., l_M)$. Fig. 1 illustrates our network scenario, where the large circle is the coverage area of macrocell, and small circles are the coverage areas of APs. In this example, the traffic of MU2 can be offloaded to AP1, the traffic of MU-3 can be offloaded to AP-2, and the traffic of MU-4 can be offloaded to AP-3.

Each AP $m \in \mathcal{P}$ has an instantaneous rate c_m which we assume to be fixed over time period and hence the maximum amount of data that it can serve within a certain time period is $c_m T$:

$$l_m \leq c_m T. \tag{1}$$

Without loss of generality, we normalize the time duration to be $T = 1$.

III. Stackelberg Game Theory Analysis

The interaction between the MBS and APs can be characterized as a two-stage non-cooperative Stackelberg game [4][5][6] model as shown in Fig. 2. The MBS publishes the economic incentive in Stage I, and APs respond with their traffic admission abilities in Stage II. All APs want to maximize their total utilities by optimizing the traffic offloaded they can admit according to the economic incentive offered by MBS. The MBS wants to maximize its utility by setting the right economic incentive to satisfy the admit abilities of APs.

In the two-stage Stackelberg game we consider the traffic which each AP can admit from macrocell as a "bid". In the second stage, each AP submits a "bid" to the MBS, then in the first stage, MBS accepts these submitted bids and determines the economic incentive (as shown in Fig. 2). APs are then offered economic incentive corresponding to their "bids". We use the notation $\boldsymbol{l}_{-m}$ to denote the vector of all bids by APs except m; i.e., $\boldsymbol{l}_{-m} = (l_1, ..., l_{m-1}, l_{m+1}, ..., l_M)$.

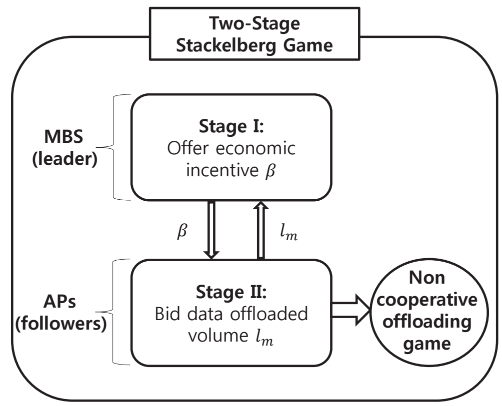

Fig. 2: Two-stage Stackelberg game: MBS's economic incentive, and APs' data offloading

In the following, we first analyse the noncooperative offloading game and best response function according to the payoff function of AP. We then prove the existence and uniqueness of Nash equilibrium for AP's strategy based on the best response function. We show that the Nash equilibrium has a close form for all AP.

A. *Stage II: Access Point's Strategy*

The APs can be modelled as followers. The aim of the the proposed scheme is to maximize the AP's overall utility. The payoff function of each AP $m \in \mathcal{P}$ can be formulated as follows

$$U_m(\boldsymbol{l}) = \beta \frac{l_m}{\sum\limits_{n \in \mathcal{P}} l_n} - \rho_m l_m, \tag{2}$$

where ρ_m and β are unit power cost for offloading traffic from MBS, and the equivalent economic incentive for offloading traffic to all APs, respectively. l_m is offloaded traffic AP m can admit. This payoff function can be explained that the economic incentive each AP achieves is proportional to the offloaded traffic it made for the MBS minus the linear cost it incurs to offload this traffic.

Given the utility function defined as above we now analyze the performance of the Stackelberg game. Given the economic incentive β decided by MBS, APs compete with each other to maximize its own utility by selecting its offloading traffic, which form a *noncooperative offloading game* (NOG).

Definition 1. A noncooperative offloading game $\mathcal{G}$ is defined as a triple $\mathcal{G} \triangleq \{\mathcal{P}, (S_m)_{m\in\mathcal{P}}, (U_m)_{m\in\mathcal{P}}\}$, where $\mathcal{P}$ is the player set (set of all APs), $(S_m)_{m\in\mathcal{P}}$ is the stratergy set of players $S_m \triangleq \{l_m|0 \le l_m \le c_m\}$, and $(U_m)_{m\in\mathcal{P}}$ is the payoff function set.

