Joint Rate and Power Control in Wireless Network: A Novel Successive Approximations Method

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***Abstract*— The conventional method of *joint rate and power control* (JRPC) relies on high *signal-to-interference ratio* (SIR) assumption which achieves only suboptimal results. By using a novel successive convex approximations method, we can attain the global optimal source rates and link powers in a distributed fashion exploiting message passing. Through simulations, our method converges faster than the previous work based on logarithm successive convex approximations.**

***Index Terms*—Utility maximization, congestion control and power allocation, distributed algorithms.**

I. Introduction

In wireless network, the rate and power control has a mutual relationship, where rate control regulates the source rates to avoid overwhelming any link capacity which depends on interference levels, which in turn decided by power control policy. The work in [2] first characterized the JRPC problem through solving a transformed convex optimization problem. By using the gradient-based algorithm, the author showed that optimal rate and power allocation could be achieved in a high-SIR approximation sense; however it is suboptimal in the general case [5]. Also in [5], a generalized convexity has been established for the same optimization problem which allowed them to propose an algorithm named Alg. A that can achieve a globally optimal solution through messaging passing without high-SIR assumption. Due to the complicated convexification, however the rate allocation of Alg. A with explicit message passing no longer preserves the existing TCP stack like that of [2], which makes it less favorable. To take into account the TCP stack preserving, they continued propose an Alg. B employing a technique called logarithmic successive convex approximations.

To avoid high-SIR assumption yet preserve TCP stack, in this letter we propose another novel successive convex approximations method to iteratively transform the original nonconvex problem of JRPC into approximated convex problem, then the global optimal solution can converge distributively with message passing. Simulation results show that our method converge faster than Alg. B.

II. System Model and Problem Definition

We consider a wireless multihop network with $\mathcal{L} = \{1, 2, \ldots, L\}$ logical links shared by $\mathcal{S} = \{1, 2, \ldots, S\}$ sources. Each source s using a fixed set of links $L(s)$ on its route has a utility $U_s(x_s)$, a function of the flow rate x_s, which is assumed to be increasing and strictly concave. We use the similar CDMA physical model of [2] where simultaneous communications can happen, which undergoes the multiple-access interference. The instantaneous capacity of link $l \in \mathcal{L}$ is a global and nonlinear function of transmit power vector $\mathbf{P}$

$$c_l(\mathbf{P}) = W \log(1 + K\text{SIR}_l(\mathbf{P})), \tag{1}$$

where W is the baseband bandwidth and K is a constant depending on modulation, coding scheme and bit-error rate (BER). The $\text{SIR}_l(\mathbf{P})$ is defined as

$$\text{SIR}_l(\mathbf{P}) = \frac{P_l G_{ll}}{\sum_{k \neq l} P_k G_{lk} + n_l}, \tag{2}$$

where G_{lk} is the instantaneous channel gain from the transmitter on link k to the receiver on link l and n_l is the noise power at receiver of link l. The JRPC problem can be formulated as the following nonconvex problem

$$\begin{aligned} \text{maximize} \quad & \sum_s U_s(x_s) \\ \text{subject to} \quad & \sum_{s: l \in L(s)} x_s \leq c_l(\mathbf{P}), \quad \forall l \end{aligned} \tag{3}$$

where the nonnegative optimization variables are source rates vector $\mathbf{x}$ and power vector $\mathbf{P}$. Different TCP protocols solve for different $U_s(x_s)$. For example, $U_s(x_s) = \alpha_s d_s \log x_s$ is shown to be associated with TCP Vegas, where α_s is the Vegas parameter and d_s is the propagation delay [3].

III. A Novel Successive Convex Approximations Method

A. Approximated Convex Optimization Problem

In order to turn the original nonconvex problem (3) to an approximated convex problem, we begin to form a new lower bound approximation to the constraint (3)

$$\sum_{s: l \in L(s)} x_s \leq \hat{c}_l(\mathbf{P}) \leq c_l(\mathbf{P}). \tag{4}$$

Henceforth, we assume $W = K = 1$ without loss of generality. We note that $c_l(\mathbf{P})$ can be rewritten in the form

$$c_l(\mathbf{P}) = \log\left(\sum_{k \in \mathcal{L}} G_{lk} P_k + n_l\right) - \log\left(\sum_{k \neq l} G_{lk} P_k + n_l\right)$$

Making use of arithmetic-geometric mean inequality, where it states that $\sum_i \theta_i u_i \geq \prod_i u_i^{\theta_i}$ with $u_i \geq 0$, $\theta_i > 0$ $\forall i$ and $\sum_i \theta_i = 1$, we have a similar inequality $\sum_i v_i \geq \prod_i (v_i/\theta_i)^{\theta_i}$

Manuscript received May 6, 2010. The associate editor coordinating the review of this letter and approving it for publication was F. Granelli.

