Dynamic Pricing for Resource Allocation in Wireless Network Virtualization: A Stackelberg Game Approach

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***Abstract*—The successful virtualization of wireless access networks is strongly affected by the way in which radio resources are managed. The infrastructure provider (InP) is required to deploy efficient and flexible resource allocation techniques to dynamically allocate the resources for the users associated with different mobile virtual network operators (MVNOs). Service contracts with different MVNOs and fairness among their users are crucial to the success of the virtualization scheme deployed by the InP. In this paper, a game-theoretic framework is proposed for resource allocation in OFDMA virtualized wireless network. The framework considers a market model consisting an InP and multiple MVNOs. Regarding the virtual resource for a virtualized wireless network as commodities, the InP wants to maximize its revenue by leasing the infrastructure to the MVNOs while meeting certain contract agreements. Moreover, MVNOs want to serve their users at the best performance and want to pay the minimum to InP. A two-stage Stackelberg game is applied to optimize the strategies of both the InP (the leader) and MVNOs (the followers). We show that this two-stage game has a unique Stackelberg equilibrium.**

***Index Terms*—wireless virtualization, resource allocation, game theory.**

I. Introduction

In the information and communication technology (ICT) sector, virtualization has become a popular concept in different areas, e.g., virtual memory, virtual machines. Virtualization involves abstraction and sharing of resources among different parties. With virtualization, the overall cost of equipment and management can be significantly reduced due to the increased hardware utilization, decoupled functionalities from infrastructure, ease migration to newer services and products, and flexible management. Wireless network virtualization can have a very broad scope ranging from spectrum sharing, infrastructure virtualization, to air interface virtualization, Similar to wired network virtualization, in which physical infrastructure owned by one or more providerss can be shared among multiple service providers, wireless network virtualization needs physical wireless infrastructure and radio resources to be abstracted and isolated to a number of virtual resource, which then can be offered to different service providers [1],[2].

Wireless network virtualization is a means by which an infrastructure provider (InP) can slice the wireless and physical resources to slices. These slices are assigned to mobile virtual network operators (MVNOs) (or service providers SPs in some literatures) so that they can serve their subscribers. The main motivation behind wireless network virtualization is cost saving of network roll-out, maximization of revenue for InPs [1],[2],[3].

Resource allocation plays a very important role in achieving energy efficiency, spectrum efficiency and quality of service (QoS) provisioning in wireless networks in general. However, in wireless network virtualization, isolation has to be taken into account in resource allocation. The problem of resource allocation in wireless virtualization has been studied in several existing works [7]-[19]. Wireless virtualization in LTE is addressed in [7],[8]. In [7], the authors develop an efficient and fast centralized heuristic to allocate the radio resource block in multi-cell LTE networks based on flexible service level agreements of each service provider (SP) expressed as a minimum bandwidth allocation in each cell. The work in [8] derives the dynamic resource allocation scheme via a flexible scheduling while keeping track of the service contracts with the SPs and also the fairness requirement between cell-center users and cell-edge users. The work in [4] addresses the two-level hierarchical resource allocation problem between InPs and MVNOs. A hierarchical combinatorial auction mechanism is provided, which is based on a truthful and sub-efficient resource allocation framework. In [5], the authors propose a virtual resource allocation scheme for OFDMA based wirless virtualization networks and show that a Pareto optimal allocation can be achieved based on the market equilibrium price theory. The problem of joint user association and power/subcarrier assignment in multi-cell OFDMA-based virtualized wireless network was studied in [9] and [10]. In these two papers, the authors apply successive convex approximation (SCA) and complementary geometric programming (CGP) to convert the problem into a computationally efficient formulation and propose an iterative algorithm for solving the non-convex optimization problem. A Lyapunov drift-plus-penalty based algorithm for joint power and sub-carrier allocation was proposed in [11] where a minimum average required data rate of each slice and a stable-queue constraint of wireless virtualized networks are preserved. In [12], the authors virtualize the wireless network and abstract the wireless network resource as the rate region, which is computed as the set of rate that can be achieved by any spectrum allocation. Another wireless resource slicing scheme called NVS was proposed in [13] where virtualization is achieved by modifying the MAC schedulers within the base station. Cellslice in [14] presents an improvement over NVS by implementing slicing in gateways. It basically depends on remote traffic shaping and it supports

