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Joint Rate Control and Spectrum Allocation under Packet Collision Constraint in Cognitive Radio Networks

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***Abstract*— We study joint rate control and resource allocation with QoS provisioning that maximizes the total utility of secondary users in cognitive radio networks. We formulate and decouple the original utility optimization problem into separable subproblems and then develop an algorithm that converges to optimal rate control and resource allocation. The proposed algorithm can operate on different time-scale to reduce the amortized time complexity.**

***Index Terms*—Utility maximization, rate control and resource allocation, cognitive radio networks.**

I. Introduction

COGNITIVE radio networks have been considered as an enabling technology for dynamic spectrum usage, which helps alleviate the conventional spectrum scarcity and improve the utilization of the existing spectrum [7]. Cognitive radio is capable of tuning into different frequency bands with its software-based radio technology. The key point of cognitive networks is to allow the secondary users (SUs) to employ the spatial and/or temporal access to the spectrum of legacy primary users (PUs) by transmitting their data opportunistically. So the most important requirement is how to devise an effective resource allocation scheme that ensures the existing licensed PUs are not affected adversely. However, without the ideal channel state information, such kind of negative effect to PUs are not avoidable. With limited channel state information assumption, the constraint turns into what is the parameter that should be applied to the quality of service (QoS) to guarantee the satisfaction of PUs. Hence, the standard spectrum access strategy in cognitive networks is to maximize the total utility of SUs while still guarantee the QoS requirement of PUs. A comprehensive survey on designing issues, new technology and protocol operations can be found in [10].

In this paper, we propose the utility maximization framework that takes into account the QoS constraint for cognitive networks. Here we choose packet collision probability as the metric for PU's QoS protection, which recently has been used widely in research community [5], [9]. Under this QoS protection requirement, the SUs must guarantee that the packet collision probability of a PU packet is less than a certain threshold specified by the PUs. We first formulate a primal utility optimization problem with appropriate constraints regarding to congestion control and PUs' QoS protection. Then we decouple this primal optimization problem into joint rate control and resource allocation subproblems, where SUs can solve the rate control problem distributively while the resource allocation is solved by the base station (BS) in a centralized manner. The resource in this context is the spectrum that would be allocated to SUs. The original decomposed resource allocation problem that entails high computational complexity is alleviated by a larger time-scale update, which significantly reduces the amortized complexity. This decomposition makes our proposal much more practical and robust in dynamic environments.

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II. Related Works

In recent time there has been a remarkably extensive research in cognitive radio networks where the major effort is on designing protocols that can maximizing the SUs spectrum utility when PUs are idle and protect PUs communications when they become active.

Generally, research on cognitive networks can be divided into two main categories. The first one is based on the assumption of static PUs channel occupation, where SUs communications are assumed to happen in a much faster time-scale than those of PUs. Hence SUs' channel allocation becomes the main issue given topologies, channel availabilities and/or interference between SUs. In [14], [15], the interference between SUs is modeled using conflict graph, with different methods and parameters to allocate channel. The authors in [4], [13] formulate the channel allocation problem as a mixed linear integer programming under the power and channel availability constraints.
The second category is based on the assumption that PUs communications temporally varies quickly so that the main issue becomes how SUs within interference range can sense and access the channel without harming PUs activity. Therefore measuring interference is the key metric in many works. In [17], both of the constraints on PUs regarding to average rate requirement and outage probability are functions of interference power caused by SUs. The work in [19] considers power control for varying states of PUs.

In previous works, under the collision packet probability constraint, researchers have tried to develop medium access

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schemes [11], [13]. Many works were based on the formulation using partially observable Markov decision process. For example, [12], [18] focus on a slotted network with single PU protection metric and the optimal access is decided after a long observation history. In [9], an overlay SUs network are consider on the multiple PUs network where PUs access decision depends on Markovian evolution.

III. System Model and Problem Definition

We consider a multi-channel spectrum sharing cognitive radio networks comprising a set of SUs' node pairs $\mathcal{M} = \{1, 2, \ldots, M\}$. Each SU's node pair consists of one dedicated transmitter and its intended receiver. SUs share a common set of $\mathcal{K} = \{1, 2, \ldots, K\}$ orthogonal channels with PUs. Each channel is occupied by each PU and PUs can send their data over their own licensed channels to the BS simultaneously. Each SU is assumed to have a utility function $U_m(x_m)$, a function of the flow rate x_m, which can be interpreted as the level of satisfaction attained by SU m [3]. The utility function of each SU is assumed to be increasing and strictly concave. Fixed link capacities of SU's and PU's are denoted by c_m and c_k, respectively.

