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Network utility maximisation framework with multiclass traffic

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Abstract: The concave utilities in the basic network utility maximisation (NUM) problem are only suitable for elastic flows. In networks with both elastic and inelastic traffic, the utilities of inelastic traffic are usually modelled by the sigmoidal functions which are non-concave functions. Hence, the basic NUM problem becomes a non-convex optimisation problem. To address the non-convex NUM, the authors approximate the problem which is equivalent to the original one to a strictly convex problem. The approximation problem is solved efficiently via its dual by the gradient algorithm. After a series of approximations, the sequence of solutions to the approximation problems converges to a local optimal solution satisfying the Karush-Kuhn-Tucker conditions of the original problem. The proposed algorithm converges with any value of link capacity. The authors also extend their work to jointly allocate the rate and the power in a multihop wireless network with elastic and inelastic traffic. Their framework can be used for any log-concave utilities.

1 Introduction

The inelastic traffic has grown tremendously beside the traditional elastic traffic in the communication networks nowadays. Traffic management in such heterogeneous networks play a key role in guarantee the quality of service of the inelastic traffic [1–11]. To allocate the resources to multiclass traffic, many works in the literature assume the constant bit rate (CBR), for example, [2, 5] or put constraints on the delay for inelastic traffic, for example, [3, 4]. Most of these works base on the assumption that the inelastic traffic is always admissible by the network, otherwise, an admission control mechanism must be proposed additionally [2–5]. One of the main research on resource allocation for multiclass traffic is to extend the well-known network utility maximisation (NUM) model for elastic traffic, [12, 13], to support multiclass traffic. Since the utilities associated with the elastic flows are modelled by strictly concave functions, the traditional NUM model with elastic traffic is a convex optimisation problem. Therefore the dual-based algorithm which solves the NUM via its dual converges to a global optimal solution [13]. However, the utility functions modelling the inelastic flows are no longer concave. Sigmoidal functions are usually used instead [1, 7–11]. As a result, the NUM becomes a non-convex optimisation problem. Sigmoidal is a function that is convex and has low value at the lower region. It is concave and has high value at the higher region. (The step function can be considered as a sigmoidal function with infinity slope at the inflection point.) Hence, the admission control scheme is naturally integrated if the non-convex NUM problem is solved. However, it is difficult to solve the non-convex NUM distributively. According to our knowledge, there is no current work in the literature solving the non-convex NUM distributively. The standard dual-based algorithm applying to the non-convex NUM does not converge anymore because of the non-zero duality gap. The primal value generated by the dual algorithm can be infeasible [7, 8].

There are several works addressing the non-convex NUM, such as [7–10]. Works in [7, 8] utilise the standard dual-based algorithm. The algorithm therein does not always converge as mentioned above. Lee *et al.* [7] propose a heuristic mechanism, called ‘self-regulating’. It is actually an admission control scheme to avoid link congestion caused by the non-concave utilities. Hande *et al.* [8] find the conditions of link capacity for which the standard dual-based algorithm converges to a global optimum. It turns out that the link capacity must be greater than a certain value in order to achieve the globally optimal convergence. In case of scarce resources, a ‘capacity provisioning’ is needed to guarantee the convergence of the standard algorithm. The work in [9] applies the technique in [8] to address the non-convex NUM framework of a random access wireless LAN with elastic and inelastic traffic.

With a different approach, Fazel and Chiang [10] apply the sum-of-squares relaxation to the non-convex NUM problem and use the semidefinite programming to solve it. The result approaches the global optimal solution when the order of polynomials in the relaxation is increased. However, the order of the relaxation is very high when the sigmoidal function is steeper at the inflection point.

On the other hand, Wang *et al.* [14] indirectly deal with the non-convex NUM by replacing the utility $U(x)$ with the function $\int(1/U(x))\mathrm{d}x$. This new function is always strictly concave as $U(x)$ is monotonically increasing. Then they derive an utility-proportional fair algorithm. The solution is always a lower bound of the globally optimal solution. More recently, based on dual-based decomposition technique, Abbas *et al.* [11] has proposed an algorithm converging to a suboptimal solution of the non-convex NUM. The dual problem is solved approximately by stochastic surrogate optimisation to ensure the zero duality gap.

We apply the successive convex approximation method to address the non-convex NUM in this paper. The proposed algorithm converges to a local optimal solution satisfying the Karush–Kuhn–Tucker (KKT) conditions of the problem for any value of the link capacity. An equivalent problem of the original one is successively approximated to a convex problem which is efficiently solved by the standard dual-based decomposition approach. After solving a series of approximation problems, the algorithm converges to a KKT solution. At the stationary point, the approximation becomes exact. The successive convex approximation method is first introduced in [15]. It is usually used with geometric programming in power control problems to approximate the capacity constraints to a convex form (see [16–19]). Chiang *et al.* [16] has a concise overview about this method. On the other hand, the dual-based decomposition approach has shown its efficiency in building a distributed algorithm. The optimisation problem is usually decomposed into several subproblems which can be solved distributively [13, 16, 20–22].