This research was supported by the MKE, Korea, under the ITRC support program supervised by the NIPA (NIPA-2010-(C1090-1021-0003)). Dr. CS Hong is the corresponding author.

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Digital Object Identifier 10.1109/LCOMM.2010.080210.100759

by letting $v_i = \theta_i u_i$, and the equality happens when $\theta_i = v_i / \sum_i v_i$.

Result 1: For each link l with a vector $\boldsymbol{\theta}^l = [\theta_1^l, \theta_2^l, \ldots, \theta_{L+1}^l]$

$$\sum_{k \in \mathcal{L}} G_{lk} P_k + n_l \geq \prod_{k=1}^{L} \left(\frac{G_{lk} P_k}{\theta_k^l} \right)^{\theta_k^l} \left(\frac{n_l}{\theta_{L+1}^l} \right)^{\theta_{L+1}^l}, \quad (5)$$

and the equality happens when

$$\theta_k^l = \frac{G_{lk} P_k}{\sum_{k \in \mathcal{L}} G_{lk} P_k + n_l}, \quad k = 1, \ldots, L$$
$$\theta_{L+1}^l = \frac{n_l}{\sum_{k \in \mathcal{L}} G_{lk} P_k + n_l} \quad (6)$$

Because $\log(.)$ is an increasing function of positive variables, by taking logarithm on both sides of (5) we have

$$\log \left(\sum_{k \in \mathcal{L}} G_{lk} P_k + n_l \right) \geq$$
$$\sum_{k=1}^{L} \theta_k^l \log \left(\frac{G_{lk} P_k}{\theta_k^l} \right) + \theta_{L+1}^l \log \left(\frac{n_l}{\theta_{L+1}^l} \right) \doteq f(\mathbf{P}, \boldsymbol{\theta}^l) \quad (7)$$

Letting $\hat{c}_l(\mathbf{P}, \boldsymbol{\theta}^l) = f(\mathbf{P}, \boldsymbol{\theta}^l) - \log \left(\sum_{k \neq l} G_{lk} P_k + n_l \right)$, we have

$$\hat{c}_l(\mathbf{P}, \boldsymbol{\theta}^l) \leq c_l(\mathbf{P}) \quad (8)$$

and the equality happens when (6) holds.
Letting $\hat{P}_l = \log P_l$, it is easy to see that

$$f(\hat{\mathbf{P}}, \boldsymbol{\theta}^l) = \sum_{k=1}^{L} \theta_k^l \hat{P}_k + \sum_{k=1}^{L} \theta_k^l \log \left(\frac{G_{lk}}{\theta_k^l} \right) + \theta_{L+1}^l \log \left(\frac{n_l}{\theta_{L+1}^l} \right)$$

is a linear function of $\hat{\mathbf{P}}$, so

$$\hat{c}_l(\hat{\mathbf{P}}, \boldsymbol{\theta}^l) = f(\hat{\mathbf{P}}, \boldsymbol{\theta}^l) - \log \left(\sum_{k \neq l} G_{lk} e^{\hat{P}_k} + n_l \right) \quad (9)$$

is a concave function of $\hat{\mathbf{P}}$ (recall that log-sum-exponent is convex). We have the approximated convex optimization problem of the original one (3) with variables $\mathbf{x}$ and $\hat{\mathbf{P}}$ ($\boldsymbol{\theta}^l$ is fixed) as following

$$\text{maximize} \quad \sum_s U_s(x_s) \quad (10)$$
$$\text{subject to} \quad \sum_{s: l \in L(s)} x_s \leq \hat{c}_l(\hat{\mathbf{P}}, \boldsymbol{\theta}^l), \quad \forall l$$

B. Optimal Solution of Approximated Convex Problem

The Lagrangian of (10) with dual variable vector (i.e. congestion control price) $\boldsymbol{\lambda} = (\lambda_1, \lambda_2 \ldots \lambda_L)$ is $L(\mathbf{x}, \hat{\mathbf{P}}, \lambda) = (\sum_s U(x_s) - \sum_l \lambda_l \sum_{s: l \in L(s)} x_s) + \sum_l \lambda_l \hat{c}_l(\hat{\mathbf{P}})$. This can be decomposed into two separate maximization problems in order to find the saddle points of the Lagrangian

$$\max_{\mathbf{x}} \left\{ L_x(\mathbf{x}, \boldsymbol{\lambda}) = \sum_s U(x_s) - \sum_s \sum_{l \in L(s)} \lambda_l x_s \right\} \quad (11)$$

$$\max_{\hat{\mathbf{P}}} \left\{ L_P(\hat{\mathbf{P}}, \boldsymbol{\lambda}) = \sum_l \lambda_l \hat{c}_l(\hat{\mathbf{P}}, \boldsymbol{\theta}^l) \right\} \quad (12)$$

The maximization (11) is the conventional rate control problem which is implicitly solved by the congestion control mechanism for different U_s [2], hence preserving the existing TCP stack. The second maximization (12) is the power control problem. And the dual problem of (10) is

$$\min_{\boldsymbol{\lambda} \geq \mathbf{0}} \left(\max_{\mathbf{x}} L_x(\mathbf{x}, \boldsymbol{\lambda}) + \max_{\hat{\mathbf{P}}} L_P(\hat{\mathbf{P}}, \boldsymbol{\lambda}) \right) \quad (13)$$

With the utility's assumption, Slater condition holds leading to strong duality [1]. We have the following result.

