Weighted Proportional Allocation Based Power Allocation in Wireless Network Virtualization for Future Wireless Networks

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***Abstract*—Wireless network virtualization has been considered as a promising technique that will avail the network operators to address the tremendous demand of mobile data services for 5G cellular networks by improving overall network utilization. With this approach, the cellular networks can be logically decoupled into infrastructure providers (InPs) and mobile virtual network operators (MVNOs). In such scenario, an InP owns wireless resources of base stations (e.g., bandwidth, transmit power, antenna, storage capacity, computation capacity) which is shared with multiple MVNOs. Then, each MVNO will allocate the obtained network slices to their mobile users. So, the resource allocation to the users of MVNOs is really challenging. In this paper, we address the power allocation problem in the network virtualization with the objective of maintaining the beneficiary network utilization in an efficient manner, and intra-slice, inter-slice isolation. To address the proposed power allocation problem we adopt the weighted proportional allocation mechanism. Under this mechanism, MVNOs are modeled as bidders who will bid for power for their mobile users and InP is modeled as an auctioneer (i.e., a seller). Finally, numerical results present our proposed algorithm outperforms equal resource sharing in virtualized wireless networks.**

***Index Terms*—Weighted proportional allocation, power allocation, wireless network virtualization, future wireless networks**

I. INTRODUCTION

Recent years have observed the rampant growth in mobile data traffic due to ubiquitous presence of mobile devices and data hungry applications (e.g., video and web applications). To cater such an overwhelming increase of network traffic, currently, wireless network virtualization has attracted the research communities. Wireless network virtualization decouples the current mobile network operators into two entities: the infrastructure providers (InPs) and the mobile virtual network operators (MVNOs). An InP owns the physical network infrastructures (i.e., base stations, cell sites) and the associated wireless resources (e.g., bandwidth, transmission power, antennas, backhaul and so on). MVNOs can lease these resources to their mobile users by creating their own virtual networks for multiple services (e.g., video telephony, live streaming, VoIP) from the multiple InPs. By this way, we see the virtualization of physical radio resources on a shared infrastructure introduces an effective way to mitigate capital expenditures (CAPEX) and operational expenditures (OPEX) for the infrastructure provider (InP), enabling the existence of multiple MVNOs [1].

Although wireless network virtualization is considered as a promising technology for the future wireless networks, it poses several challenges to address. A significant challenge amongst them is an efficient resource allocation problem which includes the issue on how to virtualize the radio resources (e.g., the transmit power), and split it in slices for multiple MVNOs who have to fulfill the dynamic demands of their mobile users or subscribers while maintaining the allocation requirements such as intra-slice, inter-slice isolation [2] [3]. An efficient resource allocation scheme will assist to improve the overall network resource utilization, the quality of services to each user and can correspondingly avoid interference among the MVNOs. Furthermore, the resource allocation problem becomes more challenging in a scenario where the MVNOs behave selfish. Therefore, we need to design a mechanism that can neutralize the impact of the greedy MVNOs, and can achieve the desired social efficiency. Primarily, there are two kinds of design implementation for the resource allocation in network virtualization. In the first design, the InP plays a governing role and as per the predetermined resource requirements directly allocates radio resources to the mobile end users of different MVNOs [4] [5] [6] [7]. In the second, MVNOs are responsible in allocating the resources obtained from the InPs to their mobile users. In this work, we also choose the first design implementation for the power allocation. Most of the existing works in wireless network virtualization consider maximizing the network utilization. However, in this work, we propose the problem from the economize point of view. Here, we try to maximize the social welfare (i.e., sum of valuation of MVNOs) and the utility of MVNOs.