Different from the previous works which allocate the resources for multiclass traffic based on NUM model, this paper proposes a unified framework which can be applied to many cross-layer NUM models to support multiclass traffic. (See [21, Section 3] and the references therein for some typical cross-layer optimisation models.) The followings are some examples to which our framework can be applied:

1. Joint rate and power control for the multiclass traffic in multihop wireless networks. In the wireless environment, the link capacity is not fixed. It is regulated by the transmission power and the interference of other transmitting sources. The joint NUM is a non-convex optimisation problem even with the elastic flows because of the non-convex form of the capacity constraints, [16–18, 23]. With both elastic and inelastic traffic, the joint NUM problem is non-convex in both the objective and the constraints. The successive approximation technique is used in this case to approximate both the objective and the constraints. This extension will be presented in Section 4.
2. Joint rate control, routing and scheduling using the node centric formulation for the multiclass traffic. The well-known node-centric formulation of a multihop wireless network is mainly for elastic flows with concave utility. By decomposing the convex optimisation problem, the scheduling, routing and end-to-end flow rate control are implemented [24–26]. We can directly extend this node-centric model to support the multiclass traffic using the proposed framework in this paper.
3. Joint allocation of flow rate and persistent probability of the multiclass users in the random access networks. The corresponding work for elastic traffic is introduced in [20]. However, to extend to multiclass traffic, we cannot directly apply the proposed framework in this paper because of the logarithmic transformation of the variables to separate the constraints. The general requirements for the utility functions are also different. The interested readers can refer to [27] for more details.

The remaining of the paper is organised as follows: Section 2 describes the network model. Section 3 presents the approximation problem and the successive approximation algorithm. Section 4 extends the framework to jointly rate and power control for the wireless networks with both elastic and inelastic traffic. Finally, the numerical results and conclusions are given in Section 5 and Section 6, respectively.

2 Network model

We consider a network that includes a set of links $\mathcal{L}$. The network is shared by a set of sources $\mathcal{N}$. We denote N and L as the cardinalities of $\mathcal{N}$ and $\mathcal{L}$, respectively. Let x_s be the rate of the flow from source s and $\boldsymbol{x} \triangleq [x_1, \ldots, x_N]$ be the rate vector of all sources. (We use italic characters to denote variables and bold characters to denote vectors in this paper.) Assume that each flow rate x_s is bounded by the constants $x_s^{\min}$ and $x_s^{\max}$. Each source s is associated with a monotonically increasing utility function $U_s(x_s)$. Let $L(s)$ be the set of links which the flow from source s uses. The value $\sum_{s:l\in L(s)} x_s$ is the total traffic of the flows in the network that use link l. This amount cannot exceed the capacity of the link, c_l. Our goal is to find the rate allocation that maximises the aggregate utility in the network. Then the basic NUM is stated as follows [12, 13]:

$$\mathbb{P}1\colon \mathrm{Max.} \sum_{s\in\mathcal{N}} U_s(x_s)$$
$$\text{s.t.} \sum_{s:l\in L(s)} x_s \le c_l,\ \forall l \in \mathcal{L},$$
$$\boldsymbol{x}^{\min} \le \boldsymbol{x} \le \boldsymbol{x}^{\max}$$

where

$$\boldsymbol{x}^{\min} = \left[x_1^{\min}, \ldots, x_N^{\min}\right] \quad \text{and} \quad \boldsymbol{x}^{\max} = \left[x_1^{\max}, \ldots, x_N^{\max}\right]$$

In the case in which U_s is strictly concave for all $s \in \mathcal{N}$, the problem $\mathbb{P}1$ is the basic NUM model described in [12, 13]. This paper deals with the multiclass traffic, so the utilities are both concave and non-concave functions (see Fig. 1). At first, we consider two groups of utilities: the concave utilities for elastic flows

$$U(x) = \begin{cases} \log(x+1), & \text{if } \alpha = 1, \\ \dfrac{(x+1)^{(1-\alpha)} - 1}{1-\alpha}, & \text{if } \alpha \in (0, 1) \cup (1, \infty) \end{cases} \tag{1}$$

and the sigmoidal utilities for inelastic flows

$$U(x) = \frac{c}{1 + \mathrm{e}^{-a(x-b)}}, \quad \forall a, b, c > 0 \tag{2}$$

The sigmoidal function has the inflection point b. It is convex if $x < b$ and is concave if $x > b$. The slope at the inflection point increases as a increases. In Fig. 1, the sigmoidal with $a = 3$ is steeper at the inflection point than the curve with $a = 1$. The steeper the slope, the more stringent the realtime