Result 2: The optimal solution $(\mathbf{x}^*, \mathbf{P}^*, \boldsymbol{\lambda}^*)$ can be achieved if the variables update iteratively as following until convergence
Rate control: The source rate updates

$$x_s^{(t+1)} = U_s^{'-1} \left(\sum_{l \in L(s)} \lambda_l^{(t)} \right) \quad (14)$$

, where $U_s^{'-1}$ is the inverse of the first derivative of utility.
Power control: The link power updates

$$P_l^{(t+1)} = \frac{\lambda_l^{(t)} \theta_l^l}{\sum_{k \neq l} G_{kl} m_k^{(t)}} \quad \text{with} \quad m_k^{(t)} = \frac{\lambda_k^{(t)} SIR_k^{(t)}}{P_k^{(t)} G_{kk}} \quad (15)$$

Congestion Price Update:

$$\lambda_l^{(t+1)} = \left[\lambda_l^{(t)} + \kappa \left(\sum_{s: l \in L(s)} x_s^{(t)} - \hat{c}_l(\hat{\mathbf{P}}^{(t)}, \boldsymbol{\theta}^l) \right) \right]^+, \quad (16)$$

where κ is the step size and $[z]^+ = \max\{z, 0\}$.

Proof: Solving (11) to have (14) is straightforward. With respect to power control, we have

$$\frac{\partial L_P(\hat{\mathbf{P}}, \boldsymbol{\lambda})}{\partial \hat{P}_l} = 0 = \lambda_l \theta_l^l - \sum_{k \neq l} \frac{\lambda_k G_{kl} e^{\hat{P}_l}}{\sum_{j \neq k} G_{kj} e^{\hat{P}_j} + n_k} \quad (17)$$

Transforming (17) back to $\mathbf{P}$ space, we obtain (15). The update of (16) shows that we apply the projected gradient-descent method to solve the dual problem (13), which guarantees the convergence of dual variable with an appropriate choice of stepsize κ [1]. ■

Finally, we comment on the distributed nature of these updates. The result (14) is the well-known congestion control solution where we can reuse existing distributed TCP algorithms (i.e. TCP Vegas) [3]. Link power update can also be implemented in a distributed fashion through message passing similar to the algorithm in [2]: each receiver of link k broadcasts its control message m_k (which is a real number and assumed to be heard by others), then each transmitter of link l receives them, estimate G_{kl} through training sequence and update its power as (15). The congestion price update (16) also only needs local information: ingress rate and received signal measurement.

C. Successive Convex Approximations: Algorithm and Optimality

We continue presenting an algorithm that can achieve the globally optimal solutions of nonconvex problem (3) by solving successively the approximated problem (10).

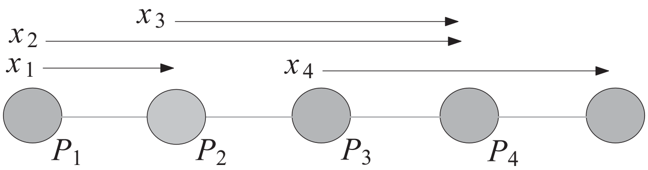

Fig. 1: Network topology.

Algorithm 1: A Novel Successive Convex Approximations

1) Initialize $(\mathbf{x}, \mathbf{P}) = 0$, $\tau = 1$
2) Form the τ-th approximated convex problem (10) of the original problem (3) by updating $\boldsymbol{\theta}^{l^{(\tau)}}$, $\forall l$ with (6).
3) Solve the τ-th approximated convex problem (10) for optimal solution $(\mathbf{x}^{*(\tau)}, \mathbf{P}^{*(\tau)})$ as in section III-B.
4) Increment τ and go to step 2 until convergence.

Theorem 1: The series of approximations of Algorithm 1 converge to the stationary points satisfying the Karush-Kuhn-Tucker (KKT) conditions of the original problem (3).