To address the proposed power allocation problem, we design the weighted proportional allocation, which is referred as a class of auction where each MVNO is allowed to submit bids for its users for the resources to the InP. Under the weighted proportional allocation mechanism, after receiving

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bids from different MVNOs, an InP allocates the resource (i.e., transmit power) directly to the mobile users of each bidding agent (MVNO) in proportion to its bidding values [8] [9]. In this mechanism, there will be two kinds of bidders (MVNOs): price taking bidders (i.e., bidders who cannot influence the market clearing price) and price anticipating bidders (i.e., bidders who can influence the market clearing price). We can achieve optimal social welfare when there are only market taking bidders. However, in the real world, there will be both price taking and price anticipating bidders. In the simulation section, we also present the gap between the optimal social welfare and the achieved social welfare under Nash equilibrium. Some of the other efficient auction such as Vickrey-Clarke-Groves (VCG) mechanism focus precisely on the truthful bidding of the bidders, which means each bidder (MVNO)will submit its true valuation. However, bidders will not submit true valuation to the seller (InP) because it is their private information. But under weighted proportional allocation, the seller (InP) can effectively induce the marginal valuation of bidders (MVNOs) without the submission of their true valuation.

The main contributions of this work are as follows:

- Firstly, we model a virtualized wireless network (VWN) and propose the power allocation scheme in VWN.
- Then, the weighted proportional allocation is adopted to address the proposed power allocation problem in virtualized wireless network.
- we carry out the detailed theoretical analysis of weighted proportional allocation such as existence of pure Nash equilibrium, unique resource allocation to each bidder (MVNO), market influence power of each MVNO.

This work considers the power trading problem in the virtualized wireless network, whereas, the basic knowledge of the weighted proportional allocation can be applied in other divisible resources trading problem (e.g., resource blocks, storage, computation resource) in the VWN.

The remainder of this paper is organized as follows: system model and VWN are introduced in Section II. Section III represents the problem formulation and solution approach for the proposed resource allocation problem. Simulation results are presented in Section IV and we conclude this research work at Section V.

II. SYSTEM MODEL

We consider a virtualized wireless network with a single infrastructure provider (InP) that has a macro base station (MBS) and a set of multiple mobile virtual network operators $\mathcal{M} = \{1, 2, \ldots, M\}$, where each MVNO $m \in \mathcal{M}$ has a set of mobile users $\mathcal{U} = \{1, 2, \ldots, U\}$. The macro base station has a set of channels $\mathcal{C} = \{1, 2, \ldots, C\}$ allocated to the users of MVNOs, and the maximum transmit power of the MBS is denoted as $P^{\max}$. A fraction of the maximum transmit power of the MBS allocated to the user u of the MVNO m, assigned on the channel $c \in \mathcal{C}$, is denoted as p_c^{mu}. In this work, we consider a hypervisor is deployed at the MBS which virtualizes the physical resources and then allocates them to the users of different MVNOs. Moreover, we assume that an InP has direct access to the users' information, such as channel station information, QoS requirement, and so on. In our model, MVNOs serve as bidders and an InP is an auctioneer (seller). Each MVNO $m \in \mathcal{M}$ submits the bidding value for each of its user to the InP in each resource allocation round. Let us denote the bidding value of MVNO $m \in \mathcal{M}$ for its user $u \in \mathcal{U}$, assigned to the channel $c \in \mathcal{C}$, as $w_c^{mu} (0 \leq w_c^{mu} < \infty)$. After receiving all of the bidding values from MVNOs, the InP allocates the resource to the users of each MVNO according to their bidding values. If an InP knows the true valuation of the MVNOs, then the resource allocation is straightforward. However, the true valuation of each MVNO is a private information.

