

## Joint Congestion Control and Power Control With Outage Constraint in Wireless Multihop Networks

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**Abstract**—We consider the problem of joint congestion control and power control with outage constraint in an interference-limited multihop wireless network. We transform the original nonconvex problem into a convex programming problem and develop a message-passing distributed algorithm that can attain the global optimal source rate and link transmit power. This algorithm however requires a larger control message size than that of the conventional scheme, which increases network overheads. We continue to develop a practical near-optimal distributed algorithm that only requires local signal-to-interference ratio (SIR) measurement to limit the size of the message. Numerical results show that both schemes have nearly identical performance and outperform the conventional scheme.

**Index Terms**—Congestion control, convex optimization, distributed algorithms, power control, utility maximization.

### I. INTRODUCTION

In a wireless multihop network, congestion control and power control have a mutual relationship. Congestion control regulates the source rates to avoid overwhelming any link capacity, which depends on the interference levels, which, in turn, are decided by link transmit power control. Based on this relationship, Chiang [3] characterized the first joint congestion control and power control (JCPC) problem by solving a transformed convex optimization problem. By using the gradient-based algorithm, the author showed that the optimal source rate and link transmit power could be attained in a distributed fashion with message passing. However, the solutions were optimally achieved in a high signal-to-interference ratio (SIR) approximation sense (link capacity approximation), and they are suboptimal in a general sense [13].

Many works later considered different aspects of this JCPC problem. Ghasemi and Faez [6] studied a cross-layer problem of joint congestion, media contention, and power control. Lee and Lim [9] proposed a new window control algorithm for the congestion control and a new power control algorithm to improve throughput and power efficiency. Both of them, however, also employed link capacity approximation. Long *et al.* [11] considered a cross-layer design of random access and power control to adapt for the congestion states with a proposed optimal algorithm, but their algorithm required complicated convexification computation. Tran and Hong [14] tackled the nonconvexity of JCPC using a successive approximation method, which also requires many computations due to the successive approximations. Dogahe *et al.* [5] considered the queuing delay in the JCPC problem, again with high SIR approximation.

Manuscript received April 24, 2011; revised August 5, 2011 and October 24, 2011; accepted November 11, 2011. Date of publication December 15, 2011; date of current version February 21, 2012. This work was supported by the Ministry of Knowledge Economy, Korea, through the Information Technology Research Center Support Program supervised by the National IT Promotion Agency under Grant NIPA-2011-(C1090-1121-0003). C. S. Hong is the corresponding author. The review of this paper was coordinated by Prof. P. Lin.

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Digital Object Identifier 10.1109/TVT.2011.2179955

The aforementioned works also assumed slowly varying wireless channels, implying that such algorithms must attain optimal solutions before the fading state changes. In case of the fast fading channel, the update rate must be fast enough to keep track of changing fading states. This leads to extravagant overheads until the schemes collapse. One solution for this issue is allowing transmission outages to occur between successive updates; as a result, the updates can proceed on a much slower time scale. This idea was first employed in [8] to solve the centralized power control problem. The first work using this idea to address the JCPC problem may have been [13]. However, this work used the outage capacity, which is an approximated link capacity, to implicitly include the outage constraint into their cross-layer design. This outage capacity utilization results in the disappearance of the SIR threshold, which is one of the most important parameters of the outage constraint. Consequently, the network quality-of-service (QoS) control, which can be characterized by tuning this parameter, was lost.<sup>1</sup>

To overcome the limitations of the aforementioned works, we study the JCPC problem without high SIR assumption. Additionally, we employ an explicit outage constraint to battle with fast fading channels and further to keep the SIR threshold alive for network QoS control. After formulating this JCPC as an optimization problem, we prove its convexity, which is not a trivial work due to the complex relationship between interfering powers in the explicit outage constraint. Next, we propose two message-passing distributed algorithms that solve this convex problem. The first algorithm can attain the global optimal solutions using a dual gradient algorithm. However, this scheme requires a larger size of the control message than that of the conventional scheme [3], which increases the network overhead. To overcome this issue, we design a second algorithm, which is near optimal yet practical due to its small size control messages as in [3]. Extensive numerical results show that the gap between the optimal and near-optimal algorithms is almost indistinguishable, and the second design demonstrates a faster convergence rate than the first design. Both schemes also outperform the suboptimal conventional scheme [3].

### II. SYSTEM MODEL AND PROBLEM FORMULATION

#### A. Network Model

We consider a wireless multihop network with  $\mathcal{L} = \{1, \dots, L\}$  logical links shared by  $\mathcal{S} = \{1, \dots, S\}$  sources. We assume that each source  $s$  emits a flow using a fixed set of links  $L(s)$  on its route. The set of sources using link  $l$  is denoted by  $S(l) = \{s \mid l \in L(s)\}$ . The operating ranges of the vectors of source rates  $\mathbf{x} = [x_1, \dots, x_S]^T$  and link transmit powers  $\mathbf{P} = [P_1, \dots, P_L]^T$  are denoted as  $\mathcal{X} = \{x_s, s \in \mathcal{S} \mid x_s^{\min} \leq x_s \leq x_s^{\max}\}$  and  $\mathcal{P} = \{P_l, l \in \mathcal{L} \mid P_l^{\min} \leq P_l \leq P_l^{\max}\}$ , respectively. Each source  $s$  always has data to transmit, and it obtains utility  $U_s(x_s)$  when transmitting a flow at data rate  $x_s$ . The utility function  $U_s(x_s)$  is assumed to be increasing and strictly concave in  $x_s$ . A large class of user fairness can be characterized by the following general  $\alpha$ -fair utility function [12]:

$$U_s^\alpha(x_s) = \begin{cases} (1 - \alpha)^{-1} x_s^{1-\alpha}, & \text{if } \alpha \geq 0, \alpha \neq 1 \\ \log x_s, & \text{if } \alpha = 1. \end{cases} \quad (1)$$

For example, it provides proportional fairness with  $\alpha = 1$ , harmonic mean fairness with  $\alpha = 2$ , and max-min fairness with  $\alpha \rightarrow \infty$ .

<sup>1</sup>See [4], which showed that a minimum success frame rate can be converted to an appropriate SIR threshold for a specific modulation and coding scheme.