The QoS constraint of PUs is denoted by γ_k, the maximum fraction of PU k's packets that can have collisions, which is set at the BS *a priori*. Hence the maximum packet collision rate that a PU k can tolerate is $\gamma_k c_k$. The collision rate of a PU is denoted by e_k. We denote the probability that channels are idle (i.e. channels are not occupied by PUs) by the vector [1] $\boldsymbol{\pi} = (\pi_1, \pi_2, \ldots, \pi_K)$, which is achieved by SUs through the knowledge of traffic statistics and/or channel probing [9].

A. Primal Problem

We formulate the utility maximization problem with PUs' QoS protection constraint in a cognitive radio network as the followings:

$(\mathbf{P})$:

$$\underset{\boldsymbol{x},\boldsymbol{\phi},\boldsymbol{e}}{\text{maximize}} \quad \sum_m U_m(x_m) \tag{1}$$

$$\text{subject to} \quad x_m \le \sum_k c_m \pi_k \phi_{mk}, \quad \forall m \tag{2}$$

$$e_k \le \gamma_k c_k, \quad \forall k \tag{3}$$

$$\sum_m \phi_{mk} = 1, \ \sum_k \phi_{mk} = 1 \quad \forall m, k, \tag{4}$$

$$0 \le x_m \le x_m^{max}, \quad \forall m \tag{5}$$

where x_m^{max} is the maximum data rate of SU m and ϕ_{mk} is the fraction of time that a given channel k is allocated to SU m. Define an allocation function at any time instant t as follows:

$$I_{mk}(t) = \begin{cases} 1 & \text{if channel } k \text{ is allocated to } m \text{ at } t \\ 0 & \text{otherwise.} \end{cases} \tag{6}$$

Then we have

$$\phi_{mk} = \lim_{t\to\infty} \frac{1}{t} \sum_{\tau=0}^{t-1} I_{mk}(\tau). \tag{7}$$

[1] In this paper, vector notation is presented by bold-face font.

Constraint (2) ensures that the source rate on a SU link cannot exceed its attainable link rate with channel-occupancy information. (3) is precisely the collision constraint rendering the QoS provisioning for PUs. Constraint (4) allows at most one SU to be allocated to channel k and at most one channel k to be allocated to one SU at any time instant. It is straightforward that $(\mathbf{P})$ is a convex optimization problem.

B. Dual Problem

In order to use the duality approach for solving problem $(\mathbf{P})$, we first form the partial Lagrangian:

$$\begin{aligned} L(\boldsymbol{x}, \boldsymbol{e}, \boldsymbol{\phi}, \boldsymbol{\lambda}, \boldsymbol{\mu}) = &\sum_m U_m(x_m) + \sum_k \mu_k(\gamma_k c_k - e_k) \\ &+ \sum_m \lambda_m \Big(\sum_k c_m \pi_k \phi_{mk} - x_m\Big), \end{aligned} \tag{8}$$

where $\boldsymbol{\lambda} = (\lambda_m, m \in \mathcal{M}) \ge 0$ and $\boldsymbol{\mu} = (\mu_k, k \in \mathcal{K}) \ge 0$, the Lagrange multipliers of constraints (2) and (3), are considered as the congestion price and collision price respectively. The dual objective function is:

$$D(\boldsymbol{\lambda}, \boldsymbol{\mu}) = \max_{\boldsymbol{x},\boldsymbol{e},\boldsymbol{\phi}} \quad L(\boldsymbol{x}, \boldsymbol{e}, \boldsymbol{\phi}, \boldsymbol{\lambda}, \boldsymbol{\mu}) \quad \text{subject to} \quad (3), (4), (5) \tag{9}$$

Then, the dual optimization problem is:

$(\mathbf{D})$:

$$\underset{\boldsymbol{\lambda}\ge\mathbf{0},\boldsymbol{\mu}\ge\mathbf{0}}{\text{minimize}} \quad D(\boldsymbol{\lambda}, \boldsymbol{\mu}) \tag{10}$$

Given the assumptions on utility function, it is not difficult to see that Slater condition is satisfied, and strong duality holds [1]. This means that the duality gap is zero between the dual and primal optimum. This allows us to solve the primal via the dual.