Definition 2. A player's bid vector $\boldsymbol{l} = (l_1, ..., l_M)$ is a Nash equilibrium of the game $\mathcal{G} \triangleq \{\mathcal{P}, (S_m)_{m\in\mathcal{P}}, (U_m)_{m\in\mathcal{P}}\}$ if, for every $m \in \mathcal{P}, U_m(l_m, \boldsymbol{l}_{-m}) \ge U_m(l_m^{'}, \boldsymbol{l}_{-m})$ for all $l_m^{'} \ge 0$, where $U_m(l_m, \boldsymbol{l}_{-m})$ is the resulting bid for the m^{th} player given the other players' bid results $\boldsymbol{l}_{-m}$.

Theorem 1. A Nash equilibrium exist in the game $\mathcal{G} \triangleq \{\mathcal{P}, (S_m)_{m\in\mathcal{P}}, U_m(\boldsymbol{l})\}$.
Proof. The following result is obtained from [9].

Proposition 2:. A Nash equilibrium exists in game $\mathcal{G} \triangleq \{\mathcal{P}, (S_m)_{m\in\mathcal{P}}, U_m(\boldsymbol{l})\}$, if for all $m \in \mathcal{P}$:
1) $(S_m)_{m\in\mathcal{P}}$ is a nonempty, convex, and compact subset of some Euclidean space $\mathbb{R}^M$.
2) $U_m(\boldsymbol{l})$ is continuous in $\boldsymbol{l}$ and concave in l_m.

Strategy space is defined to be $\mathcal{S} = \{[S_m]_{m\in\mathcal{P}} : l_m|0 \le l_m \le c_m\ \}$. So it is a nonempty, convex and compact subset of the Euclidean space $\mathbb{R}^M$.
From (2) U_m is obviously continuous in $\boldsymbol{l}$. We take the second order derivative with respect to l_m to prove its concavity.

$$\frac{\partial U_m}{\partial l_m} = \frac{\beta \sum\limits_{n\in\mathcal{P}, n\neq m} l_n}{\left(\sum\limits_{n\in\mathcal{P}} l_n\right)^2} - \rho_m, \tag{3}$$

$$\frac{\partial^2 U_m}{\partial^2 l_m} = \frac{-2\beta \sum\limits_{n\in\mathcal{P}, n\neq m} l_n}{\left(\sum\limits_{n\in\mathcal{P}} l_n\right)^3} < 0. \tag{4}$$

The second order derivative of U_m with respect to l_m is always negative, therefore U_m is concave in l_m. According to Proposition 1. A Nash equilibrium exists in game NOG. □

Theorem 2. The game NOG has a unique Nash equilibrium.

Proof. By Theorem 1, we know that there exists a Nash equilibrium in NOG. Let vector $\boldsymbol{l}$ denote the Nash equilibrium in the game NOG. By definition, the Nash equilibrium has to satisfy $\boldsymbol{l} = \boldsymbol{br}(\boldsymbol{l})$ where $\boldsymbol{br}(\boldsymbol{l}) \triangleq (br_1(\boldsymbol{l}), ..., br_M(\boldsymbol{l}))$ and $br_m(\boldsymbol{l})$ is the best response function of player m given offloading selection of other players $br_m(\boldsymbol{l}) = br_m(\boldsymbol{l}_{-m})$. The key aspect of the uniqueness proof is to realize that the best response correspondence $\boldsymbol{br}(\boldsymbol{l})$ is a standard function [7]. A function is said to be standard if it satisfies the following:

- Positivity $\boldsymbol{br}(\boldsymbol{l}) > 0$
- Monotonicity: if $\boldsymbol{l} \ge \boldsymbol{l}^{'}$ then $\boldsymbol{br}(\boldsymbol{l}) \ge \boldsymbol{br}(\boldsymbol{l}^{'})$
- Scalability: for all $\mu > 1$, $\mu\boldsymbol{br}(\boldsymbol{l}) \ge \boldsymbol{br}(\mu\boldsymbol{l})$