We consider the code-division multiple-access model. The instantaneous capacity of link  $l \in \mathcal{L}$  is described as  $c_l(\gamma_l(\mathbf{P})) = W \log(1 + K\gamma_l(\mathbf{P}))$ , where  $W$  is the baseband bandwidth and  $K$  is a constant, depending on modulation, coding scheme, and bit error rate (BER) [7]. Unless otherwise stated, we assume that  $W = K = 1$  without loss of generality.  $\gamma_l(\mathbf{P})$  is the *instantaneous* SIR of link  $l$ , which is defined as  $\gamma_l(\mathbf{P}) = P_l G_{ll} F_{ll} / \sum_{k \neq l} P_k G_{lk} F_{lk} + N_0$ , where gain  $G_{lk}$  represents a slow fading channel (e.g., log normal shadowing), gain  $F_{lk}$  models a fast fading channel from the transmitter on link  $k$  to the receiver on link  $l$ , and  $N_0$  is the thermal noise power at each receiver. We assume a nonline-of-sight radio transmission environment. Hence, we can employ a Rayleigh fading model, where exponential random variables  $F_{lk}$  are i.i.d and their mean values are normalized to unity. Over the considered time scale,  $G_{lk}$  is assumed to be constant. Then, the certainty equivalent SIR is  $\bar{\gamma}_l(\mathbf{P}) = E[P_l G_{ll} F_{ll}] / E[\sum_{k \neq l} P_k G_{lk} F_{lk} + N_0] = P_l G_{ll} / \sum_{k \neq l} P_k G_{lk} + N_0$ .

### B. Problem Formulation: JCPC With an Explicit Outage Constraint

The JCPC with outage constraint can be originally formulated as follows:

$$\begin{aligned} & \underset{\mathbf{x} \in \mathcal{X}, \mathbf{P} \in \mathcal{P}}{\text{maximize}} && \sum_s U_s(x_s) - \sum_l P_l \\ & \text{subject to} && \sum_{s \in S(l)} x_s \leq c_l(\bar{\gamma}_l(\mathbf{P})) \quad \forall l \\ & && \Pr[\gamma_l \leq \gamma_l^{\text{th}}] \leq \xi_l \quad \forall l \end{aligned} \quad (2)$$

where  $c_l(\bar{\gamma}_l(\mathbf{P})) = \log(1 + \bar{\gamma}_l(\mathbf{P}))$ ,  $\Pr[\gamma_l \leq \gamma_l^{\text{th}}]$  is the outage probability defined as the proportion of time that some SIR threshold  $\gamma_l^{\text{th}}$  is not met for a sufficient reception at link  $l$ 's receiver, and  $\xi_l \in (0, 1)$  is the outage probability threshold on link  $l$ . The objective is to maximize the network utility while minimizing the total power. For a Rayleigh fading channel, as in [8], the closed-form outage probability is  $\Pr[\gamma_l \leq \gamma_l^{\text{th}}] = 1 - \exp(-N_0 \gamma_l^{\text{th}} / P_l G_{ll}) \prod_{k \neq l} (1 + \gamma_l^{\text{th}} (P_k G_{lk} / P_l G_{ll}))^{-1}$ . Then, the second constraint of problem (2) can be rewritten as  $\prod_{k \neq l} (1 + \gamma_l^{\text{th}} (P_k G_{lk} / P_l G_{ll})) \leq \Omega_l(P_l)$ , where  $\Omega_l(P_l) = \exp(-N_0 \gamma_l^{\text{th}} / P_l G_{ll}) / (1 - \xi_l)$ .

## III. OPTIMAL ALGORITHM

Problem (2) is a nonconvex optimization problem. In this section, we first transform (2) into an equivalent convex problem and design a distributed optimal algorithm.

### A. Equivalent Convex Formulation

We first denote new variables and sets  $\hat{P}_l = \log P_l$ ,  $\hat{x}_s = \log x_s$ ,  $\hat{\mathcal{X}} = \{\hat{x}_s \mid \forall s \in \mathcal{S} \mid \log x_s^{\min} \leq \hat{x}_s \leq \log x_s^{\max}\}$ ,  $\hat{\mathcal{P}} = \{\hat{P}_l \mid \forall l \in \mathcal{L} \mid \log P_l^{\min} \leq \hat{P}_l \leq \log P_l^{\max}\}$ . We also denote  $\bar{\gamma}_l = \bar{\gamma}_l(\mathbf{P})$  and  $\hat{\gamma}_l = \bar{\gamma}_l(e^{\hat{\mathbf{P}}})$  to simplify the notation. Then, problem (2) is transformed as follows:

$$\underset{\hat{\mathbf{x}} \in \hat{\mathcal{X}}, \hat{\mathbf{P}} \in \hat{\mathcal{P}}}{\text{maximize}} \quad \sum_s U_s(e^{\hat{x}_s}) - \sum_l e^{\hat{P}_l}$$

$$\begin{aligned} \text{subject to} \quad & \log \left( \sum_{s \in S(l)} e^{\hat{x}_s} \right) \leq \log c_l(\hat{\gamma}_l) \quad \forall l \\ & \sum_{k \neq l} \log \left( 1 + e^{\hat{P}_k - \hat{P}_l} \frac{\gamma_l^{\text{th}} G_{lk}}{G_{ll}} \right) \leq \log \Omega_l(e^{\hat{P}_l}) \quad \forall l. \end{aligned} \quad (3)$$

We assume that  $U_s(\exp(\cdot))$  is a concave function with a mild condition as in [10]. Then, we have the following:

*Theorem:* Problem (3) is a convex optimization problem.

*Proof:* See Appendix A.  $\blacksquare$

### B. Dual Decomposition and Optimal Solution

The Lagrangian of (3) can be presented as  $L(\hat{\mathbf{x}}, \hat{\mathbf{P}}, \boldsymbol{\lambda}, \boldsymbol{\nu}) = L_{\hat{\mathbf{x}}}(\hat{\mathbf{x}}, \boldsymbol{\lambda}) + L_{\hat{\mathbf{P}}}(\hat{\mathbf{P}}, \boldsymbol{\lambda}, \boldsymbol{\nu})$ , where  $L_{\hat{\mathbf{x}}}(\hat{\mathbf{x}}, \boldsymbol{\lambda}) = \sum_s U_s(e^{\hat{x}_s}) - \sum_l \lambda_l \log(\sum_{s \in S(l)} e^{\hat{x}_s})$  and  $L_{\hat{\mathbf{P}}}(\hat{\mathbf{P}}, \boldsymbol{\lambda}, \boldsymbol{\nu}) = \sum_l (\lambda_l \log c_l(\hat{\gamma}_l) + \nu_l \log \Omega_l(e^{\hat{P}_l}) - e^{\hat{P}_l} - \nu_l \sum_{k \neq l} \log(1 + e^{\hat{P}_k - \hat{P}_l} (\gamma_l^{\text{th}} G_{lk} / G_{ll})))$ . Here,  $\boldsymbol{\lambda} = [\lambda_1, \dots, \lambda_L]^T$  and  $\boldsymbol{\nu} = [\nu_1, \dots, \nu_L]^T$  are the Lagrange multipliers of the first and second constraints. The partial dual functions can be represented as

$$D_1(\boldsymbol{\lambda}) = \max_{\hat{\mathbf{x}} \in \hat{\mathcal{X}}} L_{\hat{\mathbf{x}}}(\hat{\mathbf{x}}, \boldsymbol{\lambda}) \quad (4)$$

$$D_2(\boldsymbol{\lambda}, \boldsymbol{\nu}) = \max_{\hat{\mathbf{P}} \in \hat{\mathcal{P}}} L_{\hat{\mathbf{P}}}(\hat{\mathbf{P}}, \boldsymbol{\lambda}, \boldsymbol{\nu}). \quad (5)$$

