Optimal Power Allocation in Wireless Small Cells Networks

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Abstract

Heterogeneous wireless small cells networks (HetNet) is a promising technology to solve the increasing demand of mobile services. Deploying a large number of small cells (i.e., femtocells and picocells) within macrocells' coverage area has many new technical challenges. In this work, we propose power allocation problem in heterogeneous small cells networks where low-power small cells have limited available power to allocate to their associated mobile users. Moreover, we implemented a numerical method so called Newton's method to address the proposed problem. Simulation results show that our proposed solution method converges to the optimal solution and increase the network capacity.

Keywords – Newton's Method, optimal power allocation, wireless small cells network.

1. Introduction

Wireless network services and wireless devices are increasing exponentially. In order to solve this scenario, 3GPP has introduced heterogeneous wireless network (HetNet) that aims to increase spectral efficiency per unit area [1][2]. Heterogeneous small cells network consists of macrocells, relay nodes, and low power small cells.

Due to the increase of different types and number of cells, we have to face with many challenges in heterogeneous wireless small cells network. Among them, cell association, interference management, and efficient resource allocation are the most important challenges. In this work, we address the problem of resource (i.e., power) allocation with the help of numerical method so called Newton's method.

2. System Model

As shown in Fig. 1, we consider heterogeneous wireless small networks with a single macrocell, and a set of femtocells $M=\{1,2,\ldots,M\}$ where all femtocells are using the same frequency band. The bandwidth of each femtocell is divided into multiple channels $N=\{1,2,\ldots,N\}$ and serves a set of users $U=\{1,2,\ldots,U\}$ through (OFDMA) and each user requests for a minimum rate requirement $R_{u,m}^{rev}$. Here, we assume that all femtocells are closed access and deployed in suburban houses. Let $h_{u,n}^{m}$ and $p_{u,n}^{m}$ be the channel gain on channel 'n' from femtocell 'm' to user 'u' and the allocated power of femtocell 'm' to user 'u' on channel 'n', respectively. We assume that a channel is assigned to only one mobile user and a user can access only one channel. The received Signal to Interference plus

Noise Ratio (SINR) of user ‘u’ on channel ‘n’ in femtocell ‘m’ can be expressed as:

$$\gamma_{u,n}^{m} = \frac{p_{u,n}^{m} h_{u,n}^{m}}{I_{macro} + N_0} \qquad (1)$$

where I_{macro} is the interference from macrocell and N_0 is the noise power. And then, the downlink data rate of user ‘u’ is

$$R_{u,n}^{m} = \omega_n \log_2(1+\gamma_{u,n}^{m}) \qquad (2)$$

where ω_n is the bandwidth of channel ‘n’.

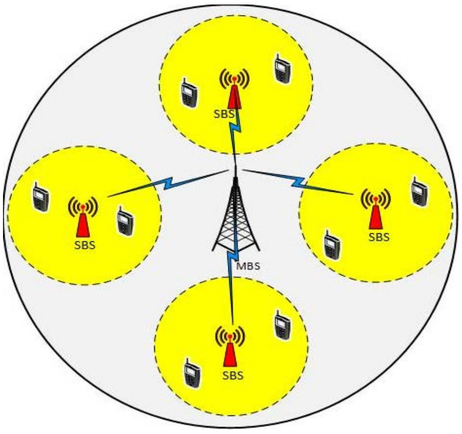

Fig. 1. System Model

3. Problem Formulation

Each femtocell aims to fulfill the rate requirement of its mobile users by allocating power. Thus, the power allocation problem in wireless small cells network can be expressed as:

$$\max_{p_{u,n}^{m}} \sum_{m=1}^{M}\sum_{u=1}^{U}\sum_{n=1}^{N} R_{u,n}^{m} \qquad (3)$$

subject to:

$$p_{u,n}^{m} \geq 0, \ \forall m,\ u,\ n \qquad (4)$$

$$\sum_{u=1}^{U}\sum_{n=1}^{N} p_{u,n}^{m} \leq p_m^{\max},\ \forall m \qquad (5)$$

$$\sum_{n=1}^{N} R_{u,n}^{m} \geq R_{u,m}^{rev},\ \forall u, m \qquad (6)$$

where constraints (1) and (2) express the transmit power on all channels is below the maximum transmit power of femtocell and constraint (3) ensures the QoS requirement of each user in each femtocell.

4. Proposed Power Allocation Algorithm

For the above problem, the Lagrangian can be expressed as:

$$\mathrm{L}(\mathrm{p}_{u,n}^{m}, \lambda_m, \mu_{u,m}) = \sum_{m=1}^{M}\sum_{u=1}^{U}\sum_{n=1}^{N} R_{u,n}^{m} + \sum_{m=1}^{M} \lambda_m \left(p_m^{\max} - \sum_{u=1}^{U}\sum_{n=1}^{N} p_{u,n}^{m} \right) + \sum_{u=1}^{U}\sum_{m=1}^{M} \mu_{u,m} \left(\sum_{n=1}^{N} R_{u,n}^{m} - R_{u,m}^{rev} \right) \qquad (7)$$

where λ_m and $\mu_{u,m}$ are non-negative Lagrangian multipliers. By taking the first-order derivative of (7) with respect to $p_{u,n}^{m}$, λ_m, $\mu_{u,m}$, the Karush-Kuhn-Tucker (KKT) conditions is