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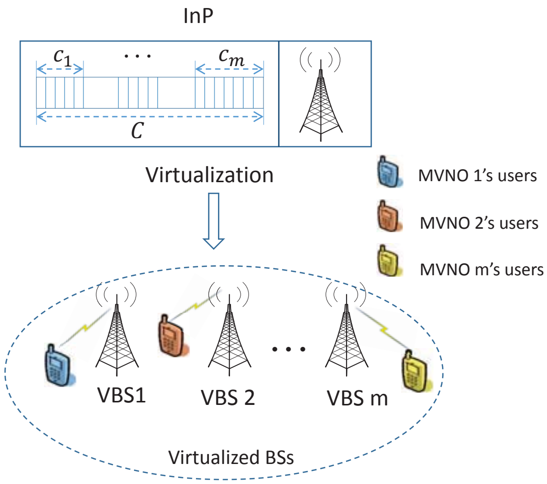


Fig. 1: System model

both downlink and uplink slicing. In [16], by limiting the number of mobile virtual networks (MVNs) embedded in the physical network, the authors showed that MVN admission control can effectively guarantee QoS experienced by users and maximize the utility of the physical network simultaneously. In [17] the authors studied a virtual resource management scheme in green cellular networks with shared full duplex relaying. The problem of energy-aware virtual resource management was formulated as a three-stage Stackelberg game and the subgame perfect equilibrium for each stage was analyzed.

In this paper, we proposed a virtual resource allocation scheme for OFDMA based wireless virtualization networks, which considers the network benefits for InP and MVNOs simultaneously. We consider both the revenue maximization for InP and cost minimization for MVNOs while guarantee service contracts agreements between InP and MVNOs. Comparing with existing works, the main contributions of this paper are summarized as follows:

- We first formulate the resource allocation problem for the wireless network virtualization as a hierarchical two-stage Stackelberg game with InP plays the leader role and MVNOs act as followers.
- We show that there exists a unique Stackelberg equilibrium for the proposed game.

The rest of this paper is organized as follows. Section II introduce wireless network virtualization and system model. The two-stage Stackelberg game is presented in Section III. Section IV discusses the Stackelberg equilibrium and proposed an algorithm for the resource allocation scheme. Numerical results are discussed in Section V. Finally, we conclude this study in Section VI.

II. System Model

We consider the downlink of a single cell consisting single base station (BS) scenario. The BS and spectrum are owned and managed by a single infrastructure provider (InP) who provides its network virtual service to a set of mobile virtual network operators (MVNOs) $\mathcal{M}$ by individual contracts. In general, the isolation between the slices of different MVNOs is required by any wireless virtualization scheme. In this paper we assume a general isolation scheme with no restriction on the assigned resource to the users of different MVNOs, which guarantees the isolation by achieving a certain minimum share of the resources.

We assume that the isolation between different MVNOs is achieved by proper allocation of spectrum resource to the users that belongs to different MVNOs. Each MVNO is assigned a minimum amount of the available resource (BW) on a service contract agreements with InP where we denote by ρ_m^{min} the minimum portion of BW allocated to MVNO m, $\rho_m^{min} \in [0,1], \forall m$, and $\sum_{m=1}^{M} \rho_m^{min} \leq 1$. The system has M MVNOs where each MVNO m provides its service to K_m users.

