# Chance-Constrained Spectrum Allocation for Fair LTE-U/Wi-Fi Coexistence

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***Abstract*—In order to meet the ever-increasing data rate demands of future mobile networks, LTE in the unlicensed band (LTE-U) has been proposed in which LTE is operated on the unlicensed spectrum along with the incumbent unlicensed spectrum technologies e.g. Wi-Fi. However, enabling LTE-U with the Wi-Fi network in the same unlicensed spectrum results in severe Wi-Fi performance degradations. This paper addressed such coexistence problem of Wi-Fi and LTE-U, by formulating a chance-constrained optimization problem such as to maintain certain Wi-Fi performance. In the first step, unlicensed spectrum is shared among Wi-Fi and LTE-U by adjusting the so-called duty-cycle according to the random Wi-Fi performance. Secondly, the LTE-U user association is performed through knapsack problem to efficiently utilize the available LTE-U duty-cycle. Simulations results show that the proposed algorithm provides fairness to the Wi-Fi system by adjusting the LTE-U duty-cycle.**

***Index Terms*—Chance-Constrained Optimization, Knapsack Optimization, LTE-U, Wi-Fi Fairness.**

## I. Introduction

Various high-demand applications in the future mobile networks have been increasing the mobile data-rate requirements exponentially. To cope with this exponential growth in the wireless traffic, the concept of small cell networks, that rely on low-cost, low-power, small cell base stations (SBSs) has recently emerged. While small cell networks can boost the capacity of existing cellular networks, they still face many challenges pertaining to the spectrum scarcity. In order to address this spectrum scarcity challenge, the idea of operating wireless cellular networks in the unlicensed band, using the so-called *LTE-U* technology has recently been proposed [1]. Listen before talk (LBT)-based LTE-U standard known as *licensed assisted access* (LAA) is presented in 3GPP Rel-13 to operate in small-cell base stations (SBS) for the downlink communication in 5GHz band [2].

However, enabling of LTE-U may cause severe performance degradation in the pertaining unlicensed spectrum technologies e.g. Wi-Fi. To reap from the benefits of LTE-U, a fair coexistence mechanism is required for better spectrum efficiency and performance maintenance in unlicensed band. It can be seen that the design of such fairness mechanism for LTE-U and Wi-Fi coexistence is not trivial due to the discrepancies in their medium access techniques. Indeed, Wi-Fi systems rely on distributed MAC based on distributed coordination function (DCF) in which the participating peers use contention-based scheme to access the channels [3]. Meanwhile, cellular system use scheduled MAC protocols in which the network resources are smartly allocated to the users for dedicated periods of time.

It is obvious that LTE-U is more spectrum efficient due to its centralized MAC scheduling protocol as compared to Wi-Fi systems. However, LTE-U can cause Wi-Fi performance degradation in the form of long delays and additional collisions [4] if no fair mechanism is applied for sharing the unlicensed spectrum. Therefore, the traditional ways of maximizing the spectrum efficiency using classic optimization theory can't perform well in the coexistence problem of Wi-Fi and LTE-U.

Hence, a fair spectrum sharing algorithm for the coexistence of Wi-Fi and LTE-U is required that can capture the uncertain performance of Wi-Fi system to dynamically allocate the redundant spectrum resources to LTE-U. To address this problem, we propose a *chance-constrained* fair spectrum management scheme for the coexistence of Wi-Fi and LTE-U in order to maximize the LTE-U rate while maintaining certain performance of Wi-Fi system. The chance-constrained optimization problem helps to capture the volatile performance of Wi-Fi system due to random collisions happening in Wi-Fi MAC. Chance-constrained optimization problem formulation is apropos for such coexistence problem to guarantee sufficient performance of Wi-Fi.

