Multi-Operator Backup Power Sharing in Wireless Base Stations

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***Abstract*—Installation of backup power supply plays a vital role in maintaining communication services which can save billions of dollars as well as human lives during natural disasters. Due to the higher capital and operational expense compared to public power, pooling and sharing the backup power supplies can be an economical solution since the backup power capacity can be sized based on the aggregate demand of co-located operators. However, how to pool and share the backup power at multi-operator cellular sites in a fair manner should be considered due to the limited capacity and high user demands. In this paper, we adopt the Nash Bargaining Solution (NBS) of a bargaining problem which can guarantee the fairness of backup power sharing and design a decentralized algorithm approach with limited information exchange among the operators. Our simulation demonstrates that the sharing the backup power reduces the average delay and requires less BS power consumption than the non-sharing approach, especially for high traffic load scenarios. In addition, we also extend the formulation with respect to admission control for very high traffic demand cases.**

***Index Terms*—Backup Power sharing, Base Station, Fair sharing, Decentralized Optimization.**

I. Introduction

Nowadays, the mobile service demand keeps increasing since there are more and more mobile devices and services. Accordingly, service availability of mobile communication becomes one of the crucial requirements for the success of all mobile network operators. However, there are many factors (e.g., aging power infrastructure, natural disasters) lead to the power outages and disrupt many mobile services. These power outages often happen and being extremely challenging for mobile operators. Communications service interruptions affected by power outages are a daily norm in many developing countries [1], while even in developed economies such as the United States, communications service outages are also proliferating and affect millions of people in 2015 [2]. To ensure communications service continuity, wireless operators have commonly installed backup power supplies alongside their BSs. The necessity of improving service availability during power outages has drawn significant attention. The Federal Communications Commission (FCC) has proposed a mandate that carriers must increase or provide sufficient emergency/backup power at their cell sites [3].

There are multiple options to supply backup power to BSs during power outages, such as diesel generator, lead-acid battery, li-ion battery and fuel cell. Currently, diesel generator is widely set up for many systems; however, it has some drawbacks, such as pollution, high noise, and heavy weight, which is not suitable for many urban wireless tower installations. In addition, battery is also a common option for backup power, but its high capital investment and maintenance cost have made it less and less appealing. More recently, fuel cells have been extensively studied in an attempt to improve technical performance, reliability and reduce environmental issues. According to these advantages, fuel cell is emerging as one of the most popular options for many applications including BS backup power.

While multiple backup power options are available, they are all very expensive. This creates an impediment to implementing the FCC regulation regarding backup power at the cellular BSs. Although FCC recommends the installation of at least 8 hours backup power installation, many BSs do not have enough backup power, even for major wireless carriers, due to the high capital cost. For that reason, sharing the precious backup power resource emerges as a key opportunity to lower the cost and benefit all participating wireless operators. Indeed, backup power sharing can be easily implemented with almost no changes to the co-located sites, where many wireless operators already shared the tower infrastructure and physically co-locate their BSs. In the report [4], tower sharing allows operators to cut CapEx, e.g., infrastructure cost for operators is reduced by 16% to 20%. In addition, independent tower companies have become prevailing in India, China, Southeast Asia, and United States since 2015 [4]–[6]. More importantly, some task forces of FCC have begun to study and recommend the sharing of power supplies [7]. Consequently, on top of tower sharing, backup power sharing among multiple wireless operators can be easily deployed and viewed as an integral element of infrastructure sharing [5].

Despite the economic advantage and benefit, a major issue is how to fairly share the backup power among multiple participating operators and making them better off. In this paper, we study the under-explored problem - *fairness of backup power sharing in multi-operator cellular towers where wireless operators can associate their own traffic loads (i.e., route their power demand) to different towers in a fair manner*. Towards this end, we adopt Nash Bargaining theory, which is designed for a cooperative game that helps participants achieve fairness and Pareto optimal solutions [8]. Intuitively, operators can make an agreement to maintain the service by using shared

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backup power in a collaborative manner if they attain greater utility than non-cooperating.

Recently, there is an increasing interest in sharing power studies for mobile networks. *First*, in the state-of-the-art on sharing renewable power [9]–[11], the authors propose hybrid power models, in which BSs can receive power from both electric grid and renewable energy. Even though BS power demand exceeds the renewable energy capacity and battery storage, operators can receive additional power from the grid. On the other hand, we explores the uninvestigated sharing backup power problem in which the available backup power is a hard constraint for each operator during an emergency grid outage. *Second*, the works [9], [10] focus on sharing power among BSs without multi-operator consideration. The recent works such as in [11], the authors consider sharing power storage among the multi-operator at a single site while in [12], [13], the authors propose the multi-operator cooperation based on roaming/offloading traffic loads and low-utilized BSs switching-off operation for multiple sites. Nevertheless, our proposed model exploits multi-operator backup power sharing and user association decision among multiple sites in a considered region. Accordingly, power demand of BSs at the co-located multi-operator sites can be regulated by routing user traffic loads among these sites. In this scenario, the amount of received backup power is the only power source of the system which needs to be shared for all operators and affects the average delay performance of each operator. Individually, each operator makes a decision on user association such that they can balance the BSs load and optimize the average delay performance based on the flow-level analysis. Different from the existing works on energy sharing, we adopt a different utility function (i.e., flow-level delay cost). *Third*, under the limitation of the available backup power, a cooperative approach using Nash Bargaining Solution can guarantee the fairness of operators' gain in terms of delay performance when participating in the cooperative solution. Therefore, the integration of user association problem and the sharing limited backup power is analyzed in our work.

In summary, the key novelty of our study is that we propose the fair backup power sharing among wireless operators as a cost-effective approach to improve the communications service quality. Concretely, we make the following contributions:

- In Section II, we apply the analytical framework of flow-level delay-optimal user association [14] among cellular BSs of a single operator into the co-located multi-operator sites in wireless networks. Accordingly, the objective of operators is minimizing their delay performance in terms of the flow-level cost and load balancing among BSs.
- In Section III, we develop a scheme to fairly share the backup power supply among the operators by applying the NBS. For practical implementation, we design a decentralized algorithm based on Jacobi-Proximal Alternating Direction Method of Multipliers (JP-ADMM) approach with limited information exchange among the operators to solve the bargaining problem. Then, we provide numerical studies based on practical settings of cellular BSs to demonstrate the effectiveness of the proposed backup power sharing scheme. Our sharing backup power approach can reduce the flow-level cost in terms of delay and improve power efficiency compared to no sharing strategy. Moreover, we extend the problem formulation considering admission control for very high traffic demands, in which the backup power provisioning is insufficient for all users.