The InP owns the frequency spectrum denoted by $\mathcal{F}$ with bandwidth size S Hz. The total bandwidth S Hz is divided into C orthogonal subcarriers with the bandwidth size ω_0 Hz. InP sells spectrum of size C_m from the total available bandwidth C and charges price p_m (per unit) to the MVNO m. Each MVNO m has a set of K_m users. Each user k of MVNO m will be allocated an amount of bandwidth $c_{m,k}$ then the achievable data rates can be formulated using Shannon capacity as follow

$$R_{m,k} = c_{m,k}\omega_0 \ln\left(1 + \frac{P_m g_{m,k}}{c_{m,k}\omega_0 n_0}\right), \tag{1}$$

where $c_{m,k}$ represents the amount of bandwidth (i.e., amount of subcarriers in OFDMA) allocated to the kth users of mth MVNO. Let P_m denote the normalized transmit power of the BS to the MVNO m's users, here, fixed equal power allocation mechanism is used, and $g_{m,k}$ is the large-scale channel power gain that includes pathloss and shadowing. n_0 is the background noise.

With virtualization, each MVNO can schedule next serving users and allocate necessary bandwidth to users based its own QoS requirements. Assuming the pre-agreed bandwidth of slice allocated to MVNO m is $\bar{R}_m$, MVNO can allocate any data rate $R_{m,k}$ to its serving user k under the constrains

$$\sum_{k=1}^{K_m} R_{m,k} \leq \bar{R}_m, \ \forall m \in \mathcal{M}. \tag{2}$$

InP conducts the allocation of substrate resource to user k of MVNO m, $\bar{R}_m$ requested by MVNO m should be quaranteed.

InP create virtual resource (VR) slices based on the request from MVNOs, and operate the VR slices and assign them to SPs [1],[2]. We simply assume that if a certain amount of spectrum resource is allocated to a MVNO, then the corresponding physical substrate resource (e.g., base station) is also available for it in the form of VR slice. Hence, we can focus on the spectrum resource in a virtualized wireless network, and the two terms VR allocation and bandwidth allocation can be used alternatively in the rest of this paper.

III. Stackelberg Game

In this section, we formulate the wireless virtual resource allocation strategy as a two-stage Stackelberg game.

The interactions between the InP and MVNOs can be characterized as a two-stage Stackelberg game model. The InP publishes the resource price in a first stage and then the MVNOs respond the resource demand in a second stage. All MVNOs want to serve their uses at the best performance and pay the minimum to InP by optimizing the resource demand

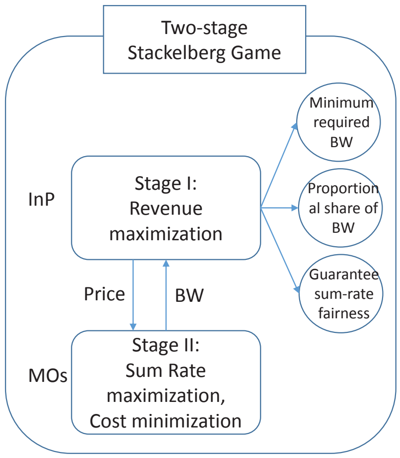


Fig. 2: Game model

according to the resource price offered by the InP. The InP wants to maximize its revenue by setting the right resource price to satisfy the demand of MVNOs while guarantees the contracts agreements, i.e., minimum required bandwidth, proportion share of bandwidth and sum rate fairness. Next, we discuss in details the strategies and modelling of MVNOs and InP respectively.

A. Stage II: MVNO model - Followers Game

We consider the net utility function of MVNO m as follows

$$\mathcal{U}_m(\boldsymbol{c}_m, p_m) = \sum_{k=1}^{K_m} R_{m,k} - p_m \sum_{k=1}^{K_m} c_{m,k} \tag{3}$$

where p_m is the price per unit of bandwidth charged by InP. The objective of the MVNO here is to maximize its users total throughput and minimize cost it has to pay to the InP.