IV. Joint Rate Control and Resource Allocation with QoS Provisioning Algorithm

A. Decomposition Structure

In this section, we present a different time-scale algorithm of joint rate control and resource allocation with QoS protection for PU. Note that by the definition of e_k, we have a relationship:

$$e_k = \sum_m \phi_{mk}(1 - \pi_k) c_k. \tag{11}$$

By substituting (11) into (8) and rearranging the order of summation, we can decompose (9) into the following two subproblems (partial dual functions):

$$D_x(\boldsymbol{\lambda}) = \max_{0 \le \boldsymbol{x} \le \boldsymbol{x}^{max}} \quad \sum_m [U_m(x_m) - \lambda_m x_m] \tag{12}$$

and

$$\begin{aligned} D_\phi(\boldsymbol{\mu}) = &\\ &\max \sum_m \sum_k \phi_{mk} \left[\lambda_m \pi_k c_m - \mu_k (1 - \pi_k) c_k\right] \\ &\text{subject to} \quad \sum_m \phi_{mk} = 1, \ \sum_k \phi_{mk} = 1 \quad \forall m, k. \end{aligned} \tag{13}$$

The maximization problem (12) can be conducted in parallel and in a distributed fashion by SUs. In contrast, if we consider (13) at an arbitrary time instant t, we have the equivalent problem:

$$\begin{aligned} \max \quad & \sum_m \sum_k I_{mk}(t)[\lambda_m(t)\pi_k c_m - \mu_k(t)(1-\pi_k)c_k] \\ \text{subject to} & \sum_m I_{mk}(t) = 1, \; \sum_k I_{mk}(t) = 1, \quad \forall m, k, \end{aligned} \tag{14}$$

which is a combinatorial optimization problem that needs to be solved in a centralized fashion by the BS. This problem is the Maximum Weighted Bipartite Matching problem on an $M \times K$ bipartite graph between M secondary users and K channels where the weight of the edge between SU m and channel k is $\lambda_m(t)c_m\pi_k - \mu_k(t)(1-\pi_k)c_k$.

B. Optimal Solutions

It is straightforward that for $\boldsymbol{\lambda}$ fixed, the maximization (12) has the optimal solution

$$x_m^* = \min\left\{[U_m^{'-1}(\lambda_m)]^+, \, x_m^{max}\right\}, \quad \forall m. \tag{15}$$

where $U_m^{'-1}$ is the inverse of the first derivative of utility function.

Similarly for $\boldsymbol{\mu}$ fixed, the optimal solution ϕ_{mk}^* of maximization (14) can be found using Hungarian method [2].

Now we can solve the dual problem (10) by using a subgradient projection method [1]. Since $D(\boldsymbol{\lambda}, \boldsymbol{\mu})$ is affine with respect to $(\lambda_m(t), \mu_k(t))$, the subgradient of it at $(\lambda_m(t), \mu_k(t))$ is

$$\frac{\partial D}{\partial \lambda_m(t)} = \sum_k c_m \pi_k I_{mk}(t) - x_m(t) \tag{16}$$

$$\frac{\partial D}{\partial \mu_k(t)} = \gamma_k c_k - \sum_m I_{mk}(t)(1-\pi_k)c_k, \tag{17}$$

and the updates of dual variables are

$$\lambda_m(t+1) = \left[\lambda_m(t) - \alpha(t)\left(\frac{\partial D}{\partial \lambda_m(t)}\right)\right]^+ \tag{18}$$

$$\mu_k(t+1) = \left[\mu_k(t) - \alpha(t)\left(\frac{\partial D}{\partial \mu_k(t)}\right)\right]^+, \tag{19}$$

where $[z]^+ = \max\{z, 0\}$ and $\alpha(t) > 0$ is the step-size with the appropriate choice satisfying

$$\sum_{t=0}^{\infty} \alpha(t)^2 < \infty, \tag{20}$$

$$\sum_{t=0}^{\infty} \alpha(t) = \infty \tag{21}$$

leads to the convergence of the optimal dual values [1].

C. Algorithm

In this section, we present our algorithm and then explain the rationale behind it. We assume that all variables are initialized to 0 and the algorithm will stop if the convergence reached.

At the BS level

1) For every iteration t, each BS updates the new and average collision prices on each channel k:

$$\mu_k(t+1) = \left[\mu_k(t) - \alpha(t)\left(\gamma_k c_k - \sum_m I_{mk}(t)(1-\pi_k)c_k\right)\right]^+, \tag{22}$$

$$\overline{\mu}_k(t+1) = (1-\beta)\overline{\mu}_k(t) + \beta\mu_k(t+1), \tag{23}$$

where $0 < \beta < 1$.