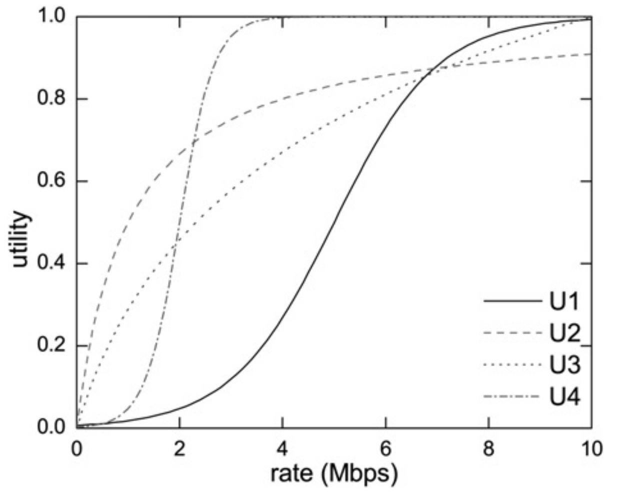

Fig. 1 *Utility functions: $U_1(x) = (1/(1 + e^{-(x-5)}))$, $U_2(x) = x/(x+1)$ ($\alpha = 2$), $U_3(x) = \log(x+1)/\log(11)$ ($\alpha = 1$), and $U_4(x) = 1/(1 + e^{-3(x-4)})$*

application's demand. On the other hand, the utility function of the elastic user in the network with multiclass traffic usually has the form as (1) in the literature. It is the α-fair utility but shifted by 1 on the x-axis. Otherwise, if we use the canonical form of α-fair utility for the elastic traffic, its utility is always negative as $\alpha > 1$, then the inelastic user always takes the advantage over the elastic user in the network.

Problem $\mathbb{P}1$ is a non-convex optimisation problem because of the sigmoidal utilities. Therefore it cannot be solved by the canonical method as in [12, 13]. In the following section, we apply a novel approach to derive a KKT solution to $\mathbb{P}1$.

3 Successive approximation method

3.1 Convex approximate problem

We replace $\mathbb{P}1$ with the equivalent problem as follows

$$
\begin{aligned}
&\mathbb{P}2\text{: Max.} \log\left(\sum_{s\in\mathcal{N}} U_s(x_s)\right)\\
&\text{s.t.} \sum_{s:l\in L(s)} x_s \le c_l, \quad \forall l \in \mathcal{L}\\
&\boldsymbol{x}^{\min} \le \boldsymbol{x} \le \boldsymbol{x}^{\max}
\end{aligned}
$$

Problem $\mathbb{P}2$ is still a non-convex optimisation problem because of the non-concave objective. For the use of the successive approximations method which requires a convex objective in a minimisation problem (or a concave objective in the maximisation problem, equivalently) [15], we transform $\mathbb{P}2$ into an epigraph-form problem [28, p.4.2.4],

$$
\begin{aligned}
&\mathbb{P}3\text{: Max.} y\\
&\text{s.t.}\; y \le \log\left(\sum_{s\in\mathcal{N}} U_s(x_s)\right)\\
&\sum_{s:l\in L(s)} x_s \le c_l, \quad \forall l \in \mathcal{L}\\
&\boldsymbol{x}^{\min} \le \boldsymbol{x} \le \boldsymbol{x}^{\max}
\end{aligned}
$$

Lemma 1: Let μ be the Lagrange multiplier associated with the first constraint of $\mathbb{P}2$ and ν and μ be the Lagrange multipliers associated with the respective first and second constraints of $\mathbb{P}3$. Suppose that $U_s(x_s)$ is increasing and positive for all $s \in \mathcal{N}$, we have the following results

1. If $(\boldsymbol{x}^*, \mu^*)$ is a KKT point of $\mathbb{P}2$, then $(\boldsymbol{x}^*, (\sum_{i\in\mathcal{N}} U_i(x_i^*))\boldsymbol{\mu}^*)$ is also a KKT point of $\mathbb{P}1$.
2. If $(\boldsymbol{x}^*, \boldsymbol{y}^*, \nu^*, \mu^*)$ is a KKT point of $\mathbb{P}3$, then $(\boldsymbol{x}^*, \mu^*)$ is also a KKT point of $\mathbb{P}2$.