The optimization problem of MVNO m is given as follows

$$\mathbf{P_{MVNO}}: \quad \underset{c_m}{\text{maximize}} \;\; \mathcal{U}_m(\boldsymbol{c}_m, p_m) \tag{4}$$

It is obviously that problem (4) is an integer-programming problem, as the variables to be optimized, i.e., $c_{m,k}$ are in terms of integers. So there exists no polynomial time-complexity algorithm to solve this problem.

B. Stage I: InP model - Leader Game

We consider the revenue function of the InP as follows

$$\mathcal{R}(\boldsymbol{c}, \boldsymbol{p}) = \sum_{m=1}^{M} c_m p_m, \tag{5}$$

where $c_m = \sum_{k=1}^{K_m} c_{m,k}, \forall m$ represents the total bandwidth sold by InP to the MVNO m.

The InP objective is to maximize its revenue by setting the right resource price to satisfy the demand of MVNOs, while guarantees the contracts agreements. The optimization problem of InP is given as follows

$$\mathbf{P_{InP}}: \quad \underset{\boldsymbol{p}}{\text{maximize}} \;\; \mathcal{R}(\boldsymbol{c}, \boldsymbol{p}) \tag{6}$$

$$\text{subject to} \;\; c_m \geq \rho_m^{min} C, \;\forall m \tag{7}$$

$$\sum_{m=1}^{M} c_m \leq C, \tag{8}$$

$$\sum_{k=1}^{K_m} R_{m,k} \leq \bar{R}_m, \;\forall m, \tag{9}$$

$$0 \leq p_m \leq p^{\max}, \;\forall m, \tag{10}$$

where constraint (7) is the minimum required BW for each MVNO, constraint (8) is the proportional share of BW among different MVNOs, (9) is the service contract constraint.

IV. Stackelberg equilibrium and Algorithm

The objective of the proposed Stackelberg game is to find the Stackelberg equilibrium (SE), in which both InP and MVNOs have no incentive to deviate. Since the strategy of one stage will affect the other stage's strategy, we employ the backward induction method to analyze it.

A. Stackelberg equilibrium

Denoting a solution to the InP's revenue maximization by p^*, we have the following definition

Definition 1. $(\boldsymbol{c}^*, \boldsymbol{p}^*)$ is a SE for the proposed game if it satisfies the following conditions for any values of $(\boldsymbol{c}, \boldsymbol{p})$

$$\mathcal{R}(\boldsymbol{c}^*, \boldsymbol{p}^*) \geq \mathcal{R}(\boldsymbol{c}^*, \boldsymbol{p}), \forall p_m \tag{11}$$

$$\mathcal{U}_m(c_m^*, p_m^*) \geq \mathcal{U}_m(c_m, p_m^*), \forall c_m. \tag{12}$$

For the proposed game in this paper, the SE can be obtained as follow: the Stage-II problem is first solved to obtain $\{c_m^*\}$, which is then used to solve the Stage-I problem to obtain p^*.

B. Optimal solution for Stage II

In this session, we solve the relaxed version of problem (4) with continuous feasible space $\mathcal{C} = \{c_{m,k} | c_{m,k} \in \mathbb{R}^+, \forall m, k\}$. Here, we assume that system operates in a high SINR regime, i.e., SINR is much larger than 1; thus, the data rate can be approximated as $R_{m,k} = c_{m,k}\omega_0 \ln\left(\frac{P_m g_{m,k}}{c_{m,k}\omega_0 n_0}\right)$. This approximation is reasonable when the signal level is much higher than the interference level.

Lemma 1. *For given price p_m, the unique optimal solution for Stage-II is:*

$$c_{m,k}^* = G_{m,k} e^{-\left(\frac{\omega_0 + p_m}{\omega_0}\right)}, \;\forall m, k \tag{13}$$

where $G_{m,k} = \frac{P_m g_{m,k}}{\omega_0 n_0}$.