It is shown in [7] that the fixed point $\boldsymbol{l} = \boldsymbol{br}(\boldsymbol{l})$ is unique for a standard function. Therefore, the Nash equilibrium of NOG is unique.
The best response for the non-cooperative offloading game is the unique optimal solution for the following optimization problem

$$\begin{aligned} \underset{l_m}{\text{maximize}} \quad & U_m(\mathbf{l}) \\ \text{subject to} \quad & l_m \le c_m, \end{aligned} \tag{5}$$

The constraint means that the offloaded traffic of an AP can not exceed its capacity. Solve problem (4), we have solution as shown in (6)
Assuming that the second condition for the second case are satisfied.

$$\rho_m \sum_{n\neq m} l_n < \beta, \tag{7}$$

and

$$\sqrt{\frac{\beta \sum\limits_{n\neq m} l_n}{\rho_m}} - \sum_{n\neq m} l_n < c_m. \tag{8}$$

Then the best response correspondence is calculated as

$$l_m(\boldsymbol{l}_{-m}) = \sqrt{\frac{\beta \sum\limits_{n\neq m} l_n}{\rho_m}} - \sum_{n\neq m} l_n. \tag{9}$$

We first prove the positivity of $br_m(\boldsymbol{l})$, it is clear that condition (7) is satisfied then $br_m(\boldsymbol{l}) > 0$. As for monotonicity, $br_m(\boldsymbol{l})$ is a quadratic function of the term $\sqrt{\sum\limits_{n\neq m} l_n}$. Therefore, when $\sum\limits_{n\neq m} l_n < \frac{1}{4}\beta$, $br_m(\boldsymbol{l})$ is monotonically increasing function.
As for scalability, we have

$$\begin{aligned} \mu br_m(\mathbf{l}) - br_m(\mu\mathbf{l}) &= \mu\left(\sqrt{\frac{\beta \sum\limits_{n\neq m} l_n}{\rho_m}} - \sum_{n\neq m} l_n\right) \\ &\quad - \left(\sqrt{\frac{\beta\mu \sum\limits_{n\neq m} l_n}{\rho_m}} - \mu\sum_{n\neq m} l_n\right) \\ &= (\mu - \sqrt{\mu})\sqrt{\frac{\beta \sum\limits_{n\neq m} l_n}{\rho_m}}. \end{aligned} \tag{10}$$

For $\forall\mu > 1$ we have $\mu - \sqrt{\mu} > 0$, therefore (10) is positive and $\mu\boldsymbol{br}(\boldsymbol{l}) \ge \boldsymbol{br}(\mu\boldsymbol{l})$ always satisfied.
The best response correspondence $br_m(\boldsymbol{l})$ which is positive, monotonic and scalable, is a standard function. Therefore, there exists a unique Nash equilibrium for NOG game $\mathcal{G} \triangleq \{\mathcal{P}, (S_m)_{m\in\mathcal{P}}, U_m(\boldsymbol{l})\}$. □

Theorem 3. The unique equilibrium for game NOG is given by

$$l_m^* = \left[\frac{\beta(M-1)}{\sum\limits_{m\in\mathcal{P}} \rho_m}\left(1 - \frac{(M-1)\,\rho_m}{\sum\limits_{m\in\mathcal{P}} \rho_m}\right)\right]_0^{c_m}. \tag{11}$$

Proof. Solve the equations set (9) for all players in set, then the result is given by (11). □

$$br_m(1) = l_m^* = \begin{cases} 0, & \text{if } \rho_m \sum\limits_{n \neq m} l_n \geq \beta \\ \sqrt{\frac{\beta \sum\limits_{n \neq m} l_n}{\rho_m}} - \sum\limits_{n \neq m} l_n, & \text{if } \rho_m \sum\limits_{n \neq m} l_n < \beta, \sqrt{\frac{\beta \sum\limits_{n \neq m} l_n}{\rho_m}} - \sum\limits_{n \neq m} l_n < c_m \\ c_m, & \text{if } \sqrt{\frac{\beta \sum\limits_{n \neq m} l_n}{\rho_m}} - \sum\limits_{n \neq m} l_n \geq c_m \end{cases} \quad (6)$$

B. *Stage I: Macrocell Base Station's Strategy*

The MBS can be modelled as leaders. The overall utility function of MBS can be formulated as follows

$$U_{MBS}(\beta) = \delta \sum_{m \in \mathcal{P}} l_m - \beta \sum_{m \in \mathcal{P}} l_m, \quad (12)$$

where δ and β are unit spectrum saved by offloading traffic to APs, and the economic incentive for offloading traffic to all APs, respectively. We consider linear utility and linear cost for all offloaded traffic to all APs.