*Proposition 1:* The following source rate update solves the maximization problem (4):

$$x_s(t+1) = [U_s'^{-1}(\lambda_s(t))]_{x_{\min}}^{x_{\max}} \quad (6)$$

where  $U_s'^{-1}$  is the inverse function of the first derivative of the utility function,  $[a]_b^c = \max\{\min\{a, c\}, b\}$ , and  $\lambda_s(t) = \sum_{l \in L(s)} (\lambda_l(t) / \sum_{f \in S(l)} x_f(t))$ .

*Proof:* See Appendix B.  $\blacksquare$

*Proposition 2:* The link power update (7), shown at the bottom of the page, where  $\delta_k(t) = \lambda_k(t) (1 / \log(1 + \bar{\gamma}_k(t))) (\bar{\gamma}_k(t) / (1 + \bar{\gamma}_k(t)))$ ,  $m_k(t) = \delta_k(t) (\bar{\gamma}_k(t) / G_{kk} P_k(t))$ , and  $\tilde{m}_k(t) = \gamma_k^{\text{th}} / G_{kk} P_k(t)$ , solves the maximization problem (5).

*Proof:* See Appendix C.  $\blacksquare$

Because both the partial Lagrangians  $L_{\hat{\mathbf{x}}}(\hat{\mathbf{x}}, \boldsymbol{\lambda})$  and  $L_{\hat{\mathbf{P}}}(\hat{\mathbf{P}}, \boldsymbol{\lambda}, \boldsymbol{\nu})$  are strictly concave, their optimal solutions are unique for a specific  $(\boldsymbol{\lambda}, \boldsymbol{\nu})$ . Dual functions  $D_1(\boldsymbol{\lambda})$  and  $D_2(\boldsymbol{\lambda}, \boldsymbol{\nu})$  are differentiable everywhere according to [1, Prop. 6.1.1], and there is no duality gap [1, Prop. 5.3.1]. By denoting the dual function  $D(\boldsymbol{\lambda}, \boldsymbol{\nu}) = D_1(\boldsymbol{\lambda}) + D_2(\boldsymbol{\lambda}, \boldsymbol{\nu})$ , we can apply the projected gradient method to solve the dual problem  $\min_{(\boldsymbol{\lambda}, \boldsymbol{\nu}) \geq 0} D(\boldsymbol{\lambda}, \boldsymbol{\nu})$  using the dual variable updates as follows:

$$\lambda_l(t+1) = [\lambda_l(t) - \kappa(t) g_l]^+(t)$$

$$\text{with } g_l(t) = \log c_l(\bar{\gamma}_l(t)) - \log \left( \sum_{s \in S(l)} x_s(t) \right) \quad (8)$$

$$P_l(t+1) = \left[ \frac{\delta_l(t) - \nu_l(t) \tilde{m}_l(t) \frac{N_0}{\log(1 - \xi_l)}}{1 + \sum_{k \neq l} (G_{kl} m_k(t) + \nu_k(t) G_{kl} \tilde{m}_k(t) 1 + G_{kl} \tilde{m}_k(t) P_l(t))} \right]_{P_{\min}}^{P_{\max}} \quad (7)$$

$$\nu_l(t+1) = [\nu_l(t) - \kappa(t)h_l(t)]^+$$

$$\text{with } h_l(t) = \log \Omega_l(P_l(t)) - \sum_{k \neq l} \log \left( 1 + \gamma_l^{\text{th}} \frac{G_{lk}P_k(t)}{G_{ll}P_l(t)} \right). \quad (9)$$

Here,  $[a]^+ = \max\{a, 0\}$ ,  $\kappa(t)$  is the positive step size, and vectors  $\mathbf{g}(t) = [g_1(t), \dots, g_L(t)]^T$  and  $\mathbf{h}(t) = [h_1(t), \dots, h_L(t)]^T$  are gradients of dual function  $D(\boldsymbol{\lambda}, \boldsymbol{\nu})$  with respect to  $\boldsymbol{\lambda}$  and  $\boldsymbol{\nu}$ .

We propose an optimal JCPC with outage constraint algorithm as follows:

**Algorithm 1:** Optimal JCPC with an Outage Constraint

- 1) **Initialize** with any  $\{\boldsymbol{\lambda}(0), \boldsymbol{\nu}(0)\} \geq 0$ .
- 2) **Repeat until converge to the global optimal**  $\{\mathbf{x}^*(\boldsymbol{\lambda}^*), \mathbf{P}^*(\boldsymbol{\lambda}^*, \boldsymbol{\nu}^*)\}$ : with  $\{\boldsymbol{\lambda}(t), \boldsymbol{\nu}(t)\}$  available at time  $t$ , the source rate and link transmit power update  $\{\mathbf{x}(t+1), \mathbf{P}(t+1)\}$  for congestion control and power control using (6) and (7), respectively; then, each link updates  $\{\boldsymbol{\lambda}(t+1), \boldsymbol{\nu}(t+1)\}$  using (8) and (9), respectively.

We address the convergence of Algorithm 1 in the following theorem:

*Theorem 2:* For any initial  $\{\boldsymbol{\lambda}(0), \boldsymbol{\nu}(0)\} \geq 0$ , the updates generated via Algorithm 1 converge to the global optimal  $\{\mathbf{x}^*(\boldsymbol{\lambda}^*), \mathbf{P}^*(\boldsymbol{\lambda}^*, \boldsymbol{\nu}^*)\}$  if the step size satisfies  $\sum_{t=0}^{\infty} \kappa(t)^2 < \infty$  and  $\sum_{t=0}^{\infty} \kappa(t) \rightarrow \infty$ .