C. Optimal solution for Stage I

We now characterize the optimal solution of the Stage-I based on the optimal solution of Stage II. Substituting (13) into problem (6), the InP problem in Stage-I can be reformulated as

$$\mathbf{P}'_{\mathbf{InP}}: \quad \max_{\boldsymbol{p}}. \quad \sum_{m=1}^{M} p_m \sum_{k=1}^{K_m} G_{m,k} e^{-(\frac{\omega_0+p_m}{\omega_0})} \tag{14}$$

$$\text{s.t.} \quad \sum_{k=1}^{K_m} G_{m,k} e^{-(\frac{\omega_0+p_m}{\omega_0})} \geq \rho_m^{min} C, \ \forall m \tag{15}$$

$$\sum_{m=1}^{M} \sum_{k=1}^{K_m} G_{m,k} e^{-(\frac{\omega_0+p_m}{\omega_0})} \leq C, \tag{16}$$

$$\sum_{k=1}^{K_m} G_{m,k} (\omega_0 + p_m) e^{-(\frac{\omega_0+p_m}{\omega_0})} \leq \bar{R}_m, \ \forall m, \tag{17}$$

$$0 \leq p_m \leq p^{\max}, \ \forall m, \tag{18}$$

We further assume that ρ_m^{min} are chosen such that there exist feasible point in problem (14). It is straightforward to show that problem (14) is a convex problem. The Lagrangian form of (14) can be decomposed into M subproblem as follows:

$$L(\boldsymbol{p}, \lambda, \mu, \nu) = \sum_{n=1}^{M} L_m(p_m, \lambda_m, \mu, \nu_m), \tag{19}$$

where λ_m, μ_m and ν_m are Lagrange multipliers and

$$\begin{aligned} L_m(p_m, \lambda_m, \mu, \nu_m) &= p_m \sum_{k=1}^{K_m} G_{m,k} e^{-(\frac{\omega_0+p_m}{\omega_0})} \\ &+ \lambda_m \sum_{k=1}^{K_m} G_{m,k} e^{-(\frac{\omega_0+p_m}{\omega_0})} - \mu \sum_{k=1}^{K_m} G_{m,k} e^{-(\frac{\omega_0+p_m}{\omega_0})} \\ &- \nu_m \sum_{k=1}^{K_m} G_{m,k} (\omega_0 + p_m) e^{-(\frac{\omega_0+p_m}{\omega_0})} - \delta_m p_m. \end{aligned} \tag{20}$$

The dual problem is then given as

$$\begin{aligned} &\max. \quad D(\lambda, \mu, \nu) \\ &\text{s.t} \quad \lambda, \mu, \nu \geq 0, \end{aligned} \tag{21}$$

where $D(\lambda, \mu, \nu) = \max_{\boldsymbol{p}} L(\boldsymbol{p}, \lambda, \mu, \nu)$ is the dual function. Problem (14) is convex; hence, there exists a strictly feasible point so Slater's condition holds, leading to strong duality [21]. This allows us to solve the primal (14) via the dual (21). The dual problem (21) can be solve using the sub-gradient method, which updates the Lagrange multipliers as follows:

$$\lambda_m^{(t+1)} = \left[\lambda_m^{(t)} - \kappa_\lambda^{(t)} \left(\sum_{k=1}^{K_m} G_{m,k} e^{-(\frac{\omega_0+p_m^{(t)}}{\omega_0})} - \rho_m^{min} C \right) \right]^+, \tag{22}$$

$$\mu^{(t+1)} = \left[\mu^{(t)} + \kappa_\mu^{(t)} \left(\sum_{m=1}^{M} \sum_{k=1}^{K_m} G_{m,k} e^{-(\frac{\omega_0+p_m^{(t)}}{\omega_0})} - C \right) \right]^+, \tag{23}$$