Based on the knowledge of the APs's behavior in the noncooperative offloading game , the MBS aims to maximize its net profit by solving the following optimization problem

$$\begin{aligned} &\underset{\beta}{\text{maximize}} && U_{MBS}(\beta) \\ &\text{subject to} && \beta \leq \beta^{max} \end{aligned} \quad (13)$$

Substituting (11) into the MBS's utility function (12) we obtain:

$$\max_{\beta}. \quad \frac{(\delta - \beta)\beta(M-1)}{\sum\limits_{m \in \mathcal{P}} \rho_m} \sum_{m \in \mathcal{P}} \left(1 - \frac{(M-1)\,\rho_m}{\sum\limits_{m \in \mathcal{P}} \rho_m}\right). \quad (14)$$

It is straightforward to show that the problem (14) is a convex optimization problem and it can be solved by some standard convex optimization algorithms. Our proposed data offloading scheme is shown in Table I.

We consider the alternative utility function of MBS as follows

$$\begin{aligned} U_{MBS}(\beta) &= \delta \sum_{m \in \mathcal{P}} \log(l_m) - \beta \sum_{m \in \mathcal{P}} l_m \\ &= \delta \log(\prod_{m \in \mathcal{P}} l_m) - \beta \sum_{m \in \mathcal{P}} l_m, \end{aligned} \quad (15)$$

We use the logarithm utility function for proportional fairness offload among APs. Substituting (11) into the MBS's utility function (15) we obtain:

$$\max_{\beta}. \quad \delta \log(\beta^M \Gamma) - \beta^2 \Phi, \quad (16)$$

where

$$\Gamma = \prod_{m \in P} \frac{(M-1)}{\sum\limits_{m \in P} \rho_m} \left(1 - \frac{(M-1)\,\rho_m}{\sum\limits_{m \in P} \rho_m}\right), \quad (17)$$

and

$$\Phi = \frac{(M-1)}{\sum\limits_{m \in P} \rho_m} \sum_{m \in P} \left(1 - \frac{(M-1)\,\rho_m}{\sum\limits_{m \in P} \rho_m}\right). \quad (18)$$

It can be seen that the MBS's utility function is concave, hence, we can obtain the optimal value of economic incentive β^* by the first order optimality condition.

$$\beta^* = \left[\sqrt{\frac{\delta M}{2\Phi}}\right]^{\beta^{max}}. \quad (19)$$

The Lagrangian of the problem (16) is given as follows

$$L(\beta, \lambda) = \delta \log(\beta^M \Gamma) - \beta^2 \Phi + \lambda(\beta - \beta^{max}), \quad (20)$$

where λ is Lagrange multiplier. Then, (16) can be solved by the dual problem given as

$$\begin{aligned} &\underset{\lambda}{\text{maximize}} && D(\lambda) \\ &\text{subject to} && \lambda \geq 0, \end{aligned} \quad (21)$$

where $D(\lambda) = \max\limits_{\beta} L(\beta, \lambda)$ is the dual function. The dual function can be solved by using the sub-gradient method which updates the Lagrange multiplier as follows

$$\lambda(t+1) = [\lambda(t) + \kappa(t)(\beta(t) - \beta^{max})]^+, \quad (22)$$

where $\kappa(t)$ is step size at iteration t which must be positive. $[X]^+ = \max\{0, X\}$. Based on Karush–Kuhn–Tucker conditions (KKT), the optimal economic incentive β^* can be obtained through the following equation

$$\frac{\partial L(\beta, \lambda)}{\partial \beta} = 0, \quad (23)$$

thus, the economic incentive optimal solution is given as follows

$$\beta^* = \left[\frac{\lambda + \sqrt{\lambda^2 + 8\delta M \Phi}}{4\Phi}\right]^+. \quad (24)$$

Based on above optimization analysis, we present the EIDO - Economic Incentive for Data Offloading algorithm