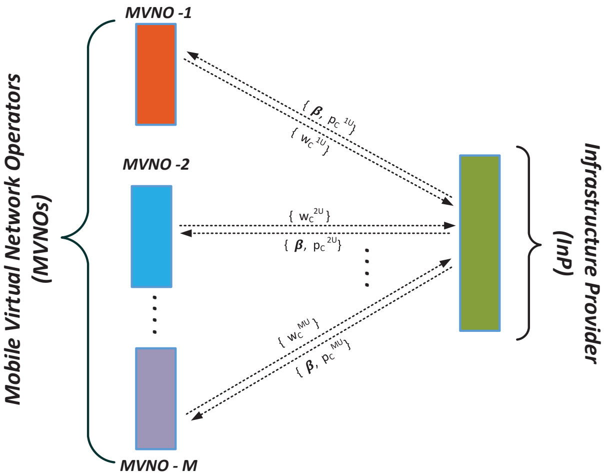

Fig. 1: System Model

III. PROBLEM FORMULATION

Each MVNO decides the demand for the resources depending on the QoS requirement of its user. Note that we consider the well known $\log(.)$ function as the valuation function of MVNOs.

Here, an InP allocates resource to the mobile users of MVNOs proportional to their bidding values. It means that users of the MVNO with higher bidding value will get more resource from the InP. Therefore, the interaction between the MVNOs and an InP can be formulated as a weighted proportional allocation, where the objective is to maximize the sum of the valuation of all MVNOs. Thus, the resource allocation problem between InP and MVNOs in the virtualized wireless network can be written as follows:

$$\max_{\mathbf{p_c^{mu}}} \qquad \sum_{m=1}^{M} \sum_{u=1}^{U} \sum_{c=1}^{C} x_c^{mu} \log \left(B \log_2 \left(1 + \frac{p_c^{mu} h_c^{mu}}{N_0} \right) \right) \tag{1}$$

$$\text{s.t.} \quad x_c^{mu} \in \{0,1\},\ \forall m \in \mathcal{M}, \forall u \in \mathcal{U}, c \in \mathcal{C}, \tag{2}$$

$$\sum_{m=1}^{M}\sum_{u=1}^{U}\sum_{c=1}^{C} x_c^{mu} p_c^{mu} \leq P^{\mathsf{max}}, \tag{3}$$

$$p_c^{mu} \geq 0,\ \forall m \in \mathcal{M}, \forall u \in \mathcal{U}, \forall c \in \mathcal{C}, \tag{4}$$

where B is the wireless bandwidth, p_c^{mu} is the amount of power allocated to the user $u \in \mathcal{U}$ of MVNO $m \in \mathcal{M}$, assigned on the channel $c \in \mathcal{C}$, h_c^{mu} is the channel gain of user u of the MVNO m on channel c, N_0 is the additive white gaussian noise power, and x_c^{mu} is the channel assignment indicator. $x_c^{mu} = 1$ when the channel $c \in \mathcal{C}$ is assigned to user u of MVNO m, and $x_c^{mu} = 0$, otherwise. In this work, we consider only one channel is assigned to each user of MVNOs. Constraint (3) and (4) ensure the total power allocated to the users of all MVNOs must be lower than the maximum transmit power of the MBS.

If an InP knows the valuation of the MVNOs, the resource allocation will not be too difficult. However, the valuation values are the private information of MVNOs. Therefore, MVNOs do not want to reveal their bidding values to the InP. Even when the InP asks to submit the true values, MVNOs misreport theirs to maximize their own utilities. Each MVNO m submits the bidding values to get resource from the InP for its mobile users. Therefore, the resource allocated to the user $u \in \mathcal{U}$ of MVNO $m \in \mathcal{M}$, assigned on channel $c \in \mathcal{C}$, can be characterized as:

$$p_c^{mu}(\mathbf{w}) = \frac{w_c^{um}}{\sum_{m=1}^{M}\sum_{u=1}^{U}\sum_{c=1}^{C} w_c^{um}} P^{\mathsf{max}},\ \forall m, u, c, \tag{5}$$

where $\sum_{m=1}^{M}\sum_{u=1}^{U}\sum_{c=1}^{C} w_c^{um} = W$ is the total bidding value received by the InP. The cost function of each user of MVNOs depends on its bidding value. Then, the utility of $u \in \mathcal{U}$ in MVNO $m \in \mathcal{M}$ can be written as follows

$$O_c^{um}(p_c^{mu}) = \log\left(B \log_2\left(1 + \frac{p_c^{mu} h_c^{mu}}{N_0}\right)\right) - w_c^{um}, \quad u \in \mathcal{U}, \forall c \in \mathcal{C}, \tag{6}$$