The random Wi-Fi performance results in uncertain LTE-U duty-cycle. In order to efficiently utilize the available LTE-U duty-cycle, we formulated the knapsack problem for the user association of cellular users. The knapsack problem efficiently packs the cellular users such that LTE-U rate is maximized while meeting the available duty-cycle bounds. The main contributions of this paper include

- Formulation of chance-constrained optimization problem to capture the volatile performance of Wi-Fi. Based on the random performance of Wi-Fi system, LTE-U duty-cycle is adjusted dynamically for fair LTE/Wi-Fi coexistence.
- The formulated problem is solved in two steps. In the first step, the duty-cycle allocation is performed by using stochastic optimization technique. In the second step, a knapsack problem is devised to pack the cellular users in the available unlicensed spectrum.
- Simulation results are drawn to show that the proposed scheme provide sufficient fairness to Wi-Fi system and efficiently associate the cellular users to maximize LTE-U rate.

The remaining of the paper is organized as follows. Section II gives a review of the literature work already presented for LTE-U. After presenting the system model in Section III, we formulate the chance-constrained optimization problem in Section III-B which is solved through stochastic optimization and knapsack problem in Section IV. Finally, Section V presents the simulation results and Section VI concludes the paper.

## II. Related Work

In recent proposals of heterogeneous networks (HetNets), SBSs are sharing the licensed band with macro base stations (MBS) using almost blank subframe (ABS) and using On/Off period, etc. A number of solutions have been proposed for meeting the spectrum demands in HetNets including device-to-device (D2D) communication, non-orthogonal multiple access (NOMA) [5], and wireless virtualization [6]. However, *spectrum scarcity* is still the bottleneck issue of wireless cellular networks.

Several approaches have been recently proposed to ensure the effective LTE-U and Wi-Fi coexistence [7]–[18]. Such approaches include listen-before-talk (LBT) [10] in which LTE-U applies clear channel assessment (CCA) before allocating the spectrum to LTE-U. Furthermore, an LBT-based fair coexistence approach is proposed in [14] to maximize the airtime of LTE-U while maintaining the performance of Wi-Fi. Another common spectrum access mechanism in LTE-U is so-called duty cycle based spectrum access in which a channel is accessed by LTE-U users for some time duration and then released for Wi-Fi systems [7]. A proportional fairness-based approach is proposed in [9] where channel duty cycles are assigned proportionally among the LTE-U and the Wi-Fi users in an effort to provide coexistence with minimum system information required. Meanwhile, the work in [11] addresses the problem of fair LTE-U and Wi-Fi coexistence by estimating the congestion window sizes from channel access probabilities. The scope of unlicensed spectrum is extended in uplink and downlink resource allocation by formulating echo state networks game in [12]. Solution for coexistence of cellular SBSs with WLAN and other cellular operators is proposed using deep learning prediction of future actions events in [13]. In order to reduce the performance degradation of Wi-Fi, an interference minimization approach is used in [15]. A ruin theoretic approach is proposed in [16] and [19] for spectrum management among Wi-Fi and LTE-U. Furthermore, a bargaining game and multi-game approach is used in [18] and [20] for the coexistence problem of Wi-Fi/LTE-U. However, most of the existing works have applied traditional resource allocation techniques which cannot contribute well to maintain the performance of Wi-Fi while maximizing the spectrum efficiency in LTE-U. Dynamic resource allocation to LTE-U based on the uncertain behavior of Wi-Fi system is not addressed properly in the literature. The main goal of LTE-U to enhance spectral efficiency without degrading the incumbent Wi-Fi system is still needed to be addressed.

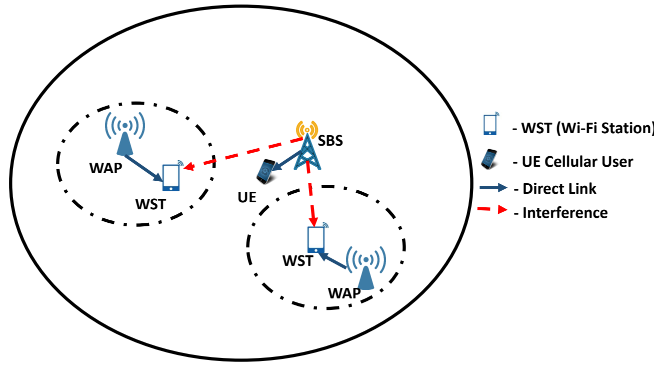


Fig. 1: LTE-U system model. Blue solid lines show the direct communication among the network nodes while the red dotted line shows interference of LTE-U on coexisting Wi-Fi network.