Proof: It is quite straightforward to verify (1) and (2) by writing down the KKT conditions of $\mathbb{P}1$, $\mathbb{P}2$ and $\mathbb{P}3$ and comparing them pair-by-pair. □

Problem $\mathbb{P}3$ is a non-convex optimisation problem because of the non-convex constraint $y \le \log\left(\sum_{s\in\mathcal{N}} U_s(x_s)\right)$. Next, we derive an inequality to approximate the non-convex constraint to a convex one.

Lemma 2: For any vector $\theta = [\theta_1, \theta_2, \ldots, \theta_N]^{\mathrm{T}} > \mathbf{0}$ and $\mathbf{1}^{\mathrm{T}}\theta = 1$,

$$
\log\left(\sum_{s\in\mathcal{N}} U_s(x_s)\right) \ge \sum_{s\in\mathcal{N}} \theta_s \log\left(\frac{U_s(x_s)}{\theta_s}\right) \tag{3}
$$

Proof: We have the arithmetic-geometric mean inequality: $\sum_{s\in\mathcal{N}} \theta_s u_s \ge \prod_{s\in\mathcal{N}} (u_s)^{\theta_s}$ for all $\boldsymbol{u} \ge \mathbf{0}$, $\boldsymbol{\theta} = [\theta_1, \theta_2, \ldots, \theta_N]^{\mathrm{T}} > \mathbf{0}$, and $\mathbf{1}^{\mathrm{T}}\boldsymbol{\theta} = 1$. Replacing u_s with $U_s(x_s)/\theta_s$ yields

$$
\sum_{s\in\mathcal{N}} U_s(x_s) \ge \prod_{s\in\mathcal{N}} \left(\frac{U_s(x_s)}{\theta_s}\right)^{\theta_s}
$$

Inequality (3) is obtained by taking the logarithm of both sides of above inequality. The equality holds if and only if

$$
\theta_s = \frac{U_s(x_s)}{\sum_{k\in\mathcal{N}} U_k(x_k)}, \quad s = 1, .., N \tag{4}
$$

□

From Lemma 2, we consider the approximation problem

$$
\begin{aligned}
&\mathbb{P}4^{\tau}\text{: Max.} y\\
&\text{s.t.}\quad y \le \sum_{s\in\mathcal{N}} \theta_s \log\left(\frac{U_s(x_s)}{\theta_s}\right)\\
&\sum_{s:l\in L(s)} x_s \le c_l, \quad \forall l \in \mathcal{L}\\
&\boldsymbol{x}^{\min} \le \boldsymbol{x} \le \boldsymbol{x}^{\max}
\end{aligned}
$$

As we has mentioned earlier, the successive approximation algorithm solves a series of approximation problems. Each approximation problem is identified by a value θ. The superscript $^{\tau}$ in $\mathbb{P}4^{\tau}$ means the τth approximation problem. The sequence of solutions to the approximation problems converges. At the stationary point, the approximation becomes exact, i.e., the equality (3) always holds. We transform $\mathbb{P}4^{\tau}$ back to the canonical form, the following

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problem is obtained

$$\mathbb{P}5^{\tau}: \quad \text{Max.} \sum_{s \in \mathcal{N}} \tilde{U}_s(x_s; \theta_s)$$
$$\text{s.t.} \sum_{s:l \in L(s)} x_s \le c_l, \quad \forall l \in \mathcal{L}$$
$$\boldsymbol{x}^{\min} \le \boldsymbol{x} \le \boldsymbol{x}^{\max}$$

where

$$\tilde{U}_s(x_s; \theta_s) \triangleq \theta_s \log\left(\frac{U_s(x_s)}{\theta_s}\right)$$

Lemma 3: The functions $\tilde{U}_s(x_s; \theta_s), \forall s \in \mathcal{N}$ are strictly concave with both elastic and inelastic utilities given by (1) and (2), respectively.

Proof: We can easily verify this fact by checking whether their second derivatives are negative. □

From Lemma 3, $\mathbb{P}5^{\tau}$ becomes a basic NUM with a strictly concave objective. Therefore it can be solved efficiently using the dual decomposition approach.

3.2 Solution to the approximation problem

We solve for the solution to $\mathbb{P}5^{\tau}$ via its dual. Since $\mathbb{P}5^{\tau}$ is a strictly convex problem, the strong duality holds and the dual optimal solution also coordinates with the primal solution. The dual function of $\mathbb{P}5^{\tau}$ is given by