$$\frac{\partial L}{\partial p_{u,n}^{m}} = (1+\mu_{u,m}) \frac{\omega_n h_{u,n}^{m}}{\ln 2(I_{macro} + N_0 + p_{u,n}^{m} h_{u,n}^{m})} - \lambda_m \leq 0 \qquad (8)$$

$$\mathrm{p}_{u,n}^{m} \left[(1+\mu_{u,m}) \frac{\omega_n h_{u,n}^{m}}{\ln 2(I_{macro} + N_0 + p_{u,n}^{m} h_{u,n}^{m})} - \lambda_m \right] = 0 \qquad (9)$$

$$\lambda_m \left[p_m^{\max} - \sum_{u=1}^{U}\sum_{n=1}^{N} p_{u,n}^{m} \right] = 0 \qquad (10)$$

$$\mu_{u,m} \left[\sum_{n=1}^{N} R_{u,n}^{m} - R_{u,m}^{rev} \right] = 0 \qquad (11)$$

For optimal power allocation, we use Newton’s method which is the efficient approximation method which converges fast to the local optimal solution [3]. Let us consider the function $f(p_{u,n}^{m})^i$ and the first order derivative $f^{'}(p_{u,n}^{m})^i$ as follows

$$f(p_{u,n}^{m})^i = (1+\mu_{u,m}^{i}) \frac{\omega_n h_{u,n}^{m}}{\ln 2(I_{macro} + N_0 + (p_{u,n}^{m})^i h_{u,n}^{m})} - \lambda_m^{i} \qquad (12)$$

$$f^{'}(p_{u,n}^{m})^i = (1+\mu_{u,m}^{i}) \frac{\omega_n (h_{u,n}^{m})^2}{\ln 2(I_{macro} + N_0 + (p_{u,n}^{m})^i h_{u,n}^{m})^2} \qquad (13)$$

where i means the i^{th} iteration. After that, by using Linear algebra, the approximation value of optimal power allocation at $(i+1)^{th}$ iteration as follows

$$(p_{u,n}^{m})^{i+1} = (p_{u,n}^{m})^{i} - \frac{f(p_{u,n}^{m})^i}{f^{'}(p_{u,n}^{m})^i} \qquad (14)$$

For the optimal power allocation solution, we need to update the Lagrangian multipliers. By using sub-gradient method and it as follows:

$$\lambda_m^{i+1} = \left[\lambda_m^i - \theta_1 (p_m^{\max} - \sum_{u=1}^{U} \sum_{n=1}^{N} p_{u,n}^m \right]^+ \qquad (15)$$

$$\mu_{u,m}^{i+1} = \left[\mu_{u,m}^i - \theta_2 (\sum_{n=1}^{N} R_{u,n}^m - R_{u,m}^{rev}) \right]^+ \qquad (16)$$

where θ_1 and θ_2 are positive constant step sizes.

5. Simulation Results

In our simulation, we consider 3 femtocells and 5 users in each femtocell's coverage area. Total transmit power of each cell is 4 Watt and the minimum data rate requirement of mobile users is between 0.2Mbps and 1Mbps. The Thermal noise power is $-174 dBm/Hz$. The long distance path loss model in our simulation is

$$PL = 40\log_{10}(d_0) - 10\log_{10}(G h_t^2 h_r^2) + 10\gamma \log_{10} \frac{d}{d_0} + X_g$$

where G is the gain product of transmitter and receiver, d_0 and d are the reference distance and actual distance between transmitter and receiver, h_t and h_r are heights of transmitter and receivers, and X_g is the random variable.

Fig. 2 shows the data rate achieved by each user in each femtocell's coverage area.

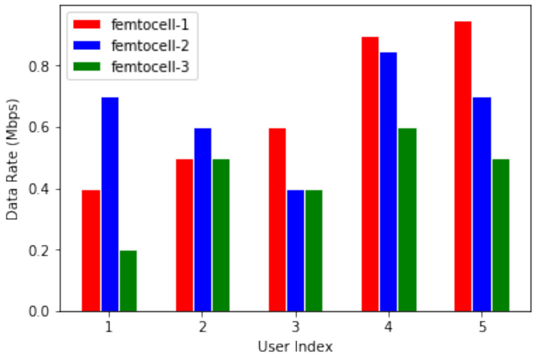

Fig. 2. Data rate achieved by each mobile user

Moreover, Fig. 3 shows the convergence rate of our proposed algorithm for optimal power allocation in wireless small cells network. The transmit power for users in femtocell-1 converges to optimal solution at iteration 20, femtocell-2 at 24, and femtocell-3 at 34, respectively. We can see that after some iterations, we proposed algorithm converges to the local optimal.

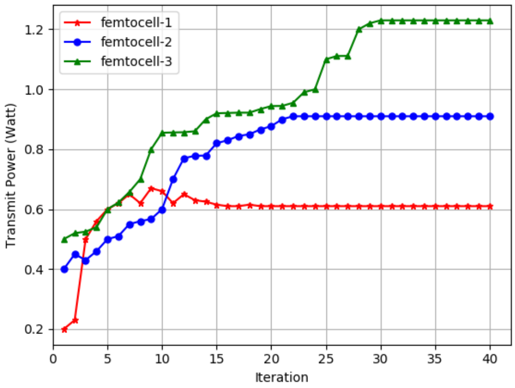

Fig. 3. Convergence rate of proposed algorithm

6. Conclusion

In this work, we have discussed power allocation problem in wireless small cells networks. Then, we have implemented Newton's method to address our proposed power allocation. In simulation, we have discussed and shown detailed results of our proposed algorithm.

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