## III. System Model and Problem Formulation:

In the system model, we consider an LTE-U network consisting of an SBS with a set $\mathcal{U}$ of $U$ cellular users. This LTE-U network is coexisting with a Wi-Fi system consisting of Wi-Fi Access Points (WAPs) and Wi-Fi users. We compositely represent WAPs and Wi-Fi users as a set $\mathcal{W}$ of $W$ Wi-Fi stations. Both LTE-U and Wi-Fi system are operated on unlicensed spectrum having a set $\mathcal{K}$ of $K$ channels. It can be seen in the Fig. 1 that LTE-U transmission using the unlicensed band can cause severe interference to the incumbent Wi-Fi system.

In order to efficiently share the unlicensed channels among Wi-Fi and LTE-U networks, we consider the so-called duty-cycle for each channel $k \in \mathcal{K}$ having one part of contention period represented by $1-\beta_k$ for Wi-Fi networks where LTE-U is on silent mode. The other part of the duty-cycle represented by $\beta_k$ is the proportion of duty-cycle of channel $k \in \mathcal{K}$ to be allocated to LTE-U. To efficiently utilize the unlicensed spectrum in such a way that LTE-U rate is maximized while maintaining sufficient Wi-Fi performance, the duty-cycle allocation among Wi-Fi and LTE-U is adjusted accordingly.

Wi-Fi network performance is volatile in nature due to the random number of collisions happening in Wi-Fi network because multiple Wi-Fi stations are contending for the channel access. To capture such randomness in the Wi-Fi networks in order to maintain certain performance level in the form of sufficient throughput of Wi-Fi in each duty-cycle, we use the chance-constraint method.

### A. Chance-constraint and Resource Allocation Model

As shown in Fig. 2, the duty-cycle is divided into two sub-duty-cycles among Wi-Fi and LTE-U for sharing the unlicensed spectrum of bandwidth capacity $C$. The LTE-U duty-cycle is represented by $\beta_k T$, where $T$ is the length of total duty-cycle and $\beta_k$ is the proportion of the duty-cycle allocated to LTE-U. In order to provide sufficient throughput to Wi-Fi, LTE-U duty-cycle i.e. $\beta_k T$ is adjusted dynamically based on the performance of Wi-Fi system. The performance of Wi-Fi

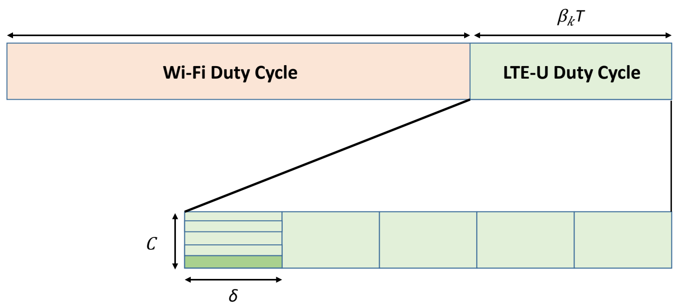


Fig. 2: Duty-cycle distribution among Wi-Fi and LTE-U. LTE-U duty-cycle is further divided into LTE sub-frames for resource allocation to cellular users.

is decreased when both the number of Wi-Fi collisions and LTE-U duty-cycle is increased. In order to maintain the Wi-Fi performance at the desired level $\vartheta$, we use the following chance constraint.

$$P[(X_t + \beta_k T) \leq \zeta] \geq \vartheta, \tag{1}$$

where, $X_t$ is the random variable with Poisson distribution representing the time wasted in Wi-Fi collisions. $\zeta$ represents the tolerance threshold of Wi-Fi which is composed of the total time duration of the number of collisions and LTE-U duty-cycle. The chance-constraint in (1) limits the collision time and LTE-U duty-cycle under a threshold. It is to be noted that in-case of large collision time, the LTE-U duty-cycle is reduced to satisfy the above constraint.