*Proof:* See Appendix D.  $\blacksquare$

*Remarks:*

- 1) Congestion control can be distributively implemented. The destination sends a message back to the source to adjust its rate according to (6), where the message accumulates  $(\lambda_l(t)/\sum_{f \in S(l)} x_f(t))$  of every intermediate link  $l$  along its path to produce a total price  $\lambda_s(t)$  at source node  $s$ .
- 2) Link power can also be updated in a distributed fashion through message passing, analogous to the algorithm in [3]. Each receiver of link  $k$  broadcasts its control message containing three real value fields, i.e.,  $m_k(t)$ ,  $\tilde{m}_k(t)$ , and  $\nu_k(t)$ . Each transmitter of link  $l$  then receives these values, estimates  $G_{kl}$  by using the training sequences, and updates its power according to (7).
- 3) The  $\boldsymbol{\lambda}(t)$  update in (8) only needs the link's local information, including the ingress rate and SIR measurement.
- 4) The  $\boldsymbol{\nu}(t)$  update in (9) requires the individual received powers of other interfering transmitters. We can reserve the fourth field containing  $P_k(t)$  in the control message broadcast by link  $k$ 's receiver.

#### IV. NEAR-OPTIMAL ALGORITHM

Due to the explicit outage constraint nature of (3), the messages broadcast by receivers contain much information, causing the overhead and energy consumption increase for decoding at transmitters. In this section, we eliminate this issue by proposing a near-optimal scheme.

Using the upper and lower bounds on the outage probability derived in [8], we apply them to the outage constraint as  $\Pr[\gamma_l \leq \gamma_l^{\text{th}}] \leq 1 - \exp(-(\gamma_l^{\text{th}}/\bar{\gamma}_l)) \leq \xi_l$  and  $(\gamma_l^{\text{th}}/\bar{\gamma}_l + \gamma_l^{\text{th}}) \leq \Pr[\gamma_l \leq \gamma_l^{\text{th}}] \leq \xi_l$ , which correspond to these SIR constraints  $\bar{\gamma}_l \geq -(\gamma_l^{\text{th}}/\log(1 - \xi_l))$  (upper bound) and  $\bar{\gamma}_l \geq \gamma_l^{\text{th}}((1/\xi_l) - 1)$  (lower bound), respectively. Hence, the second constraint of problem (2) can be approximately replaced by  $\bar{\gamma}_l \geq \eta_l$ , where  $\eta_l$  is either of those two constants. We have a new optimization problem after changing variables, i.e.,

$$\underset{\hat{\mathbf{x}} \in \hat{\mathcal{X}}, \hat{\mathbf{P}} \in \hat{\mathcal{P}}}{\text{maximize}} \quad \sum_s U_s(e^{\hat{x}_s}) - \sum_l e^{\hat{P}_l}$$

$$\text{subject to} \quad \log \left( \sum_{s \in S(l)} e^{\hat{x}_s} \right) \leq \log c_l(\hat{\gamma}_l)$$

$$\forall l - \log \hat{\gamma}_l \leq -\log \eta_l \quad \forall l. \quad (10)$$

This problem is also a convex programming problem. While the objective function and the first constraint are the same as in the convex problem (3), the second constraint  $-\log \hat{\gamma}_l = -\log(G_{ll}e^{\hat{P}_l}) + \log(\sum_{k \neq l} G_{lk}e^{\hat{P}_k} + N_0)$  is a convex function due to the sum of linear and log-sum-exp terms. The partial Lagrangians of (10) are  $L_{\hat{\mathbf{x}}}(\hat{\mathbf{x}}, \boldsymbol{\lambda}) = \sum_s U_s(e^{\hat{x}_s}) - \sum_l \lambda_l \log(\sum_{s \in S(l)} e^{\hat{x}_s})$  and  $L_{\hat{\mathbf{P}}}(\hat{\mathbf{P}}, \boldsymbol{\lambda}, \boldsymbol{\nu}) = \sum_l (\lambda_l \log c_l(\hat{\gamma}_l) + \nu_l \log \hat{\gamma}_l - e^{\hat{P}_l})$ . We see that the congestion control mechanism is the same as Algorithm 1. We focus on the power control in the following result:

*Proposition 3:* The following link power update solves the maximization problem (5):

$$P_l(t+1) = \left[ \frac{\delta_l(t) + \nu_l(t)}{1 + \sum_{k \neq l} G_{kl}m_k(t)} \right]_{P_{\min}}^{P_{\max}} \quad (11)$$

where  $\delta_k(t) = \lambda_k(t)(1/\log(1 + \bar{\gamma}_k(t)))(\bar{\gamma}_k(t)/(1 + \bar{\gamma}_k(t)))$ , and  $m_k(t) = (\delta_k(t) + \nu_k(t))(\bar{\gamma}_k(t)/G_{kk}P_k(t))$ .

*Proof:* The proof is similar to that of Proposition 2. We skip it due to the limited space.  $\blacksquare$

Using the projected gradient algorithm to solve the dual problem, dual variables update as follows:

$$\lambda_l(t+1) = \left[ \lambda_l(t) - \kappa(t) \left( -\log \left( \sum_{s \in S(l)} x_s(t) \right) + \log c_l(\bar{\gamma}_l(t)) \right) \right]^+ \quad (12)$$

$$\nu_l(t+1) = [\nu_l(t) - \kappa(t)(\log \bar{\gamma}_l(t) - \log \eta_l)]^+ \quad (13)$$

We design the near-optimal algorithm (due to the constraint approximation) as follows:

**Algorithm 2:** Near-Optimal JCPC with an Outage Constraint

- 1) **Initialize** with any  $\{\boldsymbol{\lambda}(0), \boldsymbol{\nu}(0)\} \geq 0$
- 2) **Repeat until converge to the global optimal**  $\{\mathbf{x}^*(\boldsymbol{\lambda}^*), \mathbf{P}^*(\boldsymbol{\lambda}^*, \boldsymbol{\nu}^*)\}$ : with  $\{\boldsymbol{\lambda}(t), \boldsymbol{\nu}(t)\}$  available at time  $t$ , the source rate and link transmit power update  $\{\mathbf{x}(t+1), \mathbf{P}(t+1)\}$  for congestion control and power control using (6) and (11), respectively; then, each link updates  $\{\boldsymbol{\lambda}(t+1), \boldsymbol{\nu}(t+1)\}$  using (12) and (13), respectively.

The convergence of Algorithm 2 is described by the following theorem, which can be proved in a similar manner as Theorem 2; hence, it is omitted due to the limited space.

*Theorem 3:* For any initial  $\{\boldsymbol{\lambda}(0), \boldsymbol{\nu}(0)\} \geq 0$ , the updates generated via Algorithm 2 converge to the global optimal  $\{\mathbf{x}^*(\boldsymbol{\lambda}^*), \mathbf{P}^*(\boldsymbol{\lambda}^*, \boldsymbol{\nu}^*)\}$  if the step size satisfies  $\sum_{t=0}^{\infty} \kappa(t)^2 < \infty$  and  $\sum_{t=0}^{\infty} \kappa(t) \rightarrow \infty$ .