2) For every $T \geq t$, the BS solves the following problem then broadcasts new $I_{mk}(T), \forall m, k$ on all channels.

$$\begin{aligned} \max \quad & \sum_m \sum_k I_{mk}(T)[\overline{\lambda}_m(T)\pi_k c_m - \overline{\mu}_k(T)(1-\pi_k)c_k] \\ \text{subject to} & \sum_m I_{mk}(T) = 1, \; \sum_k I_{mk}(T) = 1, \; \forall m, k, \end{aligned} \tag{24}$$

At the SU level

1) For every iteration t, each SU:
   - adjusts its source rate by solving (12)
   $$x_m(t+1) = \min\left\{[U_m^{'-1}(\lambda_m(t))]^+, \, x_m^{max}\right\}, \tag{25}$$
   where $U_m^{'-1}(.)$ is the inverse of the first derivative of U_m.
   - updates the new and average congestion prices:
   $$\lambda_m(t+1) = \left[\lambda_m(t) - \alpha(t)\left(\sum_k c_m \pi_k I_{mk}(t) - x_m(t)\right)\right]^+ \tag{26}$$
   $$\overline{\lambda}_m(t+1) = (1-\beta)\overline{\lambda}_m(t) + \beta\lambda_m(t+1) \tag{27}$$

2) For every $T \geq t$, each SU sends $\overline{\lambda}_m(T)$ to the BS, then receives the new value of $I_{mk}(T)$ from the BS.

The algorithm operates on two levels with different time-scale as follows: At the smaller time-scale t, each SU adjusts its source rate (25) using the current congestion price $\lambda_m(t)$, which is updated (26) using $I_{mk}(T)$ broadcast by BS at a periodic time $T \geq t$ (i.e. The update (26) uses the same old $I_{mk}(T)$ for consecutive T iterations). At a larger times-scale T, it sends $\overline{\lambda}_m(T)$, which is updated gradually at time-scale t (27), to the BS. At time-scale T, the BS periodically makes use of $\overline{\lambda}_m(T)$ received from SUs and its $\overline{\mu}_k(T)$ to compute $I_{mk}(T)$ (24) and broadcasts $I_{mk}(T)$ on all channels. Its periodic $\overline{\mu}_k(T)$ is updated gradually at smaller time-scale t with (22) and (23). The closed-loop in Fig. 1 shows the relationship between variables of BS and SU. The interaction between two levels with different time-scale implies that the design of our algorithm allows the BS to track just the *average* congestion price and collision price. The reason behind it is to reduce the computation burden

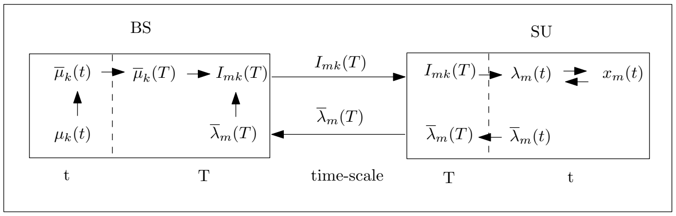

Fig. 1: Closed-loop structure between BS and SU

on the BS in terms of amortized analysis, which makes our algorithm much more implementable. For example, if the BS solves (24) by using Hungarian algorithm [2] with the complexity $O(V^3)$ for a bipartite graph $G(V, E)$ and chooses $T = V^2$, then the amortized complexity per operation is only $O(V^3)/V^2 = O(V)$.

V. Simulation Results

We consider the system of 5 SUs opportunistically accessing to 9 orthogonal channels serving 9 PUs. Link capacities of all PUs and SUs are chosen randomly, from a uniform distribution on $[0.4, 1.6]$ Mbps. We choose $U_m(x_m) = \log(x_m)$. The QoS constraint γ_k is set to 0.02 for all PUs. The values of $\alpha(t)$ and β are set to $0.2/t$ and 0.8, respectively. The Hungarian algorithm [2] is used to solve (24). We vary different values of $T = t$, $10t$, $100t$ for the comparison. In order to show that our algorithm can adapt to the change of traffic statistics, we consider two cases: high and low channel-occupancy of PUs, where the channel-idle probability $\boldsymbol{\pi}$ is assumed to have a uniform distribution on $[0.1, 0.3]$ and on $[0.7, 0.9]$ respectively.

First, we investigate that whether our algorithm can work efficiently by considering $T = t$. At the beginning, we assume that the system is under high channel-occupancy condition. Fig. 2 shows that initially all SUs transmit at their full link capacities due to price 0. After iteration 1500, all SUs flow rates converge to the average values provided in Table I. At iteration 2500, the system state changes to the low channel-occupancy condition leading to the increase of SUs flow rates. From iteration 2800, all SUs flow rates converge to the values provided in Table I.