In order to efficiently utilize the available unlicensed spectrum, the LTE-U duty-cycle is divided among cellular users based on the spectrum demand $\eta_u$ of each user $u \in \mathcal{U}$. As shown in Fig. 2, LTE-U duty-cycle is further divided into sub-frames each of time duration $\delta$. $C$ is the total bandwidth of each Wi-Fi channel $k \in \mathcal{K}$. This bandwidth can be divided into $S = C/\omega$ number of LTE-U sub-channels, where $\omega$ is the bandwidth of one LTE sub-channel. Having $\beta_k T$ the length of LTE-U duty-cycle, the number of available LTE-U resource block units can be found as follows

$$Y_k = S \left\lfloor \frac{\beta_k T}{\delta} \right\rfloor, \tag{2}$$

where $\delta$ represents the duration of each LTE-U resource block. The floor function $\lfloor . \rfloor$ is used for round-off.

As we consider a single-cell SBS in our system model, there is no interference offered to the downlink communication in the cellular network. The SNR of each cellular user can be represented as:

$$\gamma_u = \log\left(1 + \frac{P_u g_u}{\sigma^2}\right), \quad \forall u \in \mathcal{U}, \tag{3}$$

where $\sigma^2$ represents the noise temperature.

LTE-U duty-cycle is dynamically adjusted according to the performance of Wi-Fi network which means less duty-cycle is allocated to LTE-U in-case of more collisions happening in Wi-Fi system to maintain certain Wi-Fi throughput. Due to this volatile nature of LTE-U duty-cycle, the resource allocation to cellular users according to the available LTE-U duty-cycle is a challenge. In order to efficiently utilize the available LTE-U duty-cycle, we choose the high-demand users having good SNR level. To select the optimal cellular users, we introduce the following association variable:

$$x_u = \begin{cases} 1 & \text{if user } u \in \mathcal{U} \text{ is associated to the SBS,} \\ 0 & \text{Otherwise.} \end{cases}$$

### B. Problem Formulation

Our aim is to efficiently utilize the available spectrum from unlicensed band while maintaining certain throughput level for the Wi-Fi system. For this purpose, we formulate the optimization problem to maximize the rate of cellular users while satisfying the Wi-Fi fairness and cellular users demand constraints. The optimization problem is formulated as follows:

$$\max_{\boldsymbol{\beta}, \boldsymbol{x}} \sum_{k \in \mathcal{K}} \beta_k C \sum_{u \in \mathcal{U}} x_u \log(1 + \gamma_u), \tag{4}$$

$$\text{s.t.} \quad P[(X_t + \beta_k T) \leq \zeta] \geq \vartheta, \quad \forall k \in \mathcal{K}, \tag{4a}$$

$$\sum_{u \in \mathcal{U}} \eta_u x_u \leq Y_k, \quad \forall k \in \mathcal{K}, \tag{4b}$$

$$\beta_k \in [0, 1], \quad \forall k \in \mathcal{K}, \tag{4c}$$

$$x_u \in \{0, 1\}, \quad \forall u \in \mathcal{U}, \tag{4d}$$

where, the objective of the problem in (4) is the achievable LTE-U rate for the cellular users. The constraint (4a) provides the bound for the LTE-U duty-cycle $\beta_k$. This constraint ensures the fairness to Wi-Fi system by limiting LTE-U duty-cycle under a threshold. Constraint (4b) associates the suitable cellular users and pack them according to the available number of LTE-U resource-block units $Y_k$.

We can see that the problem is difficult to solve using the typical optimization solvers due to randomness in the chance constraint. Also the problem is combinatorial due to the variable $x_u$. We apply stochastic approximation of the chance-constraint to solve this problem. After that knapsack problem is formulated for the user association problem.

## IV. Stochastic Optimization and Knapsack-based Solution:

To solve the above problem, we decompose the problem into two sub-problems.