$$\begin{aligned} D_1(\lambda) &= \max_{\boldsymbol{x}}\left(\sum_{s \in \mathcal{N}} \tilde{U}_s(x_s; \theta_s) - \sum_{l \in \mathcal{L}} \lambda_l\left(\sum_{s \in \mathcal{N}} x_s - c_l\right)\right) \\ &= \max_{\boldsymbol{x}}\left(\sum_{s \in \mathcal{N}} \tilde{U}_s(x_s; \theta_s) - \left(\sum_{l \in L(s)} \lambda_l\right)x_s\right) + \sum_{l \in \mathcal{L}} \lambda_l c_l \\ &= \max_{\boldsymbol{x}} L_x(\boldsymbol{x}, \boldsymbol{\lambda}) + \sum_{l \in \mathcal{L}} \lambda_l c_l \end{aligned} \tag{5}$$

where $L_x(\boldsymbol{x}, \boldsymbol{\lambda}) \triangleq \sum_{s \in \mathcal{N}} \tilde{U}_s(x_s; \theta_s) - \left(\sum_{l \in L(s)} \lambda_l\right)x_s$.

The dual problem is

$$\min_{\boldsymbol{\lambda} \ge \boldsymbol{0}} D_1(\boldsymbol{\lambda}) \tag{6}$$

We apply the gradient projection algorithm to solve the dual problem (6). $\sum_{s:l \in L(s)} x_s(t) - c_l$ is a gradient of $D_1(\lambda)$. Hence, the gradient update is given by

$$\lambda_l^{(\tau)}(t+1) = \left[\lambda_l^{(\tau)}(t) + \kappa\left(\sum_{s:l \in L(s)} x_s^{(\tau)}(t) - c_l\right)\right]^+, \quad \forall l \in \mathcal{L} \tag{7}$$

where κ is a sufficiently small step-size for the convergence of the algorithm and $\boldsymbol{x}^{(\tau)}(t)$ is the solution to the subproblem $\max_{\boldsymbol{x}} L_x(\boldsymbol{x}, \lambda)$ given by (5) at time instant t. $L_x(\boldsymbol{x}, \lambda)$ is a concave function in terms of $\boldsymbol{x}$. Hence, the optimal point also satisfies the KKT conditions of subproblem (5). Let $q_s^{(\tau)}(t) \triangleq \sum_{l \in L(s)} \lambda_l^{(\tau)}(t)$. Solving the first derivative condition $\partial L_x(\boldsymbol{x}, \lambda)/\partial x_s = 0$ yields

$$x_s^{(\tau)}(t) = \left[\tilde{U}_s'^{-1}\left(q_s^{(\tau)}(t); \theta_s^{(\tau)}\right)\right]_{x_s^{\min}}^{x_s^{\max}}, \quad \forall s \in \mathcal{N} \tag{8}$$

where $[a]_c^b = \max(c, \min(a, b))$. Here, we use superscript $^{(\tau)}$ in notations $\lambda^{(\tau)}$, $\boldsymbol{x}^{(\tau)}$ to imply that they are the values in solving τth approximation problem. Table 1 shows the rate updates corresponding to some common utility functions.

Table 1 Rate update functions

$U(x)$	$x(t+1)$	Notes
$1/(1+e^{-a(x-b)})$	$b-(1/a)\log(q/\theta a - q)$	sigmoidal
x	θ/q	linear
$\log(x+1)$	$(\theta/q/W(\theta/q)) - 1$	logarithm ($\alpha = 1$). $W(.)$ is the Lambert W-function, the inverse function of $f(W) = We^W$
$x/x+1$	$\sqrt{\frac{1}{4}+\frac{\theta}{q}}-\frac{1}{2}$	concave ($\alpha = 2$)

3.3 Successive approximation algorithm for the non-convex NUM

Based on above analysis, the successive approximation algorithm to control the flow rate of multiclass traffic is as shown in Algorithm 1 (see Fig. 2).

We have the following theorem.

Theorem 1: Algorithm 1 (see Fig. 2) converges to the stationary point satisfying the KKT conditions of $\mathbb{P}1$.

Proof: Let $f_u(\boldsymbol{x}) \triangleq \dfrac{y}{\log\left(\sum_{s \in \mathcal{N}} U_s(x_s)\right)}$ and $\tilde{f}_u(\boldsymbol{x}) \triangleq \dfrac{y}{\sum_{s \in \mathcal{N}} \tilde{U}_s(x_s; \theta_s^{(\tau)})}$. According to [15, 16], we need to prove the following three conditions for the convergence to the KKT point of the algorithm:

1. $f_u(\boldsymbol{x}) \le \tilde{f}_u(\boldsymbol{x})$,
2. $f_u(\boldsymbol{x}^{(\tau)*}) = \tilde{f}_u(\boldsymbol{x}^{(\tau)*})$ and
3. $\nabla f_u(\boldsymbol{x})|_{\boldsymbol{x}=\boldsymbol{x}^{(\tau)*}} = \nabla \tilde{f}_u(\boldsymbol{x})|_{\boldsymbol{x}=\boldsymbol{x}^{(\tau)*}}$

where $\boldsymbol{x}^{(\tau)*}$ is the optimal solution of the τth iteration.