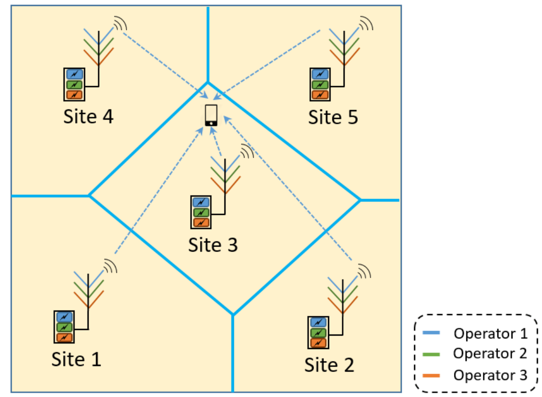


Fig. 1: User association at multi-operator sites model.

II. System Model

We apply the infrastructure-based wireless network model from multiple BSs of a single operator [14] to the co-located multi-operator sites as shown in Figure 1. Mobile users in a considered region $\mathcal{L} \in \mathbb{R}^2$ are served by a set $\mathcal{G}$ of operators. Rather than the individual deployment, each operator i has a BS set $\mathcal{H}_i$, which is located at different sites. At each site j, operators share cellular BS infrastructure and have backup power supplies. Figure 1 illustrates that a mobile terminal (MT) of operator 1 can associate with one of the co-located multi-operator sites. Even though being closest to the BS at site 3, this MT may be associated with farther BS at other sites through mobile hand-off if the BS at site 3 is heavily utilized. Since our work analyzes the sharing power of operators at co-located sites, we focus on the downlink scenario, in which BS power consumption is linearly increasing with the mobile traffic load.

At any location $x \in \mathcal{L}$, the traffic flows follow an inhomogeneous Poisson point process with arrival rate per unit area $\lambda(x)$. For simplicity, the arrival traffics can be modeled as user flows (i.e., data requests) with random sizes following independent distribution with mean $\frac{1}{\mu(x)}$. Then the *traffic load density* at the location x is defined as $\gamma(x) = \frac{\lambda(x)}{\mu(x)}$ in [14]. We assume $\gamma(x) < \infty$ for all $x \in \mathcal{L}$. The spatial traffic variability is captured in the traffic load density expression.

Following the literature [14], [15], we consider the path-loss model to capture the average channel quality between user locations and BSs. In addition, instead of dynamic inter-cell interference, we only consider the static Gaussian-like noise inter-cell interference with interference randomization or fractional frequency reuse [**?**], [16]. The fractional frequency reuse provides a strategy to mitigate interference and make interfered cells sufficiently separated if they operate on the same frequency. At location x, the transmission rate served by BS j of operator i is denoted by $c_{ij}(x)$ which follows Shannon capacity

$$c_{ij}(x) = BW \cdot \log_2 \left(1 + \frac{P_{ij} g_{ij}(x)}{\sigma^2 + I_{ij}(x)}\right) \tag{1}$$

where P_{ij} denotes the transmission power of the operator i at BS j and $g_{ij}(x)$ denotes the channel gain from the BS j of operator i to the MT at location x, including path loss, shadowing, and other factors. In addition, σ^2 denotes noise power and $I_{ij}(x)$ denotes the average interference seen by the MT at location x. Various available radio propagation models can be used to predict the path loss in dB and account for shadow fading effect. As a result, transmission rate becomes location dependent.

The *system-load density* [14] is denoted by $\beta_{ij}(x) = \frac{\gamma_i(x)}{c_{ij}(x)}$, which defines the fraction of active transmission time required to deliver the traffic load $\gamma_i(x)$ of operator i from BS j to location x. The user associated routing probability vector for each operator i is denoted by $\mathbf{p}_i(x) = \{p_{ij}(x)\}$ for all $x \in \mathcal{L}$ and $j \in \mathcal{H}_i$.

Definition 1 (Feasibility): The set $\mathcal{F}_i$ of feasible BS loads (or utilization) of the operator i, i.e., $\rho_i = \{\rho_{ij}\}$ for all $j \in \mathcal{H}_i$ is defined as follows

$$\begin{aligned}\mathcal{F}_i = \big\{\rho_i \,|\rho_{ij} = \int_{\mathcal{L}} \beta_{ij}(x) p_{ij}(x) dx \\ 0 \leq \rho_{ij} \leq 1 - \epsilon, \sum_{j \in \mathcal{H}_i} p_{ij}(x) = 1, \\ 0 \leq p_{ij}(x) \leq 1, \forall j \in \mathcal{H}_i, \forall x \in \mathcal{L}\big\},\end{aligned}$$

where ϵ is an arbitrarily small positive constant. The sum of the routing probability of a traffic flow at any location x to all the BSs should be 1. The feasible set $\mathcal{F}_i$ was proved to be convex in [14].

A. Flow-Level Cost Model

In this work, we adopt the flow-level dynamic systems [14], which consider data requests (i.e., flows and file transfers) that are initiated randomly and leave the system after serving. This will capture the network performance as the stability analysis of a queueing system. The user association problem from the dynamic flow-level model can be seen as a routing problem. By using this model, the load balancing issue of BSs is profound in [14], in which MTs can be associated with farther low-utilized BSs in order to achieve better system performance of the operator in terms of average queueing delay. Furthermore, the stochastic traffic loads are modeled as inhomogeneous spatial distributions and enable more realistic traffic characteristic for system-level analysis of mobile operators. Based on the queueing analysis [14] for the M/GI/1 multi-class processor sharing system, the expected total number of flows of the operator i is calculated by $L_i = \sum_{j \in \mathcal{H}_i} \frac{\rho_{ij}}{1-\rho_{ij}}$. Since minimizing the expected total number of flows is equivalent to minimize the average delay according to Little Law, the average delay of a typical flow D_i of the operator i is as follows

$$D_i = \frac{L_i}{\Lambda_i} = \frac{1}{\int_{x \in \mathcal{L}} \lambda_i(x) dx} \times \sum_{j \in \mathcal{H}_i} \frac{\rho_{ij}}{1 - \rho_{ij}}. \tag{2}$$

For analytical purpose, we use the cost function for flow-level performance in [14] as follows

$$\phi(\rho_i) = \sum_{j \in \mathcal{H}_i} L_i + 1 = \sum_{j \in \mathcal{H}_i} \frac{1}{1 - \rho_{ij}}. \tag{3}$$

Minimizing the cost function $\phi(\rho_i)$ is equivalent to minimizing L_i, thus minimizing the average flow delay, which helps to improve user QoS of the operator i.