### A. Stochastic Optimization Based Solution:

The first sub-problem focuses on choosing the duty-cycle based on the Wi-Fi tolerance level. The duty-cycle allocation problem is given as follows:

**Algorithm 1** Chance-constrained Knapsack Algorithm

```
Set the iteration index t = 0.
for Channel k ∈ K do
    Initialize ζ, and ϑ.
    Obtain β*_k from the first sub-problem.
    for u ∈ U do
        Obtain the user demand η_u.
        Obtain SINR level γ_u.
    end for
    Compute the Wi-Fi throughput during the duty-cycle 1 − β_k.
    Run the knapsack algorithm for user association.
    Obtain vector of user association.
    Compute the rate for selected cellular users.
end for
Compute the sum rate for each user.
t + +.
```

$$\max_{\boldsymbol{\beta}} \sum_{k\in\mathcal{K}} \beta_k, \tag{5}$$

$$\text{s.t.} \quad P[(X_t + \beta_k T) \leq \zeta] \geq \vartheta, \quad \forall k \in \mathcal{K}, \tag{5a}$$

$$\beta_k \in [0,1], \quad \forall k \in \mathcal{K}. \tag{5b}$$

We use the stochastic optimization to transform the chance constraint into deterministic constraint by assuming that the collision arrivals are Poisson distributed. This can be done as follows:

Rearranging (1), we get:

$$P[(X_t) \leq \zeta - \beta_k T] \geq \vartheta, \tag{6}$$

where, $P[(X_t) \leq \zeta - \beta_k T]$ can be considered as CDF of Poisson distribution and replaced as $F_{X_t}(\zeta - \beta_k T)$. So we get:

$$F_{X_t}(\zeta - \beta_k T) \geq \vartheta, \tag{7}$$

From here, we can easily get the following expression for $\beta_k$

$$\beta_k T \leq \zeta - \mathcal{F}_{X_t}^{-1}(\vartheta) \tag{8}$$

We can replace the Eq. (8) with Eq. (1) to formulate the following optimization problem.

$$\max_{\boldsymbol{\beta}} \sum_{k\in\mathcal{K}} \beta_k, \tag{9}$$

$$\text{s.t.} \quad \beta_k T \leq \zeta - \mathcal{F}_{X_t}^{-1}(\vartheta), \quad \forall k \in \mathcal{K}, \tag{9a}$$

$$\beta_k \in [0,1], \quad \forall k \in \mathcal{K}. \tag{9b}$$

The above problem is a bounded maximization problem, so it has a boundary solution at the equality of the constraint (9a). Therefore, we get the following optimal solution for $\beta_k$.

$$\beta_k^* = \frac{\zeta - \mathcal{F}_{X_t}^{-1}(\vartheta)}{T}, \quad \forall k \in \mathcal{K}. \tag{10}$$

TABLE I: Simulation Parameters

| Parameters | Values | Parameters | Values |
|---|---|---|---|
| $b\_wind_{min}$ | 15 | $b\_wind_{max}$ | 1023 |
| SIFS | 16 $\mu$s | Slot | 9$\mu$s |
| Ack Size | 14 Byte | Frame Size | 1000 Bytes |
| $\zeta$ | 0.5 | $\sigma$ | -97.5 dBm |
| $p_i$ | 43 dBm | Path Loss | 16.62+37.6log(d)) |

### B. Knapsack-based Solution:

After getting $\beta_k^*$, the next step is to find associated users $x_u$. For that purpose, we got the second sub-problem.

$$\max_{\boldsymbol{x}} \sum_{k\in\mathcal{K}} \beta_k^* C \sum_{u\in\mathcal{U}} x_u \log(1+\gamma_u), \tag{11}$$

$$\text{s.t.} \quad \sum_{u\in\mathcal{U}} \eta_u x_u \leq Y_k, \quad \forall k \in \mathcal{K}, \tag{11a}$$

$$x_u \in \{0,1\}, \quad \forall u \in \mathcal{U}. \tag{11b}$$

We can see that the given problem is a knapsack optimization problem which can be solved by dynamic programming. The working principle of the knapsack problem is as follows:

1) Compose a two dimensional array $V_{U*Y}$, where $U$ is the the number of cellular users in the network and $Y$ is the total unlicensed resource blocks computed from LTE-U duty-cycle.