Therefore, the utility of MVNO m is as follows:

$$O_m = \sum_{u=1}^{U}\sum_{c=1}^{C} x_c^{mu} O_c^{um}(p_c^{mu}), \quad \forall m \in \mathcal{M}. \tag{7}$$

Proposition 1: The optimal bidding value of user u of MVNO m is as follows:

$$w_c^{mu} = \frac{p_c^{mu} h_c^{mu}(1 - \alpha_c^{mu})}{\ln(2)\log_2\left(1 + \frac{p_c^{mu} h_c^{mu}}{N_0}\right)(N_0 + p_c^{mu} h_c^{mu})}, \quad \forall m \in \mathcal{M}, \forall u \in \mathcal{U}, \forall c \in \mathcal{C}, \tag{8}$$

Proof: See Appendix A. ■

The interaction between an InP and the MVNOs under the weighted proportional allocation mechanism can be expressed as follows : 1) MVNOs submit bidding values for each of their user to the InP, 2) The InP then broadcasts the price of the resource (per dBm) to all of its MVNOs. Therefore, we can express the price of resource in each allocation round as

$$\beta = \frac{\sum_{m=1}^{M}\sum_{u=1}^{U}\sum_{c=1}^{C} w_c^{mu}}{P^{\mathsf{max}}}. \tag{9}$$

After receiving the price of the resource from the InP, each MVNO will know the amount of the resource allocated to its users according to their bidding value. Moreover, as few players get involved in the resource competition game under weighted proportional allocation, the bidding value of each player has a high impact on the price of resource, and each player has the ability to change the outcome of the resource competition game. If there are many players in the competition game, the bidding value (action) of each player cannot influence much to change the outcome of the game. Whereas, when there are infinite number of players in the game, the influence power of each player to change the outcome of the competition game approaches to zero. In our model, we have finite number of MVNOs and each MVNO has a finite number of users. Therefore, the influence power of the bidding value of MVNO $m \in \mathcal{M}$ for it user $u \in \mathcal{U}$, assigned on channel $c \in \mathcal{C}$, is written as follows

$$\alpha_c^{mu} = \frac{w_c^{mu}}{\sum_{m=1}^{M}\sum_{u=1}^{U}\sum_{c=1}^{C} w_c^{mu}},\ \forall m, u, c. \tag{10}$$

From (10), we can see, the influence power of the user u in MVNO m depends on the bidding value of other MVNOs for their mobile users. However, it is not possible to know the bidding value of others, which is a private information. In this scenario, an iterative algorithm allows the user u of MVNO m to approximate its influence power by using its own information of the previous iteration. Therefore, the influence power of u of MVNO m at k^{th} iteration can be written as follows

$$(\alpha_c^{mu})^k = 1 - \frac{(w_c^{mu})^{k-1}\ln(2)\log_2\left(1 + \frac{(p_c^{mu})^{k-1} h_c^{mu}}{N_0}\right)(\rho_c^{mu})^{k-1}}{(p_c^{mu})^{k-1} h_c^{mu}}, \tag{11}$$

where $(\rho_c^{mu})^{k-1} = \left(N_0 + (p_c^{mu})^{k-1} h_c^{mu}\right)$.