Proof: Letting $h(\mathbf{x}, \mathbf{P}) = \frac{\sum_{s:l \in L(s)} x_s}{c_l(\mathbf{P})}$ and $\hat{h}(\mathbf{x}, \mathbf{P}) = \frac{\sum_{s:l \in L(s)} x_s}{\hat{c}_l(\mathbf{P})}$, we need to prove that this series of approximations satisfies the following properties according to [4]

1) $h(\mathbf{x}, \mathbf{P}) \leq \hat{h}(\mathbf{x}, \mathbf{P})$
2) $h(\mathbf{x^o}, \mathbf{P^o}) = \hat{h}(\mathbf{x^o}, \mathbf{P^o})$
3) $\nabla h(\mathbf{x^o}, \mathbf{P^o}) = \nabla \hat{h}(\mathbf{x^o}, \mathbf{P^o})$, where $(\mathbf{x^o}, \mathbf{P^o})$ is the optimal solution of the previous iteration.

Conditions 1) and 2) are clearly satisfied with (6) and (8). It is straightforward to verify condition 3) by taking derivative. Then, the globally optimal convergence of Algorithm 1 can be proved similarly as in [5]. ∎

IV. Simulation Results

We consider a network topology as in Fig. 1 with 4 flows and 5 nodes placed equally at a distance $d = 20$ m. The baseband bandwidth W is set to 125 kHz, and we use $K = -1.5/\log(5\text{BER})$ with $\text{BER} = 10^{-3}$ for MQAM modultion [2]. The channel gain is assumed to be *iid* Rayleigh random variables with mean value $h(d) = h_o(\frac{d}{15})^{-4}$ where h_o is a reference channel gain at a distance 15 m. The maximum power, noise and h_o are selected so that the average receive SNR at 15 m is 30 dB. The utility function of all users is $\log(.)$. We use two criteria to evaluate the convergence-speed performance: convergence condition of solving step 3 (i.e. inner convergence) and convergence at step 4 (i.e. outer convergence) of Algorithm 1 represented by $\max_{l \in \mathcal{L}} |P_l^{(t)} - P_l^{(t-1)}| < \epsilon$ and $\max_{l \in \mathcal{L}} |P_l^{*(\tau)} - P_l^{*(\tau-1)}| < \epsilon$ respectively, where ϵ is a small number. Table I shows the average number of iterations over 100 realizations with various values of ϵ. We see that our scheme converge faster than Alg. B (i.e. log successive approximation), especially with inner convergence. For example, the proposal converges 3 times faster with $\epsilon = 10^{-4}$. Fig. 2a shows a realization of utility convergence with $\epsilon = 10^{-6}$. Even though Alg. B and our method obtain the same convergence value which is a bit lower than the optimal one of Alg. A (convex) [5] due to the approximated nature, the relative error is small, only 0.7% approximately. Corresponding flow rate and power convergence of our method are shown in Figs. 2b and 2c.

TABLE I: Convergence Speed Comparison

ϵ	proposal		log successive convex	
	Inner Convg.	Outer Convg.	Inner Convg.	Outer Convg.
10^{-4}	42.97	9.28	142.08	10.55
10^{-5}	78.45	11.94	187.97	12.88
10^{-6}	138.97	14.26	221.01	16.44

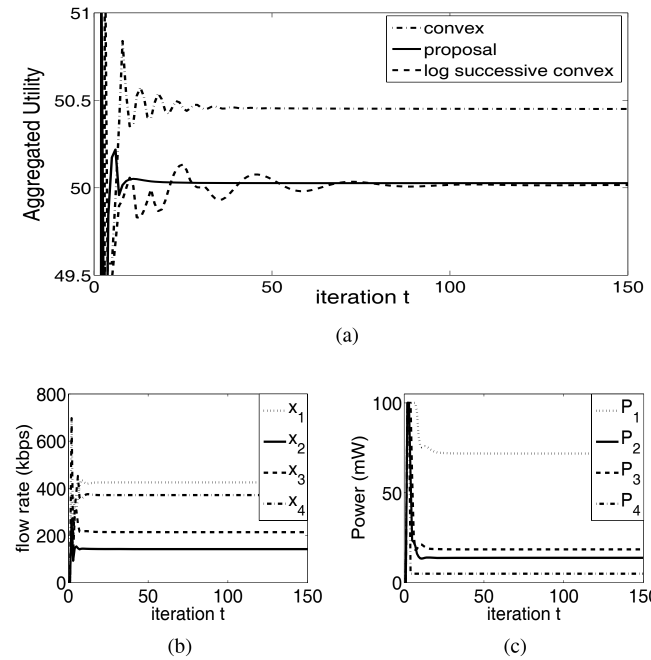

Fig. 2: Convergence of the algorithm: (a) aggregate utility comparison; (b) source rates (proposal); (c) link powers (proposal).

V. Conclusion

We propose an algorithm using successive approximations method to transform the original nonconvex problem of JCRP problem into convex problem, then the global optimal solution can converge distributively with message passing. Simulation results show that our method can outperform previous work.

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