B. Base Station Power Model

According to [15], the BS power consumption increases with the increasing BS utilization and there are two kinds of power consumptions: fixed power consumption and the power consumption that are proportional to BSs utilization. Thus, the total power consumption of a BS is given by

$$\psi_{ij}(\rho_{ij}) = (1 - m_{ij}) \rho_{ij} Q_{ij} + m_{ij} Q_{ij}. \tag{4}$$

where $m_{ij} \in [0, 1]$ is a portion of the fixed power consumption of the BS and Q_{ij} is the maximum BS's operational power when it is fully utilized, i.e., $\rho_{ij} = 1$, which includes power consumptions of transmit antennas, power amplifiers, and others. When $m_{ij} = 0$, BSs would ideally consume no power when idle, and gradually consume more power as the utilization increases.

In this paper, the operator performance is evaluated in terms of flow-level performance (3), i.e., the average delay depending on BSs utilization by queueing analysis. The utilization of these BSs determines the power usage according to (4). Therefore, when using backup power to maintain communication services (e.g., due to power outages), downsizing the maximum operational power for the economic purpose will negatively affect the BS performance. Specifically, for the user association problem with the backup power capacity of BS, some MTs cannot associate with a nearby BS with high traffic load density. Accordingly, these MTs are associated with more distant BSs with lower traffic load density. Due to the traffic density heterogeneity in different locations, the operators can have benefits of sharing the backup power to improve the delay performance. Conceivably, sharing backup power among the operators can not only reduce the capital costs but also improve the operator performance.

III. Backup Power Sharing

To enable the cooperation between operators for backup power sharing, we formulate a fair sharing problem based on the Nash Bargaining game [8], which can reduce the average flow delay better than no sharing scheme. We also design a decentralized algorithm to achieve the NBS.

A. Problem Formulation

We model the interaction between the operators at co-located sites as shown in Figure 1. Each operator minimizes its flow-level cost function (3), which is convex with respective to the BS loads. Independently optimizing user association with their own backup power would be trivial if the operators had no cooperation. In this case, the utility of operator i, denoted by $\hat{\phi}_i$, is determined by solving the following problem

No Backup Power Sharing (NBPS):

$$\begin{aligned} \min_{\mathbf{p}_i} \quad & \phi(\rho_i) && (5)\\ \text{s.t.} \quad & \psi_{ij}(\rho_{ij}) \le B_{ij}, && \forall j \in \mathcal{H}_i,\\ & \rho_{ij} \in \mathcal{F}_i, && \forall j \in \mathcal{H}_i. \end{aligned}$$

The optimal user association of the $\mathbf{NBPS}$ problem represents the probability vector. Based on this probability vector, MTs should be associated with their corresponding BSs to minimize the flow-level cost faced by operator i at every site under the limitation of the BS's maximum operational power. Therefore, downsizing the maximum operational power decreases the number of MTs that can be associated with their closest BSs. Due to the limitation of backup power, the utilization of BSs located at larger density area will be higher, which forces more MTs to associate with farther BSs, thus lower transmission rate.

The question then arises: Is there any way that the operators can cooperate on sharing backup power to improve their performance, i.e., achieve $\phi_i \le \hat{\phi}_i, \ \forall i \in \mathcal{G}$ and such that:

a) The gains from cooperation are fair at a Pareto-efficient outcome?

b) Operators do not have to reveal any private information about their traffic loads?

To deal with the first question, we will resort to the NBS in the next paragraphs. For the second question, we also design a decentralized algorithm so that operators can protect their traffic load privacy in the next subsection.

1) Backup Power Fair Sharing using NBS: When the Nash Bargaining game is applied for the backup power fair sharing, the produced NBS of this cooperative game guarantees an outcome, which is not only Pareto-efficient but also proportional-fair [8], [17]. If the Nash Bargaining game cannot produce better delay performance for all operators, their performance is still at least the solution of the $\mathbf{NBPS}$, which represents the disagreement point of this bargaining problem. Especially, if the NBS exists, it is unique and satisfies the four axioms:

a) **Pareto Efficiency**: NBS produces a Pareto optimal solution, i.e., no operators can improve its communications service quality without compromising the others'.

b) **Symmetry**: NBS provides equal gains from cooperation when the feasible region is symmetric, where the feasible region is agnostic of the player identities. As a result, the solution will be the same even if the operators utility axis are swapped.

c) **Independence of Affine Transformations**: NBS should be agnostic of any affine transformations of operator utilities. Therefore, consider an example of three operators as in Figure 1, if the NBS is given by (ϕ_1^{NB}, ϕ_2^{NB}, ϕ_3^{NB}) for some utilities (ϕ_1, ϕ_2, ϕ_3), and ϕ_1 is transformed to $a_1\phi_1 + b_1$, then the solution changes to ($a_1\phi_1^{\text{NB}} + b_1$, ϕ_2^{NB}, ϕ_3^{NB}).

d) **Independence of Irrelevant Alternatives**: The addition of irrelevant alternatives will not change the NBS. That is, for feasible regions Θ and Θ', if (ϕ_1^{NB}, ϕ_2^{NB}, ϕ_3^{NB}) $\in$ solution(Θ), $\Theta' \subset \Theta$, and (ϕ_1^{NB}, ϕ_2^{NB}, ϕ_3^{NB}) $\in \Theta'$ then (ϕ_1^{NB}, ϕ_2^{NB}, ϕ_3^{NB}) $\in$ solution(Θ').