*Remarks:*

- 1) The congestion control mechanism is the same as in Algorithm 1.
- 2) Link power control (11) is much more simplified than that of Algorithm 1, where the control message broadcast by each receiver of link  $k$  only contains  $m_k(t)$  with locally measurable quantities.
- 3)  $\boldsymbol{\nu}(t)$  update in (13) requires only its link's local SIR measurement.

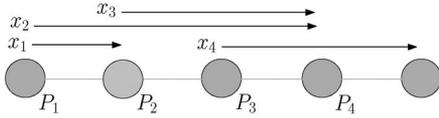


Fig. 1. Network topology with four flows and five nodes equidistantly placed.

TABLE I  
CONVERGENCE SPEED COMPARISON

$\epsilon$	$10^{-3}$	$10^{-4}$	$10^{-5}$
Algorithm 1	93.97	141.26	189.01
Algorithm 2	33.34	58.29	105.23

## V. NUMERICAL RESULTS

### A. Simulation Setting

We consider a network topology as in Fig. 1 with four flows and five nodes equidistantly placed at  $d$  meters. Baseband bandwidth  $W$  is set to 32 kHz, and we use  $K = -1.5/\log(5 \text{ BER})$  with  $\text{BER} = 10^{-3}$  for adaptive MQAM modulation [7]. The slow fading channel gain is assumed to be  $h(d) = h_o(d/100)^{-4}$ , where  $h_o$  is a reference channel gain at a distance of 100 m. The maximum power, noise, and  $h_o$  are selected, so that the average receive SNR at 100 m is 30 dB. We choose  $P_l^{\min} = 1 \text{ mW}$  and  $P_l^{\max} = 100 \text{ mW}$ , whereas  $x_s^{\min} = 0$ , and  $x_s^{\max}$  is dynamically adjusted with respect to link capacities. The step sizes are chosen to be  $0.01/t$ . The  $\alpha$ -fair utility function (1) is set to all users, which means that congestion control (6) is  $x_s(t+1) = [\lambda_s(t) - (1/\alpha)]_{x_{\min}}^{x_{\max}}$ . Since inner links suffer more interference than do outer links, we set the outage probability thresholds  $\xi_l$  of link 2 and link 3 to 0.3 and those of link 1 and link 4 to 0.2. The SIR thresholds  $\gamma_l^{\text{th}}$  of the four links are set to (0.6 0.2 0.2 0.6) dB. These are the maximal threshold values with which our simulation can converge to an optimal feasible point. For any thresholds larger than these values, link powers converge to the boundary values ( $P_l^{\min}$  or  $P_l^{\max}$ ).

### B. Optimal Gap

We investigate the impact of utility parameter  $\alpha$  on the network performance. This parameter can act as a *knob* to control the tradeoff between network efficiency and fairness in a general NUM problem [10]. We fix  $d = 80 \text{ m}$  and vary  $\alpha$  from 1 to 10 to compare the network efficiency (objective value) and fairness, where we use the Jain's fairness index as the standard fairness measurement:  $(\sum_s x_s)^2 / (S \sum_s x_s^2)$ . As shown in Fig. 2, when  $\alpha$  increases, the objective value achieves the maximum value at  $\alpha = 1.5$  and then becomes less efficient. The fairness of the system increases when  $\alpha$  increases. We observe from Fig. 2(a) that the performances of Algorithm 1 and Algorithm 2 with upper and lower bounds are almost indistinguishable due to the tight outage probability bounds. Moreover, both algorithms clearly outperform the conventional scheme [3], which spent higher power transmission due to the high SIR approximation. From Fig. 2(b), all of the compared schemes achieve nearly the same fairness performance. This can be explained that they are in the same manner of proportional allocation of congestion control (i.e., all schemes use the same update (6) for solving congestion control).

### C. Algorithm Convergence

The criterion used to evaluate the convergence speed is  $\max_l (|P_l(t) - P_l^{(t-1)}| / P_l^{(t-1)}) < \epsilon$ , where  $\epsilon$  is an arbitrary small number. We fix  $\alpha = 1$  (i.e.,  $U_s(x_s) = \log x_s$ , provided the *proportional fairness*, which is a well-known fairness criterion in telecommunication literature) and  $d = 80 \text{ m}$  for these scenarios. Table I shows the

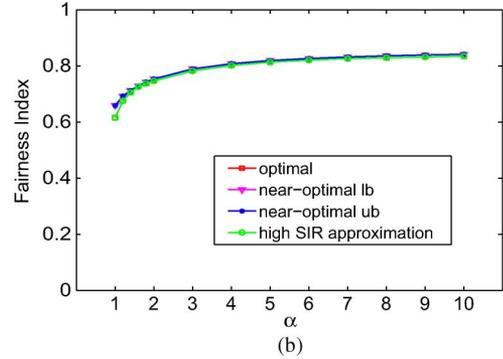
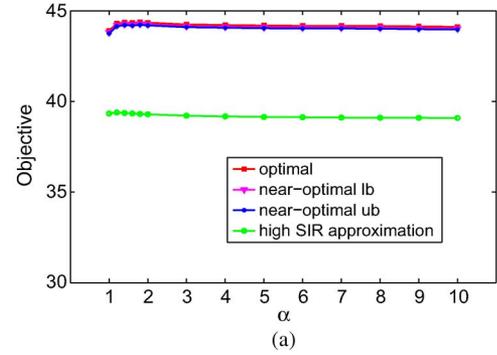


Fig. 2. Impact of a variable utility parameter  $\alpha$  on the network efficiency and fairness of Algorithm 1 (optimal), Algorithm 2 with lower bound (near-optimal lb) and upper bound (near-optimal ub) SIR constraints, and Chiang's algorithm (high SIR approximation). (a) Objective value. (b) Jain's fairness index.

average number of iterations over 100 realizations with various values of  $\epsilon$ . The convergence speed of the near-optimal algorithm is the same for both the upper bound and lower bound and is uniformly presented as Algorithm 2. We see that the near-optimal scheme converges faster than does the optimal scheme. This is a significant point, as Algorithm 2, which can achieve a near-optimal solution with smaller control message size and faster convergence, would be efficiently practical. Fig. 3 shows a convergence realization of source rates, link powers, and outage probabilities of Algorithm 1. The convergence of Algorithm 2 is almost similar to that of Algorithm 1, except that its power control is somewhat more aggressive due to the constraint approximation; hence, it is not shown due to the limited space. The outage probabilities of both schemes also converge to the desired values.