Algorithm 1 EIDO - Economic Incentive for Data Offloading

Step 1: Initialization
Set $\beta(t)$ be any feasible point in feasible set $0 \leq \beta \leq \beta^{max}$;
Set $\lambda(t) = 0$; Set $t = 0$.
Step 2: Lagrange multiplier update
Update Lagrange multiplier $\lambda(t+1)$ according to (22);
Step 3: Economic incentive update
Update economic incentive $\beta(t+1)$ according to

$$\beta(t+1) = \left[\frac{\lambda(t) + \sqrt{\lambda(t)^2 + 8\delta M \Phi}}{4\Phi}\right]^+; \quad (25)$$

We now propose the Data Offloading Scheme as shown in Table I.

TABLE I
DATA OFFLOADING SCHEME

Proposed Data Offloading Scheme
1) MBS calculates economic incentive β^* by Algorithm 1 EIDO and broadcasts this information to all APs (e.g., via the backhaul links) 2) Each AP m computes its optimal offloaded traffic it can admit l_m^* based on the received economic incentive β_m, and transmits offloaded data l_m^*.

IV. NUMERICAL RESULTS

In order to validate our proposition, we did a number of simulations to evaluate the performance of the proposed scheme. For ease of illustration, we consider the scenario as shown in Fig. 1 with a simple network which consists of one macrocell and multiple APs. We vary the number of APs in the system to evaluate the performance of our proposal and find the optimal number of APs supported by the network. We, for our simulations, gradually increase the number of APs from the range of 1 to 20. The power cost for offloading ρ_m is set equal to 1 for all APs. For the EIDO algorithm simulation, we set the boundary of the economic incentive as $\beta_{max} = 3$ and $\beta_{min} = 1$, step size is $\kappa(t) = 0.1$.

Fig. 3 illustrates the increase in offloaded traffic when the number of APs increase. Significant increase of offloaded traffic has been observed when the number of APs is less than 5 and gradually the amount of offloaded traffic increases when the number of APs is greater than 5. However it never exceeds the economic incentive threshold offered by MBS, i.e., $\beta = [1, 1.5, 2]$.

Fig. 4 shows the convergence of the EIDO algorithm for Stackelberg equilibrium. We can observe that for the given offloading traffic offered by APs, the economic incentive achieves the equilibrium after limited iteration steps.

Fig. 5a shows the MBS's payoff achieved with different value of economic incentive in case of linear utility and linear cost, and three cases of unit spectrum saving. It can be seen that lower value of δ give us a lower utility for MBS. Fig. 5b shows the variation of the optimal economic incentive β^* versus number of APs. It can be observed that optimal economic incentive increases significantly when the number of APs is less than 5, compared to economic incentive when number of APs is greater than 5 where a gradual increase was seen, similar results have also been observed in Fig 3.

Fig. 6 demonstrates the macrocell payoff and optimal economic incentive in case of logarithm utility and linear cost. It can be seen in Fig. 6a that the macrocell payoff is lower in comparison with linear utility case. In Fig. 6b the increase of economic incentive when the number of APs increases.

V. CONCLUSION

In this paper, we have developed a game theoretic model for solving data offloading problem in heterogeneous networks, in which macro base station wishes to offload its volume of traffic to Wifi which underlay in its coverage. We have formulated the data offloading problem as a two-stage Stackelberg game. In Stage I, the macro base station offers the economic incentive to the Wifi access points. In Stage II, Wifi access points decide volume of traffic they can admit from macro base station. We also prove that there exists a unique Nash equilibrium in Stage II of the Stackelberg game, this stage is also call non-cooperative game among Wifi access points. We did a number of simulations by varying the number of APs in the network to investigate the variation of optimal economic incentive which can be offered by macro base station.