The NBS of the backup power fair sharing problem can be achieved by solving the following problem

Backup Power Fair Sharing (BPFS):

$$\begin{aligned} \max_{\mathbf{p}} \quad & \prod_{i \in \mathcal{G}} \left[\hat{\phi}_i - \phi(\rho_i)\right]^{\omega_i} && (6)\\ \text{s.t.} \quad & \sum_{i \in \mathcal{G}} \psi_{ij}(\rho_{ij}) \le \sum_{i \in \mathcal{G}} B_{ij}, && \forall j \in \mathcal{H}_i, \quad (7)\\ & \phi(\rho_i) \le \hat{\phi}_i, && \forall i \in \mathcal{G}, \quad (8)\\ & \rho_{ij} \in \mathcal{F}_i, && \forall i, j. \quad (9) \end{aligned}$$

The $\mathbf{BPFS}$ problem maximizes the product of operators' gains in delay performance over the disagreement point, $\hat{\phi}_i$, which is a constant in the $\mathbf{BPFS}$ problem. The different power coefficients ω_i represent the operator heterogeneity in the fairness design. The inequality constraint (7) will not allow the total power consumption of the BSs of all the operators greater than their total backup power capacity at every site. The constraint (8) enforces the benefit of cooperation for sharing over no sharing. The constraint (9) guarantees the feasibility of BS loads. The optimal user association distribution and BS loads of this problem guarantee a better performance than or equal to the disagreement point. For that reason, disagreement points can be considered as the substitute solutions when all operators cannot achieve better flow-level performance.

B. Decentralized Solution Method

Since solving the $\mathbf{BPFS}$ problem by a centralized controller requires traffic load information of all operators, we derive a decentralized algorithm based on the Jacobi-Proximal Alternating Direction Method of Multipliers (JP-ADMM) approach [18] to protect each operator's private traffic information.

The $\mathbf{BPFS}$ problem is transformed into an equivalent problem as follows

$\mathbf{BPFS'}$:

$$\begin{aligned} \max_{\mathbf{p}} \quad & \sum_{i} \omega_i \ln \left[\hat{\phi}_i - \phi(\rho_i)\right] && (10)\\ \text{s.t.} \quad & \sum_{i \in \mathcal{G}} \psi_{ij}(\rho_{ij}) + b_j = \tilde{B}_j, && \forall j \in \mathcal{H}_i, \quad (11)\\ & b_j \ge 0, \rho_{ij} \in \mathcal{F}_i, && \forall i, j. \quad (12) \end{aligned}$$

Note that the solution of the $\mathbf{BPFS'}$ problem always satisfies the constraint (8). In addition, we introduce slack variables b_j to transform the inequality sharing backup power constraint

Algorithm 1

Initialization: Initialize $k = 0, \epsilon$, $b^{(1)}$, and $\lambda^{(1)}$;
Each operator i computes $\hat{\phi}_i$ from $\mathbf{NBPS}$ problem (5);
repeat
  $k \leftarrow k + 1$
  Each operator i receives $\lambda^{(k)}, b^{(k)}$;
  Compute $\psi_{ij}^{(k+1)}$ from subproblem (14);
  Send $\psi_{ij}^{(k+1)}$ to the coordinator of BSs;
  Each site j updates the slack variable according to (16) and the dual variable according to (19);
until $\|\lambda^{(k+1)} - \lambda^{(k)}\| \leq \epsilon$.
Operator i uses $\mathbf{p}_i^{(k+1)}(x)$ for user association.

(7) into the equality constraint (11), where the aggregate power capacity at each site j is defined as $\tilde{B}_j = \sum_{i \in \mathcal{G}} B_{ij}$. As a result, the optimal solution of $\mathbf{BPFS'}$ is also the optimal solution of $\mathbf{BPFS}$ problem. It is straightforward to see that $\mathbf{BPFS'}$ is a concave optimization problem.

There are several decentralized methods can split the resource sharing $\mathbf{BPFS'}$ problem into the individual subproblem of operators that can keeps the operator privacy, i.e., dual decomposition [19], and ADMM [20]. In this work, we adopt one of the state of the art ADMM variants, which is Jacobi-Proximal ADMM [18]. This approach is proposed to cope with faster convergence than dual decomposition method while providing a parallelization structure for subproblems update in the conventional Gauss-Seidel ADMM method [18]. Although Gauss-Seidel ADMM requires fewer iterations for convergence than JP-ADMM as shown in the simulation results of [21], Gauss-Seidel ADMM needs to perform alternatively its subproblems update, thus weakening the scalability in practice. Different from the original Jacobi-ADMM technique, JP-ADMM includes additional proximal terms in subproblems and a new parameter, $\alpha > 0$, for dual variable updates as shown later in update steps.

The augmented Lagrangian is derived for the $\mathbf{BPFS'}$ problem as follows

$$\begin{aligned}\mathcal{L}_{\mathcal{A}} = &- \sum_{i \in \mathcal{G}} \omega_i \ln\left[\hat{\phi}_i - \phi(\rho_i)\right] - \sum_{j \in \mathcal{H}_i} \lambda_j \Big(\sum_{i \in \mathcal{G}} \psi_{ij} + b_j - \tilde{B}_j\Big) \\ &+ \frac{\rho}{2} \sum_{j \in \mathcal{H}_i} \Big(\sum_{i \in \mathcal{G}} \psi_{ij} + b_j - \tilde{B}_j\Big)^2. \end{aligned} \tag{13}$$

The summary of the JP-ADMM based algorithm for backup power sharing is presented in Algorithm 1. At each iteration k of Algorithm 1, operator i receives the dual variables, slack variable and estimated BSs power usage from the previous iteration then individually solves its subproblem to obtain a solution for the user association vector $\mathbf{p}_i^{(k+1)}(x)$ as follows

$$\begin{aligned}\min_{\mathbf{p}_i} \quad & -\omega_i \ln\left[\hat{\phi}_i - \phi(\rho_i)\right] + \frac{\tau_i}{2} \sum_{j \in \mathcal{H}_i} (\psi_{ij} - \psi_{ij}^{(k)})^2 \\ & + \frac{\rho}{2} \sum_{j \in \mathcal{H}_i} \Big(\psi_{ij} + \sum_{n \neq i} \psi_{nj}^{(k)} + b_j^{(k)} - \tilde{B}_j - \frac{\lambda_j^{(k)}}{\rho}\Big)^2 \end{aligned} \tag{14}$$

$$\text{s.t.} \quad \rho_{ij} \in \mathcal{F}_i, \qquad \forall j \in \mathcal{H}_i.$$

The square differences between the power consumption variables and the previous iteration solutions are known as proximal terms.