In our resource competition game in VWN under weighted proportional allocation, each MVNO will choose the best strategy for each of its user w_c^{mu} to maximize its utility. The strategy profile $w_c^{mu*}, \forall m, u, c$ is the Nash equilibrium of the resource competition game under weighted proportional allocation mechanism when the following relation is satisfied.

$$O_c^{mu}(w_c^{mu*}; \mathbf{w}_{-w_c^{mu}}) \geq O_c^{mu}(w_c^{mu}; \mathbf{w}_{-w_c^{mu}}),\ \forall w_c^{mu} \geq 0, \tag{12}$$

where $\mathbf{w}_{-w_c^{mu}}$ is the bidding values of all users from all MVNOs excepts the user u of MVNO m. We can ensure that our resource competition game achieves a unique Nash equilibrium because the optimization problem (1) under the constraints (2) to (4) is a convex problem.

In order to characterize the equilibrium conditions of the resource competition game under weighted proportional allocation mechanism, we introduce the following function

$$\hat{v}_c^{mu}(p_c^{mu}) = \left(1 - \frac{p_c^{mu}}{P^{\mathsf{max}}}\right) v_c^{mu}(p_c^{mu}) + \frac{1}{P^{\mathsf{max}}} \int_0^{p_c^{mu}} v_c^{mu}(y)d(y), \tag{13}$$

where $v_c^{mu}(p_c^{mu}) = \log\left(B \log_2\left(1 + \frac{p_c^{mu} h_c^{mu}}{N_0}\right)\right)$. The effectual resource allocation to users of MVNOs in this resource competition game under the weighted proportional allocation mechanism can be analyzed using the following problem

$$\max_{\mathbf{p_c^{mu}}} \quad \sum_{m=1}^{M}\sum_{u=1}^{U}\sum_{c=1}^{C} x_c^{mu} \hat{v}_c^{mu}(p_c^{mu}) \tag{14}$$

$$\text{s.t.} \quad x_c^{mu} \in \{0,1\},\ \forall m \in \mathcal{M}, \forall u \in \mathcal{U}, c \in \mathcal{C}, \tag{15}$$

$$\sum_{m=1}^{M}\sum_{u=1}^{U}\sum_{c=1}^{C} x_c^{mu} p_c^{mu} \le P^{\mathsf{max}}, \tag{16}$$

$$p_c^{mu} \ge 0,\ \forall m \in \mathcal{M}, \forall u \in \mathcal{U}, \forall c \in \mathcal{C}. \tag{17}$$

The Lagrangian function of the optimization problem in (14) is as follows

$$\begin{aligned} L(p_c^{mu}, \lambda) = {} & \sum_{m=1}^{M}\sum_{u=1}^{U}\sum_{c=1}^{C} x_c^{mu} \left(1 - \frac{p_c^{mu}}{P^{\mathsf{max}}}\right) v_c^{mu}(p_c^{mu}) \\ & + \sum_{m=1}^{M}\sum_{u=1}^{U}\sum_{c=1}^{C} \frac{x_c^{mu}}{P^{\mathsf{max}}} \int_0^{p_c^{mu}} v_c^{mu}(y)d(y) \\ & + \lambda \left(P^{\mathsf{max}} - \sum_{m=1}^{M}\sum_{u=1}^{U}\sum_{c=1}^{C} x_c^{mu} p_c^{mu}\right), \end{aligned} \tag{18}$$

where $\lambda \ge 0$ is the Lagrangian multiplier for constraint (16). Then, by taking first-order derivative of (18) with respect to p_c^{mu} and λ, we get the Karush-Kuhn-Tucker (KKT) conditions [10] as follows:

$$\begin{aligned} \frac{\partial L(p_c^{mu}, \lambda)}{\partial p_c^{mu}} = {} & -\lambda x_c^{mu} + (1 - \alpha_c^{mu}) \\ & \left(\frac{x_c^{mu} h_c^{mu}}{\ln(2)\log_2\left(1 + \frac{p_c^{mu} h_c^{mu}}{N_0}\right)(N_0 + p_c^{mu} h_c^{mu})}\right), \\ & \text{if } p_c^{mu} \ge 0, \forall m \in \mathcal{M}, \forall u \in \mathcal{U}, \forall c \in \mathcal{C}, \end{aligned} \tag{19}$$