Conditions (1) and (2) are clearly satisfied with Lemma 3. Condition (3) is verified by taking the derivative and applying (4). Therefore Algorithm 1 (see Fig. 2) converges to a local optimal solution which satisfies the KKT conditions of $\mathbb{P}3$. Suppose $\boldsymbol{x}^*$ is the stationary point of Algorithm 1 (see Fig. 2). It is also a KKT solution of $\mathbb{P}2$ as well as of $\mathbb{P}1$ according to Lemma 1. The theorem is thereby proved. □

We have two notifications in practical implementation of Algorithm 1 (see Fig. 2) as follows. First, as we have seen, Algorithm 1 (see Fig. 2) has two levels of convergence: the outer iterations update θ (step 5) and the inner iterations update the source rates and link prices (step 6). The initial value of a new outer iteration is the stationary value of the previous iteration. To update θ in step 5, each user needs the information of the total utility of all the users. Therefore

Algorithm 1

1: Initialise from a feasible point;
2: $\tau := 0$;
3: **loop**
4: $\tau := \tau + 1$;
5: Update $\boldsymbol{\theta}^{(\tau)}$ according to (4) using the stationary values of the previous iteration;
6: Repeatedly update the rate and the multipliers according to (8) and (7), respectively until convergence;
7: **end loop**

Fig. 2 *Successive approximation algorithm for the non-convex NUM*

Algorithm 2

1: Initialise from a feasible point;
2: $\tau := 0$; iterations := 0;
3: **loop**
4: $\tau := \tau + 1$;
5: Update $\boldsymbol{\theta}^{(\tau)}$ and $\boldsymbol{\eta}^{(\tau)}$ according to (4) and (15) using the stationary values of the previous iteration;
6: Repeatedly update the rate, power, and multipliers according to (8), (21), and (19), respectively until convergence.
7: **end loop**

Fig. 3 *Successive approximation algorithm for the joint NUM*

each user broadcasts its utility value after each outer iteration to all the other users in the network. Second, the inner iterations must converge before updating θ theoretically. However, in a large network, it is difficult to know the stationary of all the sources. In order for Algorithm 1 (see Fig. 2) to be more practical, we fix the number of inner iterations in each outer iteration to a value. This value is large enough to solve for the solution of $\mathbb{P}5^{\tau}$. We cannot guarantee the convergence of the heuristic algorithm theoretically. Nevertheless, from many experiments, the heuristic implementation also converges and leads to similar results as Algorithm 1 (see Fig. 2) does.

3.4 Log-concave utilities

Finally, we find the conditions of the utility functions for which the above analysis can still be applied. First of all, $U(x_s)$ must be a positive, continuously differentiable, and increasing function for all $s \in \mathcal{N}$. The important condition is that the function $\theta \log(U(x_s)/\theta)$ must be concave. In the other words, $U(x_s)$ is a log-concave function, or

$$UU'' \le U'^2 \tag{9}$$

For example, the following utilities are log-concave:

- concave functions satisfying conditions (1) and (2);
- polynomials having all real roots and satisfying conditions (1) and (2) such as the convex functions x, x^2, $x(x+1)$, …;
- sigmoidal-like functions $(x^a)/(k + x^a)$, $\forall a > 1, \quad k > 0$.

4 Extension: jointly rate and power control in multihop wireless networks

In this section, we apply the framework in Sections 2 and 3 to jointly control the rate and power in a wireless multihop network with multilass traffic. Let us consider a multihop wireless network which includes a set of nodes forming a logical topology. Some nodes are the sources of the flows, and some nodes act as the 'relay' nodes. We also denote $\mathcal{N}$ as the set of source nodes and $\mathcal{L}$ as the set of (logical) links in the network.

Without perfect orthogonal channels, the receiving nodes are interfered by the transmission powers of all the transmitting nodes in the network. The link capacity is no longer constant. It depends on the transmission power vector $\boldsymbol{P} = [P_1, P_2, ..., P_L]$ and the channel condition.

$$c_l(\boldsymbol{P}) = W \log\big(1 + K\mathrm{SIR}_l\big), \quad \forall l \in \mathcal{L} \tag{10}$$