2) Set $\mathbf{V}^{U*Y}$ to zero. Choose $V[u,y]$ according to the following rule.

$$V[u,y] = max(V[u-1,y], \gamma_u + V[u-1, y-\eta_u]) \tag{12}$$

In this way, the users $u \in \mathcal{U}$ are selected while satisfying the demand bound and maximizing the value $V$ in terms of cellular rate of users. The output of the knapsack algorithm is a vector of suitable cellular users to be associated to the SBS. These users are selected in such a way to maximize the sum rate of LTE-U while utilizing the available number of resource blocks $Y_k$ from the unlicensed spectrum.

The detailed procedure of the proposed scheme is presented in Algorithm 1.

## V. Simulation Results

To draw the simulation results, we built the system model of 1 SBS coexisting with 3 WAPs. Both of the cellular and Wi-Fi networks are coexisting in a geographical area of $400 \times 400$ $m$. Cellular users and Wi-Fi stations are uniformly deployed in their respective cells. The system parameters are listed in Table I.

Fig. 3 shows the topology of the system model built in python. The black triangle represents the SBS while other triangles are representing WAPs. The corresponding users of SBS and WAPs are displayed in the same color dots. It can be seen that the cellular network transmission may interfere all of the coexisting Wi-Fi transmission in the area. Therefore, SBS can opportunistically access any of the unlicensed channel for the transmission of cellular users.

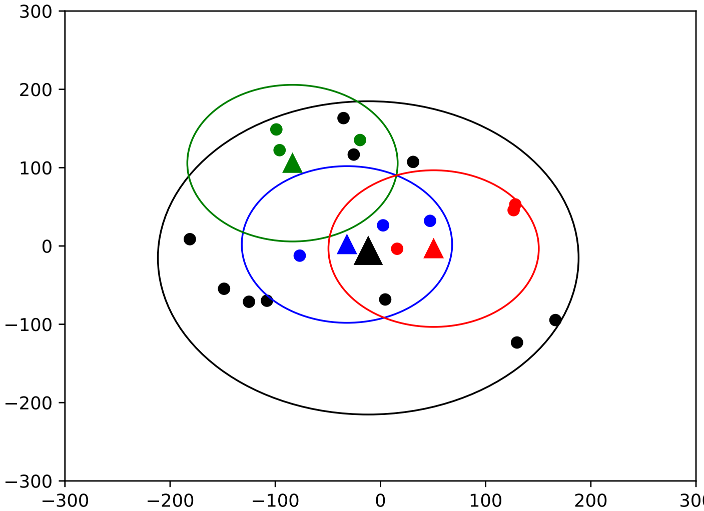


Fig. 3: System model topology for the coexistence of single SBS with 3 WAPs.

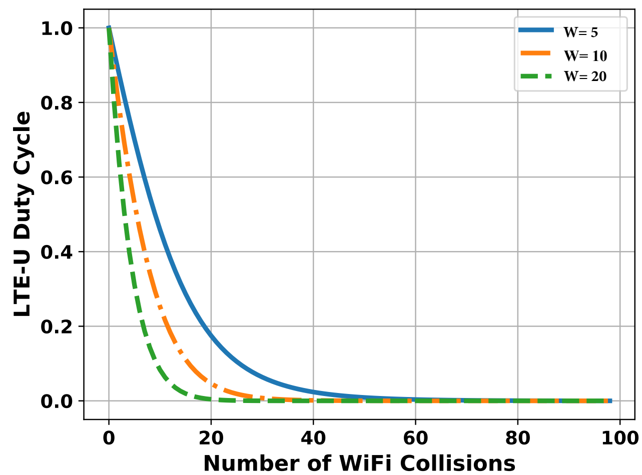


Fig. 4: Plot of Wi-Fi collisions vs. LTE-U duty-cycle

Fig. 4 shows the plot of number of collisions happening in Wi-Fi system verses the LTE-U duty-cycle. It can be seen that the proportion of LTE-U duty-cycle is reduces as the number of Wi-Fi collisions are increased. This dynamic allocation of duty-cycle provides fairness to Wi-Fi system and Wi-Fi is given more chance to cover the loss caused by extra collisions. Moreover, as the number of Wi-Fi stations $W$ increases, the number of collisions happening in Wi-Fi system are also increased. Hence, LTE-U duty-cycle is further reduced to tackle these additional collisions.