Sites updates: After solving the subproblem (14), each operator sends its estimated power consumption $\psi_{ij}^{(k+1)}$ given the user association solutions at the current iteration to the coordinators at co-located sites. Then the coordinator at each site j updates the slack variables $b_j^{(k+1)}$ as follows:

$$\min_{b_j \geq 0} \quad \frac{\rho}{2}\Big(\sum_{i \in \mathcal{G}} \psi_{ij}^{(k+1)} + b_j - \tilde{B}_j - \frac{\lambda_j^{(k)}}{\rho}\Big)^2 + \frac{\tau_j}{2}(b_j - b_j^{(k)})^2. \tag{15}$$

This slack variable update also needs an additional proximal term due to its appearance in consensus sharing constraint.

Lemma 1. *The optimal solution of the problem (15) is achieved by using Karush-Kuhn-Tucker (KKT) condition [22] as follows*

$$b_j^{(k+1)} = \Big[\frac{\rho\big(\tilde{B}_j - \sum_{i \in \mathcal{G}} \psi_{ij}^{(k+1)}\big) + \lambda_j^{(k)} + \tau_j b_j^{(k)}}{\rho + \tau_j}\Big]^+. \tag{16}$$

Proof: We derive the Lagrangian of problem (15) with Lagrangian multiplier $\mu \geq 0$ as follows

$$\begin{aligned}\mathcal{L}(b_j, \mu_j) = &\frac{\rho}{2}\Big(\sum_{i \in \mathcal{G}} \psi_{ij}^{(k+1)} + b_j - \tilde{B}_j - \frac{\lambda_j^{(k)}}{\rho}\Big)^2 \\ &+ \frac{\tau_j}{2}(b_j - b_j^{(k)})^2 - \mu_j b_j. \end{aligned}$$

Using KKT condition, we first have the following criterion

$$\begin{aligned}&\frac{\partial \mathcal{L}}{\partial b_j} = 0 \\ &\Leftrightarrow \rho\Big(\sum_{i \in \mathcal{G}} \psi_{ij}^{(k+1)} + b_j^* - \tilde{B}_j - \frac{\lambda_j^{(k)}}{\rho}\Big) + \tau_j(b_j^* - b_j^{(k)}) = \mu_j \\ &\Leftrightarrow b_j^* = \Big(\frac{\rho\big(\tilde{B}_j - \sum_{i \in \mathcal{G}} \psi_{ij}^{(k+1)}\big) + \lambda_j^{(k)} + \tau_j b_j^{(k)} - \mu_j}{\rho + \tau_j}\Big). \end{aligned} \tag{17}$$

From complementary slackness criterion, we also have

$$\mu_j^* b_j^* = 0, \mu_j^* \geq 0, b_j^* \geq 0. \tag{18}$$

Therefore, from (17) and (18), we get the closed-form of the slack variable as (16). ∎

Finally, the coordinator updates dual variables as follows

$$\lambda_j^{(k+1)} = \lambda_j^{(k)} - \alpha\rho\Big(\sum_{i \in \mathcal{G}} \psi_{ij}^{(k+1)} + b_j^{(k+1)} - \tilde{B}_j\Big). \tag{19}$$

The algorithm keeps iteratively updating variables until the dual variables differences below the predefined threshold.

Under the mild conditions, i.e., the splittable objective functions are closed proper convex and the existence of a saddle point of problem which satisfies KKT condition, the sufficient condition of JP-ADMM for the global convergence to the saddle point according to Theorem 2.1 in [18] can be guaranteed by choosing parameters such that

$$\tau > \rho\Big(\frac{|\mathcal{G}|}{2-\alpha} - 1\Big), \text{ and } 0 < \alpha < 2,$$

where $|\mathcal{G}|$ is the total number of operators. Moreover, with additional running conditions, JP-ADMM achieves $o(1/k)$ convergence rate from Theorem 2.2 in [18], where k denotes the number of iterations.

The decentralized algorithm only needs to share the dual variables, slack variables, and estimated BSs power usage with other operators while keeping traffic flows and user association information of each operator private.

C. Case Studies

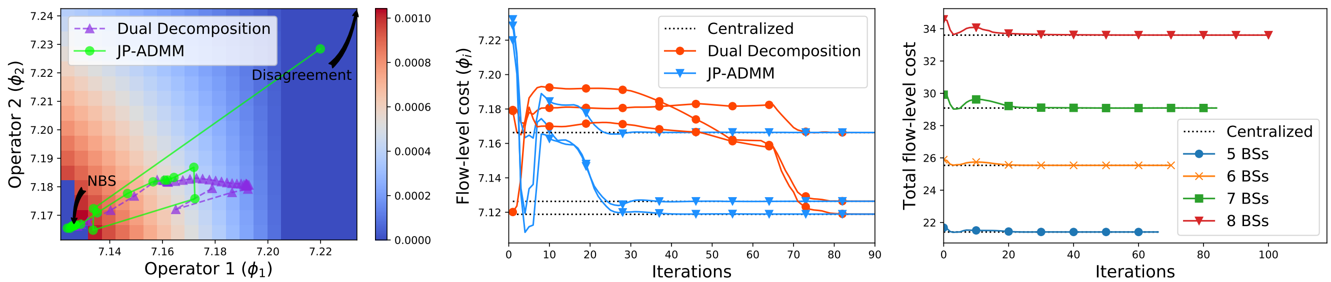
1) Simulation Settings: For an example scenario, we consider three operators which are co-located at five sites and share their infrastructure as in Figure 1. In this scenario, user traffic flows can be associated with all BSs and affect to the BS utilization. These sites are located randomly in a $1 \times 1\, km^2$ region, which is divided into 100 unit squares. The location x of data requests is determined at the bottom left corner of each unit area. According to the communication model of urban macro cells with simulation parameters in the WiMAX evaluation methodology document [23], we use the used COST 231 path loss model with BS height $32\, m$ and MT height $1.5\, m$. In the simulation, we consider no inter-operator interference and static Gaussian-like noise inter-cell interference with lognormal shadow fading with standard deviation $8\, dB$ and the maximum BSs transmission power is $40\, W$. The backup power capacity at the multi-operator sites is downsized to $388\, W$ per operator while the maximum BS operational power is $865\, W$ [15].