Algorithm 1 Dual based Resource Allocation

input: $\epsilon > 0$
initialize: $t = 0$; $p_m^{(0)}$; $\lambda_m^{(0)}, \mu^{(0)}, \nu_m^{(0)} \geq 0$; $\kappa_\lambda^{(0)}, \kappa_\mu^{(0)}, \kappa_\nu^{(0)} > 0$
repeat
  $t \leftarrow t + 1$;
  Update $\lambda_m^{(t+1)}, \mu^{(t+1)}, \nu_m^{(t+1)}$ according to (22-24);
  Update $p_m^{(t+1)}$ according to (26);
until $|p_m^{(t+1)} - p_m^{(t)}| \leq \epsilon$;
Each MVNO calculates $c^*_{m,k}$ according to (13), and rounds $c^*_{m,k}$ according to (27);

$$\nu_m^{(t+1)} = \left[\nu_m^{(t)} + \kappa_\nu^{(t)} \left(\sum_{k=1}^{K_m} G_{m,k} (\omega_0 + p_m^{(t)}) e^{-(\frac{\omega_0+p_m^{(t)}}{\omega_0})} - \bar{R}_m \right) \right]^+ \tag{24}$$

$$\delta_m^{(t+1)} = \left[\delta_m^{(t)} + \kappa_\delta^{(t)} \left(p_m^{(t)} - p^{\max} \right) \right]^+, \tag{25}$$

where $\kappa_\lambda^{(t)}$,$\kappa_\mu^{(t)}$, $\kappa_\nu^{(t)}$, and $\kappa_\delta^{(t)}$ are positive step sizes, and $[X]^+ = \max\{X, 0\}$. Based on the KKT condition [21], the optimal price $\{p_m\}$ which InP offers to each MVNO m can be obtained through $\frac{\partial L_m}{\partial p_m} = 0$ as follows:

$$p_m^{(t+1)} = \left[\frac{(\omega_0 \nu_m^{(t)} + \mu^{(t)} - \lambda_m^{(t)}) \Lambda^{(t)}}{(1 - \nu_m^{(t)}) \Lambda^{(t)} - \delta_m^{(t)}} \right]^+, \tag{26}$$

where $\Lambda^{(t)} = \sum_{k=1}^{K_m} G_{m,k} e^{-(\frac{\omega_0+p_m^{(t)}}{\omega_0})}$.

Lemma 2. *The unique optimal solution for Stage-I $\boldsymbol{p}^*$ is given as in (26).*

In the context of OFDMA-based spectrum allocation schemes, the common method to deal with this problem is to map the continuous solutions to the largest previous integer as in [15]. Following this method, we can tailor the continuous solution of problem (4) to the discrete one by the following operation

$$\bar{c}^*_{m,k} = \lfloor c^*_{m,k} \rfloor, k = 1, ..., K_m, m = 1, ..., M, \tag{27}$$

where $\lfloor x \rfloor$ denotes the largest integer no more than x.