Here, W is the symbol bandwidth and K is a constant depending on the modulation and bit-error rate (BER). The signal-to-interference ratio of link l is calculated by $\mathrm{SIR}_l(\boldsymbol{P}) = P_l G_{ll} / \big(\sum_{k \ne l} P_k G_{lk} + n_l\big)$, where G_{lk} is the path loss from the transmitter of link k to the receiver of link l. We assume the transmitting power vector are bounded, that is, $\boldsymbol{P}^{\min} \le \boldsymbol{P} \le \boldsymbol{P}^{\max}$. The NUM problem for joint rate and power control is given by the authors in [17, 18, 23]

$$\mathbb{P}6\text{: Max.} \sum_{s \in \mathcal{N}} U_s(x_s)$$
$$\text{s.t.} \sum_{s: l \in L(s)} x_s \le c_l(\boldsymbol{P}), \quad \forall l \in \mathcal{L}$$
$$\boldsymbol{P}^{\min} \le \boldsymbol{P} \le \boldsymbol{P}^{\max}$$

Even with the concave objective, the joint NUM is a non-convex problem because of the non-convexity of the link capacity constraints. Previous studies [17, 18, 23] address problem $\mathbb{P}6$ with elastic utilities by deriving it to a convex one. Using high-SIR assumption, Chiang [23] transforms the NUM problem into a convex form and solves it using the dual decomposition method. The papers in [17, 18] apply the successive approximation method to approximate the capacity constraints to a convex form without using high-SIR assumption. With both elastic and inelastic traffic, the joint NUM problem is non-convex in both the objective and the constraints. In this section, we combine the technique in [18] with our approach to derive an algorithm to jointly allocate the rate and the power for multiclass traffic.

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4.1 Convex approximation problem

We replace $\mathbb{P}6$ with the following equivalent problem

$$\begin{aligned}&\mathbb{P}7\text{: Max.}\log\left(\sum_{s\in\mathcal{N}}U_s(x_s)\right)\\&\text{s.t.}\sum_{s:l\in L(s)}x_s\le c_l(\boldsymbol{P}),\\&\boldsymbol{P}^{\min}\le\boldsymbol{P}\le\boldsymbol{P}^{\max}\end{aligned}$$

Problem $\mathbb{P}7$ is also transformed into an epigraph form in order to move the non-concave objective to the constraints

$$\begin{aligned}&\mathbb{P}8\text{: Max.}y\\&\text{s.t. }y\le\log\left(\sum_{s\in\mathcal{N}}U_s(x_s)\right)\\&\sum_{s:l\in L(s)}x_s\le c_l(\boldsymbol{P}),\quad\forall l\in\mathcal{L}\\&\boldsymbol{P}^{\min}\le\boldsymbol{P}\le\boldsymbol{P}^{\max}\end{aligned}$$

We now derive an inequality to approximate the non-convex constraints of $\mathbb{P}8$. To clearly represent the capacity formula, we assume that $W = 1$ and that K is included in the channel gain G_{ll}. The capacity formula (10) is rewritten as follows

$$\begin{aligned}c_l(\boldsymbol{P})&=\log(1+\mathrm{SIR}_l)\\&=\log\left(\sum_{k\in\mathcal{L}}G_{lk}P_k+n_l\right)-\log\left(\sum_{k\neq l}G_{lk}P_k+n_l\right),\\&\forall l\in\mathcal{L}\end{aligned}\tag{11}$$

Lemma 4: For all vectors $\boldsymbol{\eta}^l=[\eta_1^l,..,\eta_{L+1}^l]$, $\forall l = 1, \ldots, L$, such that $\boldsymbol{\eta}^l > \mathbf{0}$ and $\mathbf{1}^T\boldsymbol{\eta}^l = 1$. Let us define

$$\hat{c}_l(\boldsymbol{P};\boldsymbol{\eta}^l)\triangleq\sum_{k\in\mathcal{L}}\eta_k^l\log\left(\frac{G_{lk}P_k}{\eta_k^l}\right)+\eta_{L+1}^l\log\left(\frac{n_l}{\eta_{L+1}^l}\right)-\log\left(\sum_{k\neq l}G_{lk}P_k+n_l\right)\tag{12}$$

We have the following inequality

$$c_l(\boldsymbol{P})\ge\hat{c}_l(\boldsymbol{P};\boldsymbol{\eta}^l),\quad l=1,\ldots,L+1\tag{13}$$

Proof: Similarly to the proof of Lemma 2, from the arithmetic-geometric mean inequality $\sum_{k=1}^{L+1}\eta_k v_k\ge\prod_{k=1}^{L+1}(v_k)^{\eta_k}$, $\forall\boldsymbol{v}=[v_1,v_2,\ldots,v_{L+1}]^{\mathrm{T}}\ge\mathbf{0}$ and $1^{\mathrm{T}}v^l=1$, we replace v_k by $G_{lk}P_k/\eta_k^l$, $k = 1, \ldots, L$ and v_{L+1} by n_l/η_{L+1}^l. The following inequality is obtained

$$\sum_{k\in\mathcal{L}}G_{lk}P_k+n_l\ge\prod_{k\in\mathcal{L}}\left(\frac{G_{lk}P_k}{\eta_k^l}\right)^{\eta_k^l}\left(\frac{n_l}{\eta_{L+1}^l}\right)^{\eta_{L+1}^l},\quad\forall l=1,\ldots,L$$