Next we investigate the impact of parameter T. In Fig. 2, with high channel-occupancy the value of T does not affect much on the system performance. While we cannot see the difference between $T = t$ and $T = 10t$, there is a very small oscillation of SUs flow rates with $T = 100t$. However with low channel-occupancy, while the difference between $T = t$ and $T = 10t$ is very little, the SUs flow rates strongly oscillate with $T = 100t$ due to the long delay of information for updating the prices. So our algorithm is more robust to the high channel-occupancy than low channel-occupancy condition. This effective property can help the SUs tune the appropriate value of T to achieve fast convergence by observing channel statistics. Fig. 3 shows the convergence of absolute value of total utility objective (the original value is negative due to function $\log(.)$) in case of $T = 10t$ with similar characteristic as we discussed above.

TABLE I: Convergent rates of all SUs

flow rate (Mbps)	SU 1	SU 2	SU 3	SU 4	SU 5
high channel occupancy	0.261	0.217	0.318	0.242	0.276
low channel occupancy	0.898	0.745	1.094	0.832	0.952

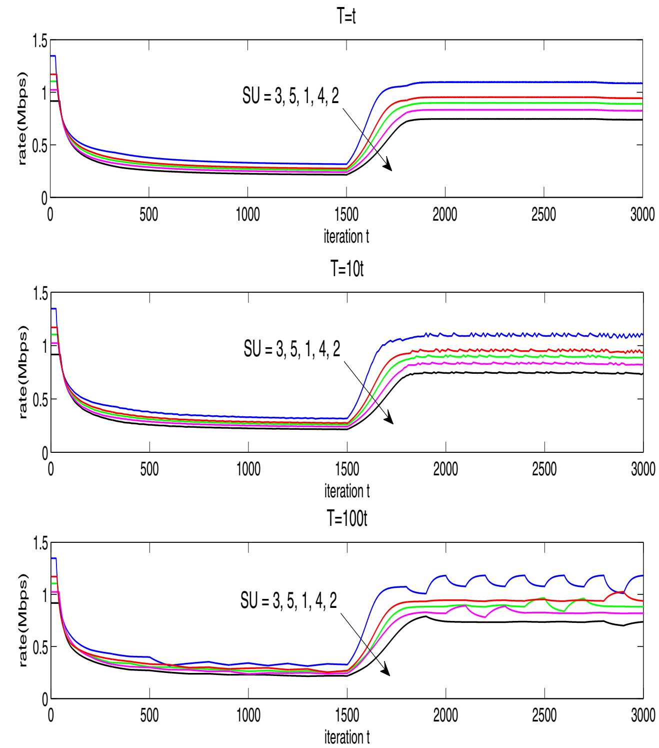

Fig. 2: The convergence of 5 SUs flow rates with different values of T

VI. Conclusion

In this work, in terms of utility maximization framework, we propose a joint rate control and resource allocation scheme with QoS provisioning in cognitive radio networks. Our algorithm operates the SU level and BS level on different time-scale, which reduces significantly the computational burden on the BS.

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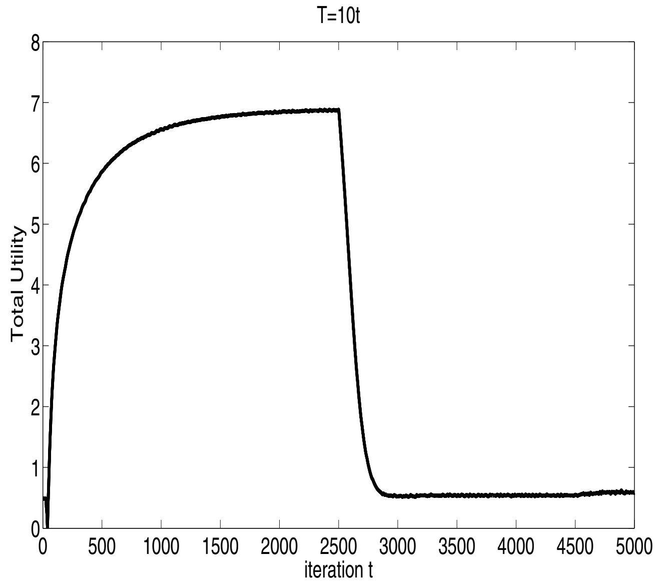

Fig. 3: The convergence of total utility

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