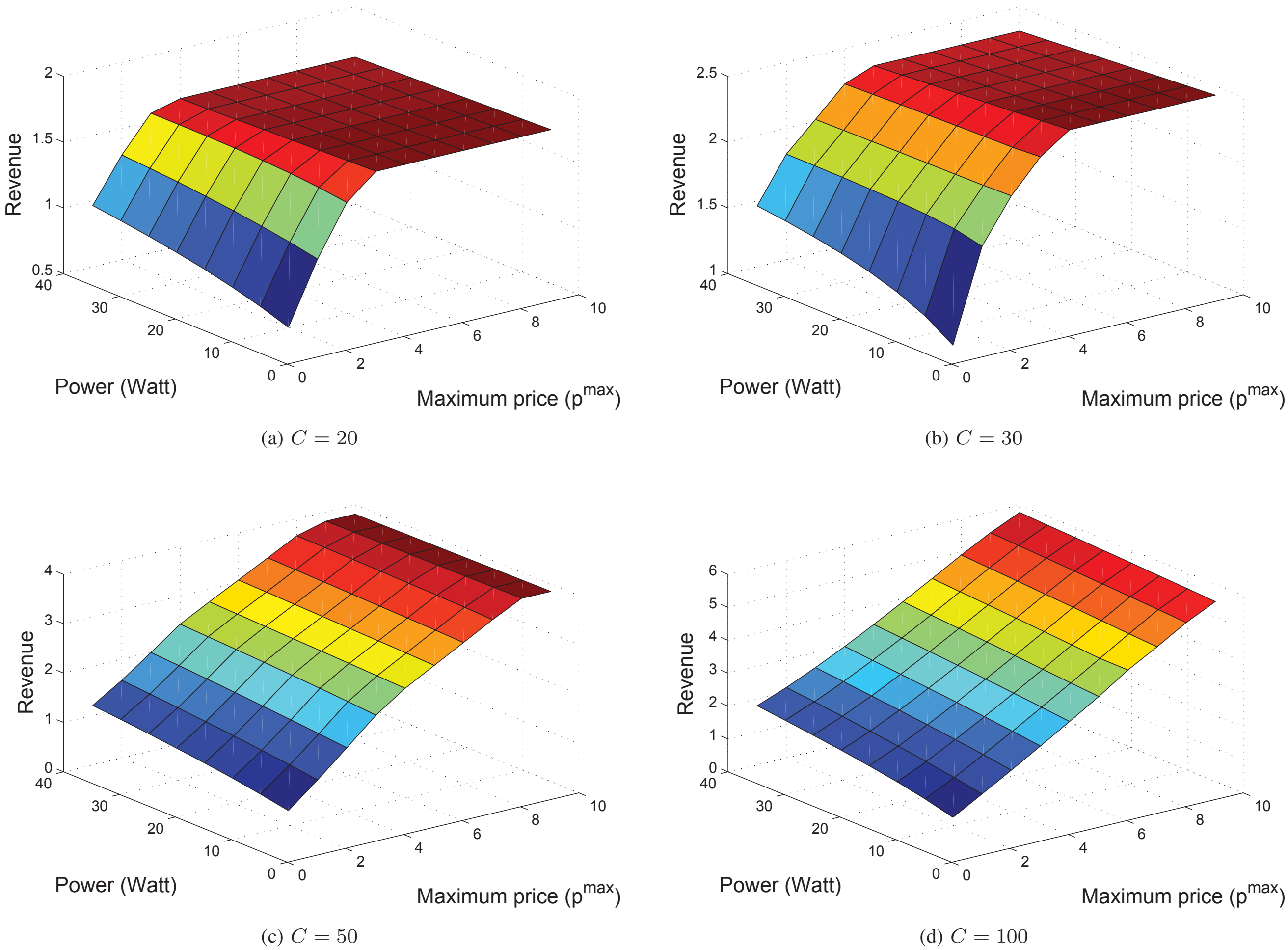
Theorem 1. *The Stackelberg equilibrium for the Stackelberg game formulated in problems (4) and (6) is $(\boldsymbol{c}^*, \boldsymbol{p}^*)$, where $\boldsymbol{c}^*$ is given by (13) and $\boldsymbol{p}^*$ is given by (26).*

Based on above optimization analysis, we present the optimal resource allocation algorithm as shown in Algorithm 1.

V. Numerical Analysis

A. Simulation Setting

In this section, we evaluate the system performance of the proposed framework using simulations. We consider a cellular network where MVNOs' subscribed users are randomly located inside a macro cell of radius 500m belonging to the InP. We consider three MVNOs with 5 subscribed users for each MVNO. We assume the InP owns $C = 20, 30, 50$



(a) $C = 20$

(b) $C = 30$

(c) $C = 50$

(d) $C = 100$

Fig. 4: Revenue versus power (Watt) and maximum price ($p^{\max}$) for different number of subchannels C.