Fig. 5 shows the plot of number of LTE-U users vs. the Per user LTE-U rate. It can be seen from the plot that LTE-U rate converges as the number of LTE-U users are increased. This is due to the limited LTE-U duty-cycle obtained from the unlicensed spectrum. It can also be observed that high rate can be achieved when more LTE-U duty-cycle is obtained from the unlicensed spectrum. We can see that further increase in the number of cellular users does not decrease the LTE-U rate because appropriate cellular users are selected using the knapsack algorithm.

Fig. 6 shows the performance of Wi-Fi network against the time axis. It can be seen that Wi-Fi network performance get stabled with the number of iterations. Moreover, the overall Wi-Fi throughput is increased as the Wi-Fi duty-cycle is increased. This result is intuitive because, more transmission time given to the Wi-Fi network increases the overall throughput of Wi-Fi.

## VI. Conclusion

In this paper, we discussed the problem of unlicensed spectrum sharing between Wi-Fi and LTE-U networks. To provide fairness to Wi-Fi system while capturing the randomness of Wi-Fi network due to random collisions, we formulated the chance-constrained optimization problem. We decomposed the formulated problem and solved through stochastic optimization and knapsack problem for the LTE-U duty-cycle management and cellular users resource allocation.

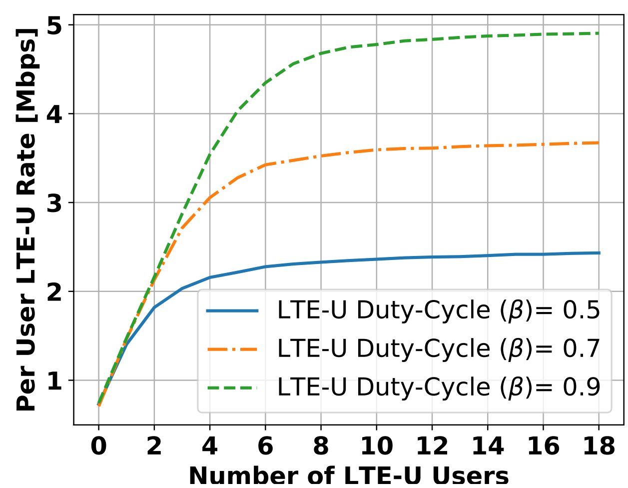


Fig. 5: Plot of number of LTE-U users vs. Per User LTE-U Rate

## Acknowledgment

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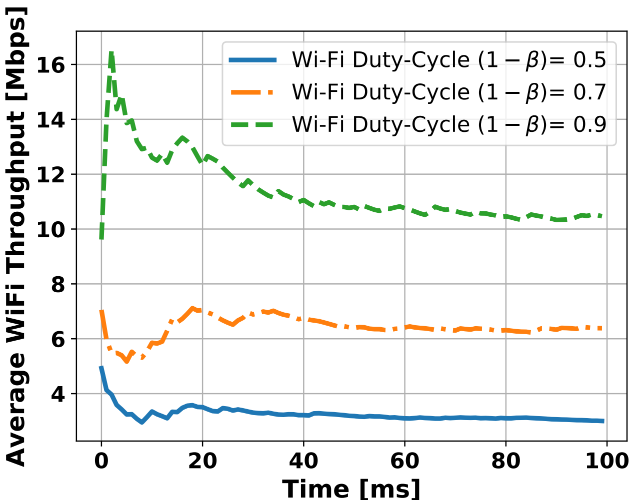


Fig. 6: Plot of Time vs. Wi-Fi Throughput


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