$$\begin{aligned} \frac{\partial L(p_c^{mu}, \lambda)}{\partial \lambda} = {} & P^{\mathsf{max}} - \sum_{m=1}^{M}\sum_{u=1}^{U}\sum_{c=1}^{C} x_c^{mu} p_c^{mu},\ \text{if } p_c^{mu} \ge 0, \\ & \forall m \in \mathcal{M}, \forall u \in \mathcal{U}, \forall c \in \mathcal{C}, \end{aligned} \tag{20}$$

Algorithm 1 Distributed Weighted Proportional Allocation for Optimal Power Allocation in Virtualized Wireless Networks

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1: Initialize: $(w_c^{mu})^0, \alpha_c^{mu} \leftarrow 0, \forall m \in \mathcal{M}, \forall u \in \mathcal{U}, \forall c \in \mathcal{C}$
2:  Set $k \leftarrow 1$
3:  Each MVNO $m \in \mathcal{M}$ calculate its influence ability upon the outcome of the game according to (11);
4:  Each MVNO $m \in \mathcal{M}$ updates the bidding value for each of its users according to (8) then submits to the InP;
5:  According to bidding value, an InP decides the amount of resource to be allocated to each user of each MVNO according to (5) ;
6:  Increment: $k \leftarrow k + 1$
7:  Repeat (3) to (6) until convergence
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When $\lambda > 0$,

$$\frac{h_c^{mu}}{\ln(2)\log_2\left(1 + \frac{p_c^{mu} h_c^{mu}}{N_0}\right)(N_0 + p_c^{mu} h_c^{mu})}(1 - \alpha_c^{mu}) = \lambda, \tag{21}$$

From (8) and (21), we can see that $\lambda = w_c^{mu}/p_c^{mu}$. It is also clear that the optimal bidding strategy of each MVNO for its users shown in (8) is also satisfied. Therefore, the optimal power allocation to each user of each MVNO under the unique Nash equilibrium is

$$p_c^{mu} = \frac{w_c^{mu} \ln(2)\log_2\left(1 + \frac{p_c^{mu} h_c^{mu}}{N_0}\right)(N_0 + p_c^{mu} h_c^{mu})}{h_c^{mu}(1 - \alpha_c^{mu})}. \tag{22}$$

Then, we can conclude the inspection that we find from the optimization problem from (14) as follows

$$\begin{cases} \lambda = w_c^{mu}/p_c^{mu} & \\ \lambda > 0, & \text{when } \sum_{m=1}^{M}\sum_{u=1}^{U}\sum_{c=1}^{C} p_c^{mu} \le P^{\mathsf{max}} \\ \rho = 0, & \text{when } \sum_{m=1}^{M}\sum_{u=1}^{U}\sum_{c=1}^{C} p_c^{mu} \le P^{\mathsf{max}} \end{cases} \tag{23}$$

IV. SIMULATION RESULTS

In this section, we present the performance of the proposed mechanism, weighted proportional allocation, by using simulations. We consider a virtualized wireless network with a single InP, who owns a macro base station, and four MVNOs with [14, 7, 9, 5] mobile users randomly located within the coverage area of the macro base station. Here, we set the radius of the base station as 500m. We also assume that the InP owns 40 OFDMA sub-channels and each channel has the total bandwidth 180kHz. The maximum transmit power of the basestation of the InP is 43dBm and the background noise power is -174dBm/Hz [11]. Fig. 2 shows the achieved valuation of the MVNOs at the equilibrium. it is observed that MVNO-1 achieves the highest valuation because it has more mobile users. The transmit power achieved by MVNOs in the resource competition game is shown in Fig. 3. We