## VI. CONCLUSION

We have reconsidered joint congestion control and power control in wireless multihop networks with explicit outage constraint. The first proposed algorithm is optimal, but its control message contains a large amount of information. The second algorithm is near optimal yet practical because it has small size control messages to reduce overheads. Numerical experiments have showed that the network performances of both schemes were nearly identical and outperformed the conventional scheme, and the near-optimal scheme had a faster convergence speed.

### APPENDIX A PROOF OF THEOREM 1

We first show the convexity of the first constraint. It is clear that  $\log(\sum_{s \in S(l)} e^{\tilde{x}_s})$  is convex due to the log-sum-exponent function.

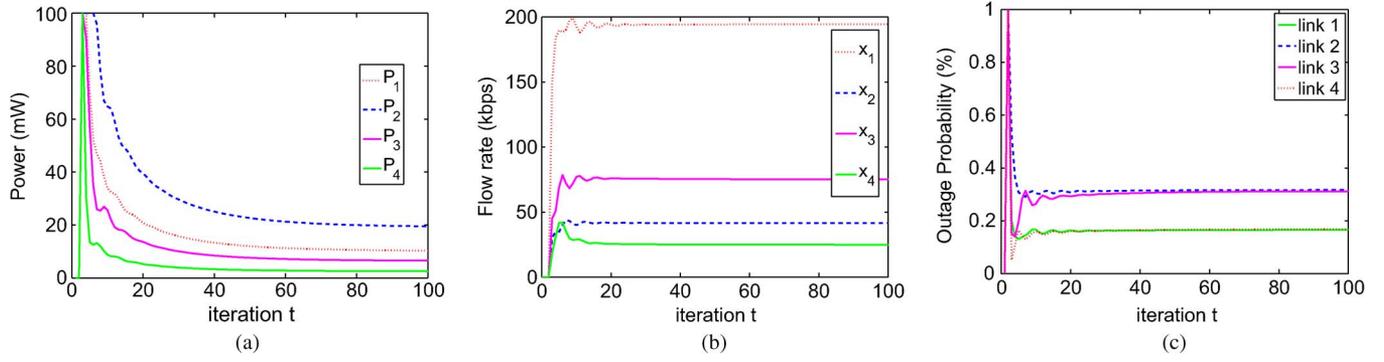


Fig. 3. Convergence of Algorithm 1. (a) Flow rates. (b) Link powers. (c) Outage probabilities.

Consider that  $\log c_l(\hat{\gamma}_l) = \log(\log(1 + \hat{\gamma}_l))$ , which can be represented as a composition function  $u(v(\hat{\gamma}_l))$ , where  $u(y) = \log(\log(1 + e^y))$ ,  $y \in \mathbb{R}$ , and  $v(\hat{\gamma}_l) = \log(\hat{\gamma}_l)$ . We have  $(\partial u(y)/\partial y) = (e^y/(1 + e^y) \log(1 + e^y)) \geq 0$ , and  $(\partial^2 u(y)/\partial y^2) = (e^y(\log(1 + e^y) - e^y)/(1 + e^y)^2 \log^2(1 + e^y)) \leq 0$  due to  $z \geq \log(1 + z) \forall z \geq 0$ . Therefore,  $u(y)$  is nondecreasing and concave in  $y \in \mathbb{R}$ .  $v(\hat{\gamma}_l)$  is clearly concave. We conclude that  $u(v(\hat{\gamma}_l))$  is concave due to the composition property (see [2, p. 84]).

Next, we will prove that  $f_l(\hat{\mathbf{P}}) \triangleq \sum_{k \neq l} \log(1 + e^{\hat{P}_k - \hat{P}_l} (\gamma_l^{\text{th}} G_{lk}/G_{ll}))$  is a convex function because  $\log \Omega_l(e^{\hat{P}_l})$  is a concave function of  $\hat{P}_l$ . Denoting  $z_k = (e^{\hat{P}_k - \hat{P}_l} (\gamma_l^{\text{th}} G_{lk}/G_{ll}) / (1 + e^{\hat{P}_k - \hat{P}_l} (\gamma_l^{\text{th}} G_{lk}/G_{ll})))^2 > 0$ , we have the following derivatives for link  $l$ :  $(\partial^2 f_l(\hat{\mathbf{P}})/\partial \hat{P}_l^2) = \sum_{k \neq l} z_k$ ; for any link  $j \neq l$ :  $(\partial^2 f_l(\hat{\mathbf{P}})/\partial \hat{P}_l \partial \hat{P}_j) = -z_j$ ,  $\partial^2 f_l(\hat{\mathbf{P}})/\partial \hat{P}_j^2 = z_j$ ,  $(\partial^2 f_l(\hat{\mathbf{P}})/\partial \hat{P}_j \partial \hat{P}_l) = \partial^2 f_l(\hat{\mathbf{P}})/\partial \hat{P}_l \partial \hat{P}_j$ ; and for  $i \neq l$ ,  $j$ :  $(\partial^2 f_l(\hat{\mathbf{P}})/\partial \hat{P}_j \partial \hat{P}_i) = 0$ . The Hessian matrix  $\nabla^2 f_l(\hat{\mathbf{P}})$  can be shown as follows:

$$\nabla^2 f_l(\hat{\mathbf{P}}) = \begin{pmatrix} \sum_{k=1, k \neq l}^{L-1} z_k & -z_1 & -z_2 & \cdots & -z_{L-1} \\ -z_1 & z_1 & 0 & \cdots & 0 \\ -z_2 & 0 & z_2 & \cdots & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ -z_{L-1} & 0 & 0 & \cdots & z_{L-1} \end{pmatrix}.$$

Then, with all  $\mathbf{v} \in \mathbb{R}^L$ , we have  $\mathbf{v}^T \nabla^2 f_l(\hat{\mathbf{P}}) \mathbf{v} = \sum_{k=1}^{L-1} z_k (v_1 - v_{k+1})^2 \geq 0$ , which shows that  $\nabla^2 f_l(\hat{\mathbf{P}})$  is a positive semidefinite matrix.

#### APPENDIX B

##### PROOF OF PROPOSITION 1

Using first-order optimal condition, we have  $(\partial L_{\hat{\mathbf{x}}}(\hat{\mathbf{x}}, \boldsymbol{\lambda})/\partial \hat{x}_s) = 0 = e^{\hat{x}_s} (U'_s(e^{\hat{x}_s}) - \sum_{l \in L(s)} (\lambda_l / \sum_{f \in S(l)} e^{\hat{x}_f}))$ . The result is then obtained via transformation back to the  $\mathbf{x}$  space.