We assume that each data request has the size that is log normally distributed with mean $1/\mu(x) = 1$. As an example of the heterogeneity of service demands, the traffic loads of operators in the considered region are generated by decreasing arrival rate from the top left and bottom right corner to the secondary diagonal for operator 1 while in the reverse direction for operator 3. On the other hand, operator 2 has the high arrival rate near the central BS (i.e., BS3) while low arrival rate near the other BSs. Finally, in the simulation results, we consider operators are homogeneous in fairness objective, i.e., $\omega_i = 1$.

2) Simulation Results: Figure 2 shows the convergence of flow-level cost of the decentralized algorithms compared with the optimal solution of the centralized algorithm (i.e., IpOpt solver [24]). Using the same setting of traffic loads and initial parameters, our JP-ADMM based decentralized algorithm produces faster convergence to the optimal solution than dual decomposition method as shown in Figure 2b. Note that the estimated power usage of each iteration solution can be over the limited capacity of backup power. In this case, the partial augmented Lagrangian has high penalty values. As a result, the JP-ADMM algorithm passed through low cost values, especially, at the iteration 4, 5, and 6. We also observe that different initial parameters strongly affect to dual decomposition convergence while it is more consistent in case of JP-ADMM. Specifically, in Figure 2a, we can observe different convergence trajectories between two decentralized algorithms. In this simulation result, we fix the flow-level cost of operator 3, which belongs the NBS of $\mathbf{BPFS}$ problem and vary flow-level cost of the remaining two operators. Accordingly, the color region represents the flow-level cost region of operator 1 and 2, which can vary from the NBS point to the disagreement point and the color values present the product of operators' gains according to the objective (6) of $\mathbf{BPFS}$ problem. This product increases along with the increment of both operators' cost and achieves the maximum value at NBS, i.e., a Pareto solution. As we expected, both algorithms converge to the NBS point by solving $\mathbf{BPFS}'$ problem. Although at the beginning, JP-ADMM solution is far from NBS, it moves quickly to the NBS after several iterations and converges faster than dual decomposition method. Furthermore, Figure 2c indicates the JP-ADMM algorithm requires more number of iterations when we increase the number of BSs in the region and keep the same stopping condition threshold for all scenarios. The more required number of iterations leads to the more running time (i.e., computational time of the subproblem, variables exchange, and site update time) for the decentralized algorithm to converge. In this result, after 40 iterations, the total cost improvement is negligible for all of the cases.

As a result of NBS, compared with no sharing, the sharing approach reduces average delay, averaged over all operators, by 4.6% in Figure 3. In addition to the improvement of the average delay in Figure 3, the backup power usage, averaged over all operators, can be reduced by 2.5% as shown in Figure 4.

Figure 5 illustrates the user association distribution of the central BS3 and the coverage of other BSs belong to operator 1. In this result, we examine the coverage of user association distribution according to heavy traffic load areas near BS2 and BS4. The red areas show the user association distribution of BS3 while the yellow areas are the coverage of the remaining BSs. The orange squares illustrate the locations where traffic flows can be probabilistically associated with multiple BSs. Due to the path loss effect, user flows try to associate with the closer BSs to receive higher transmission rates. In addition, the BSs load balancing and backup power constraint design forces some user flows to associate with the farther low-utilized BSs. As a result, the BSs near heavy traffic load density areas will have small coverages. Specifically, when the backup power capacity of BS3 is set to $388\, W$, the BS2 and BS4 have smaller coverage than other BSs, as illustrated in Figure 5a. With power sharing, there are more MTs being able to associate with



(a) Convergence trajectory of algorithms (b) Convergence rate of algorithms (c) Effects of varying the number of BSs

Fig. 2: Operator's cost convergence.

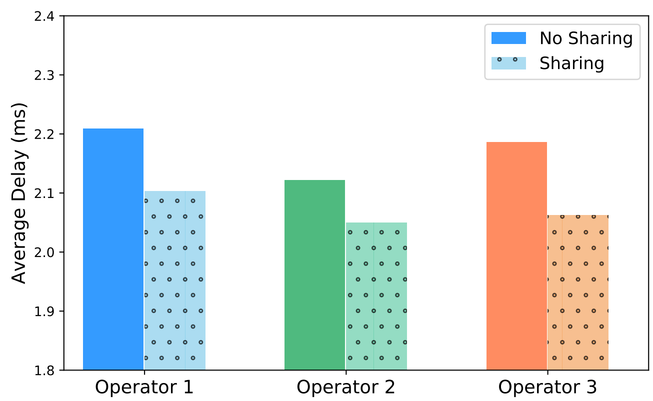


Fig. 3: Average flow delay of each operator.

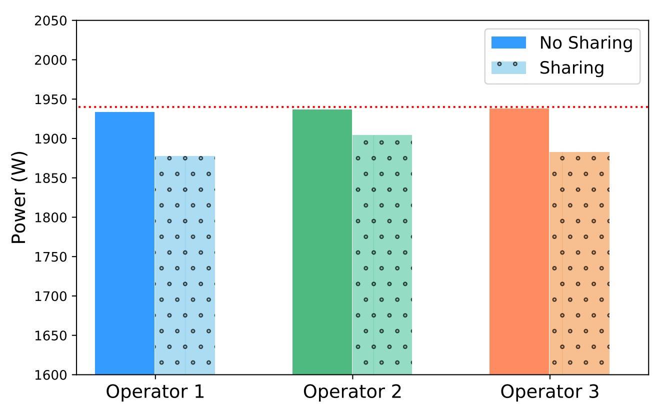
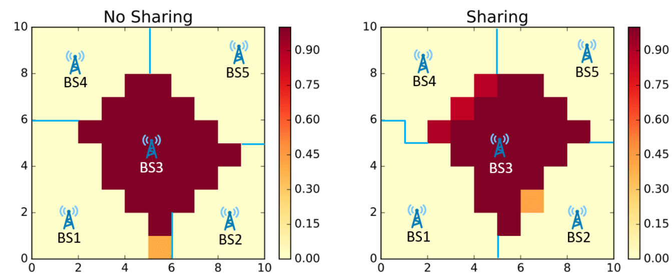


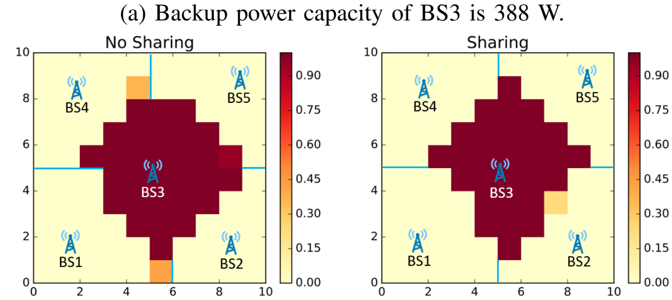
Fig. 4: Power efficiency of operators.

their closest BS. Hence, BS2 and BS4 coverage become larger with the sharing backup power approach. Figure 5b illustrates the user association distribution of the BS3 and the coverage of the other BSs of operator 1 when backup power capacity is increased to $500\,W$. The higher power capacity allows more MTs to associate with BS2 and BS4 and the coverage of these BSs become larger. The coverage of BS1 and BS5 that are near the low traffic areas are reduced for both sharing and no sharing cases.