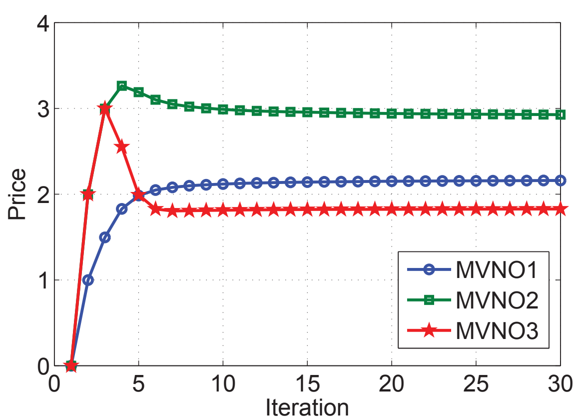


Fig. 3: Convergence of Algorithm 1

and 100 OFDMA subchannels, each of which has a total bandwidth of 180 KHz. The noise power is assumed to be 10^{-13}W. The small-scale fading coefficients of the BS-to-user links are generated as independent and identically distributed (i.i.d.) Rayleigh random variables with unit variance. Channel gains are set as $g_{m,k} = \chi d_{k,m}^{-\beta}$, where χ is a random value generated according to the Rayleigh distribution, $d_{k,m,i}$ is the geographical distance between MBS and user k of MVNO m, and $\beta = 3$ is the pathloss exponent. Here, all results presented are averaged over a large number of independent runs of random locations of users and resource gains.

B. *Numerical Results*

In Fig. 3, we present the convergence of Algorithm 1 in terms of price paid by the MVNOs. In this simulation, we consider that the InP has 30 OFDMA subchannels, i.e., $C = 30$. We can see that Algorithm 1 converges for all the three MVNOs in a limited iteration. i.e., less than 15. Moreover, we can observe that MVNO 2 has an higher price at convergence compared to the remaining MVNOs. This is because of the fact that MVNO 2 request more resources and is charged more by the InP compared to others.

Next in Fig. 4, we run simulation for four different cases when the number of OFDMA subchannels are equal to $20, 30, 50$ and 100. Here, we vary the maximum price threshold charged to MVNO and transmission power of the InP's BS to observe the revenue achieved by the InP. From Fig. 4a and

4b, we observe that when the network has limited number of subcahnnels (i.e., 20 and 30), the revenue gets saturated after a threshold of maximum price $p^{\max} > 4$. Beyond this point, the change in power level is also does not effect the revenue. However, before this thershold, increase in power level increases the revenue of the InP. Similarly an increasing trend in revenue is observed before approaching this threshold while increasing the price. This trend is observed due to the fact InP owns limited resources which cannot fulfill the demands of all MVNOs. Therefore, increasing the price of the subchannel beyond a point will not increase the revenue.

In Figs. 4c and 4d, we simulate for the cases where the subcahnnels are considered to be large enough, i.e., 50 and 100. Here, we observe that the revenue is increasing as the maximum price is increasing. This is because the demands of all the MVNOs are met and more number of subchannels are available to be sold as a service to the MVNOs. However, as the maximum price is increased, the change in power level of BS does not contribute to the revenue of the InP. Additionally, for the case of 50 subchannels, we can observe from Fig. 4c that the threshold of maximum price is achieved beyond which varying the power will not affect the revenue achieved by the InP.

VI. Conclusion

In this paper, we have developed a dynamic pricing mechanism for resource allocation in wireless network virtualization. A hierarchical structure is adopted to model the business interaction between InP and multiple MVNOs. In this model, the InP wants to maximize its revenue by leasing the infrastructure to the MVNOs while meeting certain contract agreements. Moreover, MVNOs want to serve their users at the best performance and want to pay the minimum to InP. A two-stage Stackelberg game is applied to optimize the strategies of both the InP (the leader) and MVNOs (the followers). A dual based algorithm has been proposed to solve the InP's problem. We have shown that the proposed game achieves a unique Stackelberg equilibrium. Numerical results have confirmed that the proposed algorithm converges fairly fast in all considered setups.

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