#### APPENDIX C

##### PROOF OF PROPOSITION 2

The partial Lagrangian  $L_{\hat{\mathbf{P}}}(\hat{\mathbf{P}}, \boldsymbol{\lambda}, \boldsymbol{\nu})$  can be rewritten as

$$L_{\hat{\mathbf{P}}}(\hat{\mathbf{P}}, \boldsymbol{\lambda}, \boldsymbol{\nu}) = \sum_l \left( \lambda_l \log c_l(\hat{\gamma}_l) + \nu_l \log \Omega_l(e^{\hat{P}_l}) - e^{\hat{P}_l} \right) - \sum_l \sum_{k \neq l} \nu_k \log \left( 1 + e^{\hat{P}_l - \hat{P}_k} \frac{\gamma_l^{\text{th}} G_{kl}}{G_{kk}} \right). \quad (14)$$

This results in these following derivatives:  $(\partial/\partial \hat{P}_l)(\sum_l \lambda_l \times \log c_l(\hat{\gamma}_l)) = \lambda_l (\partial(\log c_l(\hat{\gamma}_l))/\partial \hat{\gamma}_l) (\partial \hat{\gamma}_l / \partial \hat{P}_l) + \sum_{k \neq l} \lambda_k (\partial \times$

$(\log c_k(\hat{\gamma}_k))/\partial \hat{\gamma}_k) (\partial \hat{\gamma}_k / \partial \hat{P}_l) = \lambda_l (1/\log(1 + \hat{\gamma}_l)) (\hat{\gamma}_l / (1 + \hat{\gamma}_l)) + \sum_{k \neq l} \lambda_k (1/\log(1 + \hat{\gamma}_k)) (1/(1 + \hat{\gamma}_k)) (-\hat{\gamma}_k^2 / G_{kk} e^{\hat{P}_k}) G_{kl} e^{\hat{P}_l}$ ;  $(\partial/\partial \hat{P}_l)(\sum_l \sum_{k \neq l} \nu_k \log(1 + e^{\hat{P}_l - \hat{P}_k} (\gamma_l^{\text{th}} G_{kl}/G_{kk}))) = \sum_{k \neq l} \nu_k \times (e^{\hat{P}_l - \hat{P}_k} (\gamma_l^{\text{th}} G_{kl}/G_{kk}) / (1 + e^{\hat{P}_l - \hat{P}_k} (\gamma_l^{\text{th}} G_{kl}/G_{kk})))$ ;  $(\partial/\partial \hat{P}_l) \times (\sum_l \nu_l \log \Omega_l(e^{\hat{P}_l})) = -\nu_l (\gamma_l^{\text{th}} / G_{ll} e^{\hat{P}_l}) (N_0 / \log(1 - \xi_l))$ . Substituting these derivatives and  $(\partial(e^{\hat{P}_l})/\partial \hat{P}_l) = e^{\hat{P}_l}$  into  $(\partial L_{\hat{\mathbf{P}}}(\hat{\mathbf{P}}, \boldsymbol{\lambda}, \boldsymbol{\nu})/\partial \hat{P}_l) = 0$ , the result is obtained after transformation back to  $\mathbf{P}$  space.

#### APPENDIX D

##### PROOF OF THEOREM D

Using  $\|\cdot\|$  to denote the Euclidean norm, we use the Lyapunov function  $V(\boldsymbol{\lambda}, \boldsymbol{\nu}) = \|\boldsymbol{\lambda} - \boldsymbol{\lambda}^*\|^2 + \|\boldsymbol{\nu} - \boldsymbol{\nu}^*\|^2$ , where  $(\boldsymbol{\lambda}^*, \boldsymbol{\nu}^*)$  is the optimal dual solutions. From (8) and (9), and the fact that  $(\max\{a, 0\})^2 \leq a^2$ , we have

$$\|\boldsymbol{\lambda}(t+1) - \boldsymbol{\lambda}^*\|^2 \leq \|\boldsymbol{\lambda}(t) - \boldsymbol{\lambda}^*\|^2 - 2\kappa(t) [\boldsymbol{\lambda}(t) - \boldsymbol{\lambda}^*]^T \mathbf{g}(t) + \kappa(t)^2 \|\mathbf{g}(t)\|^2 \quad (15)$$

$$\|\boldsymbol{\nu}(t+1) - \boldsymbol{\nu}^*\|^2 \leq \|\boldsymbol{\nu}(t) - \boldsymbol{\nu}^*\|^2 - 2\kappa(t) [\boldsymbol{\nu}(t) - \boldsymbol{\nu}^*]^T \mathbf{h}(t) + \kappa(t)^2 \|\mathbf{h}(t)\|^2. \quad (16)$$

Since  $x_s(t)$  and  $P_l(t)$  are bounded for all  $s, l$ , and  $t$ ,  $\|\mathbf{g}(t)\|^2$  and  $\|\mathbf{h}(t)\|^2$  are also bounded from (8) and (9). Supposing that  $\|\mathbf{g}(t)\|^2 + \|\mathbf{h}(t)\|^2 \leq C$ , from (15) and (16), we have

$$V(\boldsymbol{\lambda}(t+1), \boldsymbol{\nu}(t+1)) \leq V(\boldsymbol{\lambda}(t), \boldsymbol{\nu}(t)) + \kappa(t)^2 C - 2\kappa(t) ([\boldsymbol{\lambda}(t) - \boldsymbol{\lambda}^*]^T \mathbf{g}(t) + [\boldsymbol{\nu}(t) - \boldsymbol{\nu}^*]^T \mathbf{h}(t)).$$

Employing telescoping sums, we have

$$V(\boldsymbol{\lambda}(t+1), \boldsymbol{\nu}(t+1)) \leq V(\boldsymbol{\lambda}(0), \boldsymbol{\nu}(0)) + C \sum_{\tau=0}^t \kappa(\tau)^2 - 2 \sum_{\tau=0}^t \kappa(\tau) ([\boldsymbol{\lambda}(\tau) - \boldsymbol{\lambda}^*]^T \mathbf{g}(\tau) + [\boldsymbol{\nu}(\tau) - \boldsymbol{\nu}^*]^T \mathbf{h}(\tau)). \quad (17)$$