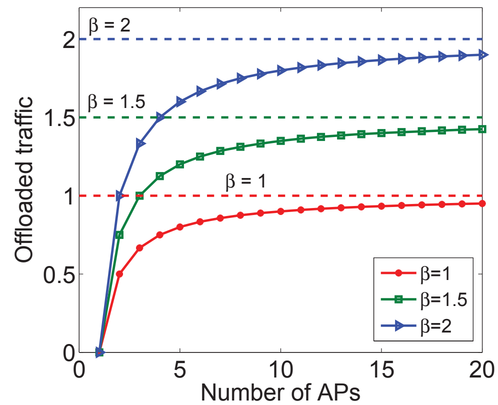

Fig. 3: Optimal offloaded traffic vs. Number of APs

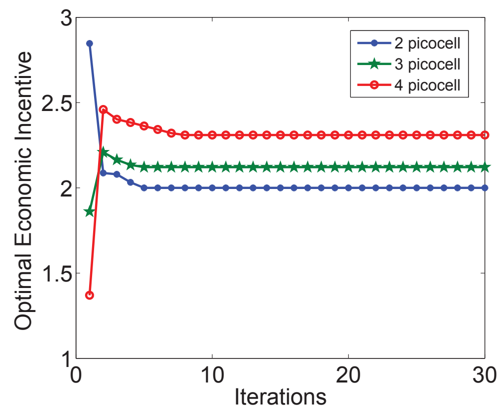

Fig. 4: Convergence of Economic incentive β^* under Algorithm EIDO

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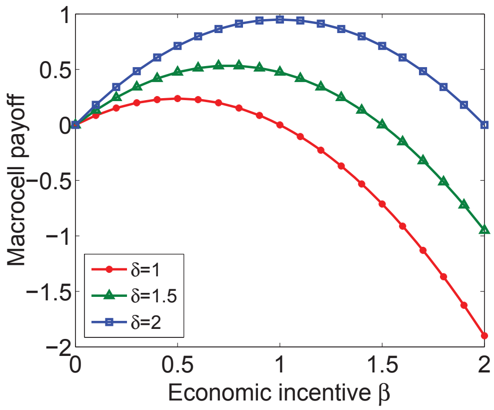
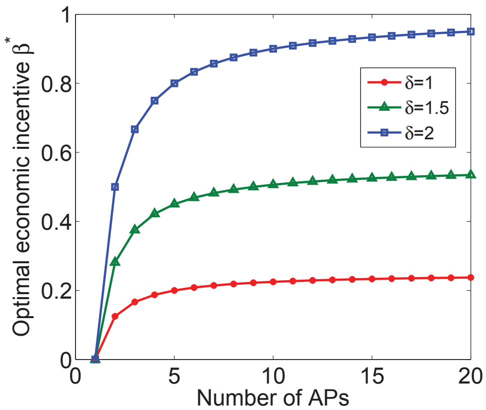
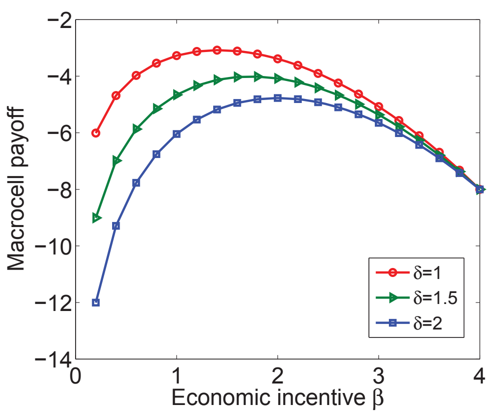


(a)

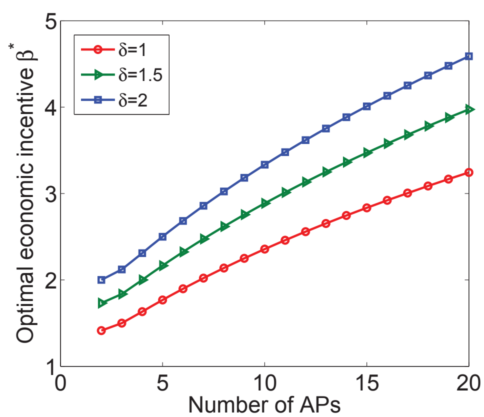


(b)

Fig. 5: MBS's linear utility and linear cost: (a) Macrocell utility Vs economic incentive β, (b) Optimal economic incentive β^* Vs number of APs.



(a)



(b)

Fig. 6: MBS's logarithm utility and linear cost: (a) Macrocell utility Vs economic incentive β, (b) Optimal economic incentive β^* Vs number of APs.

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