We next investigate the effect of increasing backup power



(a) Backup power capacity of BS3 is 388 W.



(b) Backup power capacity of BS3 is 500 W.

Fig. 5: User association distribution of the BS3 of operator 1 over the region $\mathcal{L}$.

capacity. In the **NBPS** and **BPFS** problems, the flow-level performance is determined by the user association solution, which depends on the limitation of BS backup power. Figure 6 illustrates that the total flow-level cost of operators decreases along with the increasing of the backup power capacity for both no sharing and sharing backup power case. The lower backup power capacity produces the higher improvement in flow-level performance by sharing backup power compared to no sharing scheme. However, when the backup power capacity becomes greater than $440\,W$, sharing and no sharing approaches have almost similar performance since all BSs become low utilized. Accordingly, the sharing approach does not show the benefit for low-utilized scenarios, such as excessive backup power capacity provision or low traffic loads in night hours. However, during emergency situations, user traffic loads are generally high because many people may be panic and try to search necessary information. Therefore, the study on fair

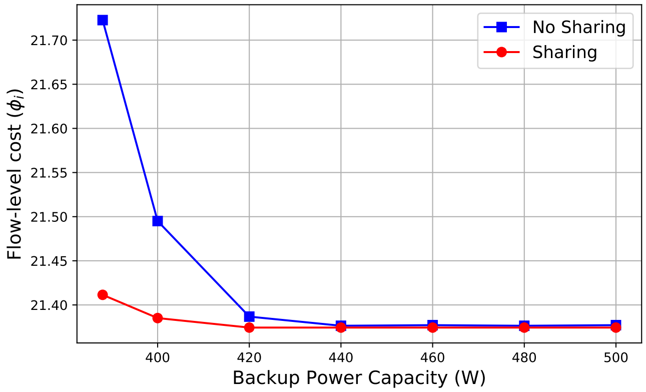


Fig. 6: Total flow-level cost when increasing backup power capacity of BSs.

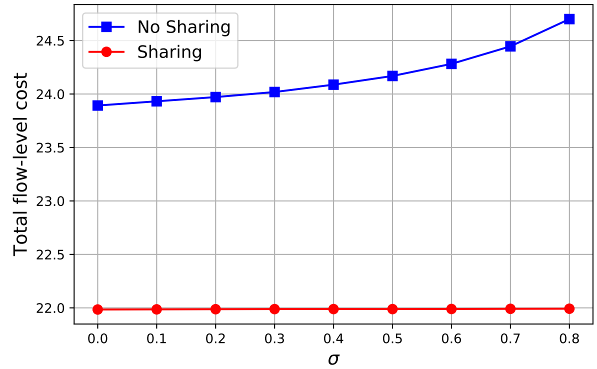


Fig. 7: Total flow-level cost by varying σ.

sharing becomes the vital issue in highly-utilized scenarios.

D. Admission Control

In the previous results, we consider the feasible traffic demands for the **NBPS** problem, thus we can obtain disagreement points and fair sharing solutions of the **BPFS** problem. For a more general analysis, we extensively investigate very high traffic demand scenarios. In this case, operators cannot individually maintain the connectivity for all users due to insufficient backup power capacities of BSs, hence some user flows cannot associate with any BSs in the considered region. Therefore, we introduce the blocking probability of traffic loads and extend the feasible set in Definition 2 [14].

Definition 2 (Feasibility): The set $\mathcal{F}_i'$ of feasible BS loads (or utilization) of the operator i, i.e, $\rho_i' = \{\rho_{i0}, \rho_i\}$ is defined as follows

$$\begin{aligned}
\mathcal{F}_i' = \Big\{\rho_i' \,|\rho_{ij} &= \int_{\mathcal{L}} \beta_{ij}(x) p_{ij}(x)dx, \quad \forall j \in \mathcal{H}_i \\
\rho_{i0} &= \int_{\mathcal{L}} \gamma_i(x) p_{i0}(x)dx, \\
0 &\le \rho_{ij} \le 1-\epsilon, \qquad\qquad \forall j \in \mathcal{H}_i \\
&\sum_{j\in\mathcal{H}_i} p_{ij}(x) = 1 - p_{i0}(x), \quad \forall x \in \mathcal{L} \\
0 &\le p_{i0}(x) \le 1-\sigma_i, \qquad \forall x \in \mathcal{L} \\
0 &\le p_{ij}(x) \le 1, \quad \forall j \in \mathcal{H}_i, \forall x \in \mathcal{L}\Big\},
\end{aligned}$$

where ϵ is an arbitrarily small positive constant. In Definition 2, $p_{i0}(x)$ denotes for the probability such that the traffic at location x cannot associate with any BSs which also known as blocking probability. This probability is limited by a threshold value (i.e., $1-\sigma_i$) and determine the QoS of operators.

When the blocking probability is greater than zero, operator i receives an additional traffic blocking cost due to the user dissatisfaction. Accordingly, the flow-level cost function of operator i (3) becomes

$$\zeta(\rho_i') = \kappa\rho_{i0} + \phi(\rho_i) = \kappa\rho_{i0} + \sum_{j\in\mathcal{H}_i} \frac{1}{1-\rho_{ij}}, \qquad (20)$$

where κ is the unit price of the blocked traffic loads. The similar sharing analysis can be readily applied for the new cost function $\zeta(\rho_i')$ and feasible set $\mathcal{F}_i'$.