Note that  $C \sum_{\tau=0}^{\infty} \kappa(\tau)^2 < \infty$ ; hence,  $\sum_{\tau=0}^t \kappa(\tau) ([\boldsymbol{\lambda}(\tau) - \boldsymbol{\lambda}^*]^T \mathbf{g}(\tau) + [\boldsymbol{\nu}(\tau) - \boldsymbol{\nu}^*]^T \mathbf{h}(\tau)) < \infty$ . We also have  $[\boldsymbol{\lambda}(\tau) - \boldsymbol{\lambda}^*]^T \mathbf{g}(\tau) + [\boldsymbol{\nu}(\tau) - \boldsymbol{\nu}^*]^T \mathbf{h}(\tau) \geq D(\boldsymbol{\lambda}, \boldsymbol{\nu}) - D(\boldsymbol{\lambda}^*, \boldsymbol{\nu}^*) \geq 0$ . Hence, due to the condition  $\sum_{\tau=0}^{\infty} \kappa(\tau) = \infty$ , there must exist a subsequence  $\{t_n\}$  such that  $\lim_{n \rightarrow \infty} [\boldsymbol{\lambda}(t_n) - \boldsymbol{\lambda}^*]^T \mathbf{g}(t_n) + [\boldsymbol{\nu}(t_n) - \boldsymbol{\nu}^*]^T \mathbf{h}(t_n) = 0$ . Since  $\boldsymbol{\lambda}(t_n)$  and  $\boldsymbol{\nu}(t_n)$  are bounded sequences, there exist limit points  $\boldsymbol{\lambda}^o$  and  $\boldsymbol{\nu}^o$  such that  $\boldsymbol{\lambda}(t_n) \rightarrow \boldsymbol{\lambda}^o$  and  $\boldsymbol{\nu}(t_n) \rightarrow \boldsymbol{\nu}^o$  as  $n \rightarrow \infty$ .

Then, for any  $\epsilon > 0$ , there exists  $N$  such that, for all  $n \geq N$ , we have  $[\lambda(t_n) - \lambda^*]^T \mathbf{g}(t_n) + [\nu(t_n) - \nu^*]^T \mathbf{h}(t_n) < \epsilon$ , which leads to

$$D(\lambda^*, \nu^*) \geq D(\lambda(t_n), \nu(t_n)) + [\lambda^* - \lambda(t_n)]^T \mathbf{g}(t_n) + [\nu^* - \nu(t_n)]^T \mathbf{h}(t_n) \geq D(\lambda(t_n), \nu(t_n)) - \epsilon.$$

Since  $D(\lambda(t), \nu(t))$  is continuous, we have  $D(\lambda^*, \nu^*) \geq D(\lambda^\circ, \nu^\circ) - \epsilon$ . Since this is true for all  $\epsilon$  and the fact that  $D(\lambda^*, \nu^*) \leq D(\lambda^\circ, \nu^\circ)$ , we must have  $D(\lambda^*, \nu^*) = D(\lambda^\circ, \nu^\circ)$ . Hence,  $(\lambda^\circ, \nu^\circ)$  is also an optimal point of  $D(\lambda, \nu)$ . Replacing  $(\lambda^\circ, \nu^\circ)$  by  $(\lambda^*, \nu^*)$ , and summing over  $t > t_n$ , from (17)

$$V(\lambda(t+1), \nu(t+1)) \leq V(\lambda(t_n), \nu(t_n)) + C \sum_{\tau=t_n+1}^t \kappa(\tau)^2. \quad (18)$$

Since  $\sum_{\tau=t_n+1}^t \kappa(\tau)^2 \rightarrow 0$  as  $n \rightarrow \infty$ , we have  $\lim_{t \rightarrow \infty} V(\lambda(t), \nu(t)) \leq \lim_{n \rightarrow \infty} V(\lambda(t_n), \nu(t_n)) = 0$ , which shows that  $(\lambda(t), \nu(t)) \rightarrow (\lambda^*, \nu^*)$  as  $t \rightarrow \infty$ . Hence, the primal optimal points  $\mathbf{x}^*(\lambda^*)$  and  $\mathbf{P}^*(\lambda^*, \nu^*)$  are achieved due to the zero-duality gap.

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## Low-Complexity Near-Optimum Multiple-Symbol Differential Detection of DAPSK Based on Iterative Amplitude/Phase Processing

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**Abstract**—Differentially encoded and noncoherently detected transceivers exhibit low complexity since they dispense with a complex channel estimation. In pursuit of high bandwidth efficiency, differential amplitude/phase (A/P)-shift keying (DAPSK) was devised using constellations of multiple concentric rings. To increase resilience against the typical high-Doppler-induced performance degradation of DAPSK and/or to enhance the maximum achievable error-free transmission rate for DAPSK-modulated systems, multiple-symbol differential detection (MSDD) may be invoked. However, the complexity of the maximum a posteriori (MAP) MSDD exponentially increases with the detection window size and hence may become excessive upon increasing the window size, particularly in the context of an iterative detection-aided channel-coded system. To circumvent this excessive complexity, we conceive a decomposed two-stage iterative A/P detection framework, where the challenge of having a nonconstant-modulus constellation is tackled with the aid of a specifically designed information exchange between the independent A/P detection stages, thus allowing the incorporation of reduced-complexity sphere detection (SD). Consequently, a near-MAP-MSDD performance can be achieved at significantly reduced complexity, which may be five orders of magnitude lower than that of the traditional MAP-MSDD in the 16-DAPSK scenario that was considered.

**Index Terms**—Differential amplitude/phase (A/P)-shift keying (DAPSK), multiple symbol differential detection, quadrature amplitude modulation (QAM).

## I. INTRODUCTION

Future wireless communications will have to support a high grade of mobility. The major candidates for the next generation of broadband wireless access systems, such as Third-Generation Partnership Project Long-Term Evolution and IEEE 802.16m, are expected to deliver a data rate of at least 100 Mb/s for high-velocity mobile users (up to 350 km/h) [1], [2]. Differential phase-shift keying (DPSK) relying on low-complexity noncoherent detection constitutes an attractive solution for high-mobility wireless communications, particularly in scenarios such as cooperative communications, since it is robust against the phase ambiguities induced by rapid fading while dispensing with a channel estimation for mobile-to-mobile links. Thus, the employment of pilot symbols may be avoided in differentially encoded and noncoherently detected schemes. Differential amplitude/phase (A/P)-shift keying (DAPSK) [3], [4]—also known as star quadrature amplitude modulation (star-QAM)—has received substantial research attention from the communication community as a benefit of its low-complexity detection and its low peak-to-mean power, as compared with the maximum–minimum distance square-QAM constellation. Basically, DAPSK expands the single-ring constellation of the traditional DPSK to multiple amplitude rings; thus, the information bits are mapped to both the A/P differences between successively transmitted symbols.

Manuscript received June 12, 2011; revised October 11, 2011; accepted December 3, 2011. Date of publication January 2, 2012; date of current version February 21, 2012. This work was supported in part by the Research Councils U.K. under the auspices of the U.K.–India Advanced Technology Centre of Wireless Communications and of the China–U.K. Science Bridge in 4G wireless communications and in part by the European Union Concerto Project. The review of this paper was coordinated by Prof. T. Tsiftsis.

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Digital Object Identifier 10.1109/TVT.2011.2181192