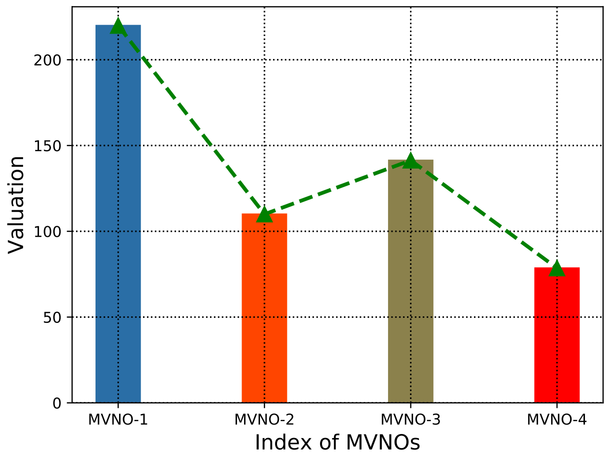

Fig. 2: Valuation of MVNOs

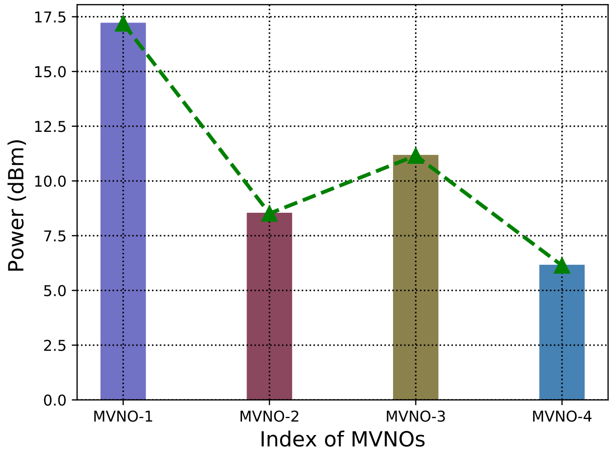

Fig. 3: Power allocation to MVNOs

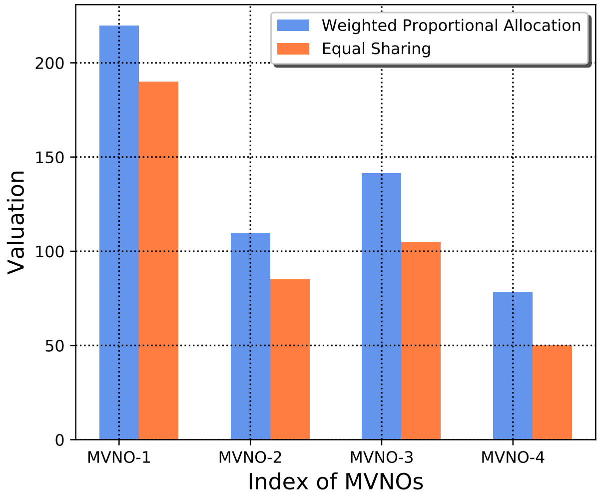

Fig. 4: Performance comparison between proposed weighted proportional allocation and equal sharing