As in Figure 7, the higher σ values induce the lower blocking probability can receive, thus the higher cost of operators using no sharing strategy. Moreover, sharing strategy provides lower total costs compared to no sharing due to the sufficient sharing backup power. Using the sharing strategy, the total cost does not seem to change when we vary the values of σ because the blocking probabilities are almost zeros.

IV. Conclusion

In this paper, we investigate an under-explored problem of backup power sharing for co-location BSs to improve the network performance and service availability during power outages. The fairness of sharing backup power supply among the operators at multi-operator sites is tackled by using Nash Bargaining solution, which can help to mitigate the flow-level cost and reduce power usage using a proposed decentralized algorithm. Simulation results show that the backup power fair sharing guarantees better delay reduction than that of no sharing approach. In addition to the delay reduction, the cooperative fair backup power sharing also decreases the operator's BS power consumption in both scenarios. In the future work, we advocate dealing with multiple time slot model using Model Predictive Control for a long period of time analysis.

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References

[1] K. Sudeep, "Blackouts in the power system," *International Journal of Electrical and Electronics Research*, vol. 3, no. 4, pp. 1–7, 2015.

[2] "Blackout Tracker: United States Annual Report 2015," Eaton, Tech. Rep., 2015. [Online]. Available: http://www.sustainablepowersystems.com/wp-content/uploads/2016/03/US_BlackoutTracker_2015_Final.pdf

[3] "Order on reconsideration, FCC 07-177," Federal Communications Commission, Tech. Rep., 2007. [Online]. Available: http://njslom.org/FCC-07-177A1.pdf
[4] "Mobile Base Stations Report 2016 Ed 3," Statplan Energy Ltd, Tech. Rep., 2016. [Online]. Available: https://www.reportlinker.com/p04230308/Mobile-Base-Stations-Report-Ed-3.html
[5] "Mobile Infrastructure Sharing," GSMA, Tech. Rep., 2012. [Online]. Available: https://www.gsma.com/publicpolicy/wp-content/uploads/2012/09/Mobile-Infrastructure-sharing.pdf
[6] "Passive infrastructure sharing," vol. Allen and Overy, Tech. Rep., 2012.
[7] "Infrastructure sharing during emergencies," CSRIC IV, Federal Communications Commission, Tech. Rep., 2014. [Online]. Available: https://transition.fcc.gov/pshs/advisory/csric4/CSRIC_IV_WG-9_Report_061814.pdf
[8] M. J. Osborne and A. Rubinstein, *Bargaining and markets*. Academic press, 1990.
[9] M. J. Farooq, H. Ghazzai, A. Kadri, H. ElSawy, and M.-S. Alouini, "A Hybrid Energy Sharing Framework for Green Cellular Networks," *IEEE Transactions on Communications*, vol. 65, no. 2, pp. 918–934, feb 2017.
[10] J. Xu and R. Zhang, "Cooperative Energy Trading in CoMP Systems Powered by Smart Grids," *IEEE Transactions on Vehicular Technology*, vol. 65, no. 4, pp. 2142–2153, apr 2016.
[11] M. Oikonomakou, A. Antonopoulos, L. Alonso, and C. Verikoukis, "Fairness in multi-operator energy sharing," in *2017 IEEE International Conference on Communications (ICC)*. IEEE, may 2017, pp. 1–6.
[12] J. Xu, L. Duan, and R. Zhang, "Energy Group Buying With Loading Sharing for Green Cellular Networks," *IEEE Journal on Selected Areas in Communications*, vol. 34, no. 4, pp. 786–799, apr 2016.
[13] A. Bousia, E. Kartsakli, A. Antonopoulos, L. Alonso, and C. Verikoukis, "Game-Theoretic Infrastructure Sharing in Multioperator Cellular Networks," *IEEE Transactions on Vehicular Technology*, vol. 65, no. 5, pp. 3326–3341, may 2016.
[14] H. Kim, G. de Veciana, X. Yang, and M. Venkatachalam, "Distributed alpha-Optimal User Association and Cell Load Balancing in Wireless Networks," *IEEE/ACM Transactions on Networking*, vol. 20, no. 1, pp. 177–190, feb 2012.
[15] K. Son, H. Kim, Y. Yi, and B. Krishnamachari, "Base Station Operation and User Association Mechanisms for Energy-Delay Tradeoffs in Green Cellular Networks," *IEEE Journal on Selected Areas in Communications*, vol. 29, no. 8, pp. 1525–1536, sep 2011.
[16] G. L. T. Evolution, "Physical layer aspects of utra high speed downlink packet access," *Technical Report TR25.814, 2006*, 2006.
[17] J. F. Nash Jr, "The bargaining problem," *Econometrica: Journal of the Econometric Society*, pp. 155–162, 1950.
[18] W. Deng, M.-J. Lai, Z. Peng, and W. Yin, "Parallel Multi-Block ADMM with o(1/k) Convergence," *Journal of Scientific Computing*, vol. 71, no. 2, pp. 712–736, may 2017.
[19] S. Boyd, L. Xiao, A. Mutapcic, and J. Mattingley, "Notes on decomposition methods," *Notes for EE364B, Stanford University*, pp. 1–36, 2007.
[20] S. Boyd, N. Parikh, E. Chu, B. Peleato, and J. Eckstein, "Distributed Optimization and Statistical Learning via the Alternating Direction Method of Multipliers," *Foundations and Trends® in Machine Learning*, vol. 3, no. 1, pp. 1–122, 2010.
[21] T. Lin, S. Ma, and S. Zhang, "On the Global Linear Convergence of the ADMM with MultiBlock Variables," *SIAM Journal on Optimization*, vol. 25, no. 3, pp. 1478–1497, jan 2015.
[22] S. Boyd and L. Vandenberghe, *Convex optimization*. Cambridge university press, 2004.
[23] R. Srinivasan, J. Zhuang, L. Jalloul, R. Novak, and J. Park, "IEEE 802.16m Evaluation Methodology Document (EMD) IEEE 802.16m-08/004r5," IEEE 802.16 Broadband Wireless Access Working Group Std., Tech. Rep., 2009. [Online]. Available: http://ieee802.org/16
[24] A. Wächter and L. T. Biegler, "On the implementation of an interior-point filter line-search algorithm for large-scale nonlinear programming," *Mathematical Programming*, vol. 106, no. 1, pp. 25–57, mar 2006.