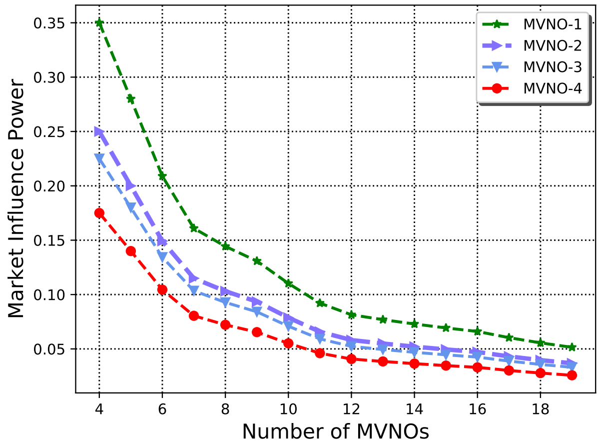

Fig. 5: Market Influence of MVNOs

also compare proposed algorithm, distributed weighted proportional allocation, with equal resource sharing among MVNOs. Fig. 4 demonstrates that our proposed algorithm achieved better performance than equal resource sharing. After that, the nature of the ability of MVNOs to influence the outcome of the resource competition is shown Fig. 5. Here, MVNOs compete with each other to get more network resource from the InP. If there are only four MVNOs in the game, the influence power of all MVNOs are higher. Then, we increase the number of MVNOs in the competition game. As a result, the influence power of all MVNOs are decreasing. When the number of MVNOs is sufficiently larger in the game, the influence power of all MVNOs approaches to zero.

V. CONCLUSION

Wireless network virtualization is one of the approaches to address the explosive growth of mobile devices, data services, and the scarcity of the wireless resource. Moreover, it helps to increase the overall network capacity, and to reduce CAPEX and OPEX of the mobile network operators. In this research work, we propose power allocation problem in the virtualized wireless network from the economic point of view where VWN includes with a single infrastructure provider (InP) and multiple MVNOs. And then, we adopt the weighted proportional allocation to solve the proposed resource allocation problem. The objective of the proposed problem is to maximize the social welfare (i.e., sum of the valuation of MVNOs) of the resource competition game. Simulation results show the power allocation under the proposed algorithm, and fairness of the resource allocation problem. Moreover, we analyze the properties of the weighted proportional allocation. In future works, we will consider multiple resources allocation between multiples InPs and multiple MVNOs.

APPENDIX A
PROOF OF PROPOSITION 1

By differentiation (6) w.r.t the bidding value of user u of MVNO m, we can get the stationary condition and it is as follows

$$\left(\frac{h_c^{mu}}{\ln(2)\log_2\left(1+\frac{p_c^{mu}h_c^{mu}}{N_0}\right)\left(N_0+p_c^{mu}h_c^{mu}\right)}\right)\frac{\partial p_c^{um}}{\partial w_c^{um}}-1=0 \tag{A.1}$$

where

$$\frac{\partial p_c^{um}}{\partial w_c^{um}}=\frac{\left(\sum_{m=1}^{M}\sum_{u=1}^{U}\sum_{c=1}^{C}w_c^{mu}-w_c^{mu}\right)P^{\mathsf{max}}}{\left(\sum_{m=1}^{M}\sum_{u=1}^{U}\sum_{c=1}^{C}w_c^{mu}\right)^2} \tag{A.2}$$

From (A.1) and (A.2), we can get

$$\left(\frac{h_c^{mu}}{\ln(2)\log_2\left(1+\frac{p_c^{mu}h_c^{mu}}{N_0}\right)\left(N_0+p_c^{mu}h_c^{mu}\right)}\right)\times \left(\frac{\left(\sum_{m=1}^{M}\sum_{u=1}^{U}\sum_{c=1}^{C}w_c^{mu}-w_c^{mu}\right)P^{\mathsf{max}}}{\left(\sum_{m=1}^{M}\sum_{u=1}^{U}\sum_{c=1}^{C}w_c^{mu}\right)^2}\right)=1 \tag{A.3}$$

After simplifying (A.3), we get

$$\frac{p_c^{um}}{w_c^{mu}}\left(\frac{h_c^{mu}}{\ln(2)\log_2\left(1+\frac{p_c^{mu}h_c^{mu}}{N_0}\right)\left(N_0+p_c^{mu}h_c^{mu}\right)}\right)(1-\alpha_c^{mu})=1 \tag{A.4}$$

$$w_c^{mu}=\frac{p_c^{mu}h_c^{mu}(1-\alpha_c^{mu})}{\ln(2)\log_2\left(1+\frac{p_c^{mu}h_c^{mu}}{N_0}\right)\left(N_0+p_c^{mu}h_c^{mu}\right)} \tag{A.5}$$

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