Taking the logarithm of both sides of the above inequality yields

$$\log\left(\sum_{k\in\mathcal{L}}G_{lk}P_k+n_l\right)\ge\sum_{k\in\mathcal{L}}\eta_k^l\log\left(\frac{G_{lk}P_k}{\eta_k^l}\right)+\eta_{L+1}^l\log\left(\frac{n_l}{\eta_{L+1}^l}\right),\quad\forall l=1,\ldots,L$$

From the link capacity formula (11), we establish (13).

The equality holds if and only if

$$\begin{aligned}\eta_k^l&=\frac{G_{lk}P_k}{\sum_{k\in\mathcal{L}}G_{lk}P_k+n_l}=\frac{G_{kk}\mathrm{SIR}_k}{G_{lk}+G_{kk}\mathrm{SIR}_k},\quad\forall k=1,\ldots,L,\\\eta_{L+1}^l&=\frac{n_l}{\sum_{k\in\mathcal{L}}G_{lk}P_k+n_l}\end{aligned}\tag{14}$$

Particularly, when $k = l$

$$\eta_l^l=\frac{\mathrm{SIR}_l}{\mathrm{SIR}_l+1},\quad\forall l=1,\ldots,L\tag{15}$$

□

From Lemmas 2 and 4, we approximate $\mathbb{P}8$ to a new problem as follows

$$\begin{aligned}&\mathbb{P}9^{\tau}\text{: Max.}y\\&\text{s.t. }y\le\sum_{s\in\mathcal{N}}\tilde{U}_s(x_s;\theta_s)\\&\sum_{s:l\in L(s)}x_s\le\hat{c}_l(\boldsymbol{P};\eta^l),\quad\forall l\in\mathcal{L}\\&\boldsymbol{P}^{\min}\le\boldsymbol{P}\le\boldsymbol{P}^{\max}\end{aligned}$$

Transforming back to the canonical form, we obtain the problem

$$\begin{aligned}&\mathbb{P}10^{\mathrm{t}}\text{: Max.}\sum_{s\in\mathcal{N}}\tilde{U}_s(x_s;\theta_s)\\&\text{s.t.}\sum_{s:l\in L(s)}x_s\le\tilde{c}_l(\tilde{\boldsymbol{P}};\eta^l),\quad\forall l\in\mathcal{L}\\&\tilde{\boldsymbol{P}}^{\min}\le\tilde{\boldsymbol{P}}\le\tilde{\boldsymbol{P}}^{\max}\end{aligned}$$

where $\tilde{P}_l^{\min}\triangleq\log(P_l^{\min})$, $\tilde{P}_l^{\max}\triangleq\log(P_l^{\max})$ and

$$\tilde{c}_l(\tilde{\boldsymbol{P}};\eta^l)\triangleq\sum_k\eta_k^l\tilde{P}_k+\sum_k\eta_k^l\log\left(\frac{G_{lk}}{\eta_k^l}\right)+\eta_{L+1}^l\log\left(\frac{n_l}{\eta_{L+1}^l}\right)-\log\left(\sum_{k\neq l}G_{lk}\mathrm{e}^{\tilde{P}_k}+n_l\right),\quad\forall l\in\mathcal{L}\tag{16}$$

Lemma 5: The function $\tilde{c}_l(\tilde{\boldsymbol{P}};\eta^l)$, $\forall l\in\mathcal{L}$ is a strictly concave function.

Proof: The function $\tilde{c}_l(\tilde{\boldsymbol{P}};\eta^l)$ is strictly concave because it is in the form of an affine function minus a log-sum-exp function, which is strictly convex according to [28, p. 3.1.5]. □

From Lemmas 3 and 5, $\mathbb{P}10^{\tau}$ is a convex optimisation problem. We utilise the Lagrange dual decomposition method to solve $\mathbb{P}10^{\tau}$ in the next subsection.

4.2 Solution to the approximation problem

Problem $\mathbb{P}10^{\tau}$ is solved via its dual. The dual function is

$$\begin{aligned} D_2(\boldsymbol{\lambda}) &= \max_{\mathbf{x},\tilde{\boldsymbol{P}}} L(\boldsymbol{x},\tilde{\boldsymbol{P}},\boldsymbol{\lambda}) \\ &= \max_{\boldsymbol{x},\tilde{\boldsymbol{P}}}\left(\sum_{s\in\mathcal{N}}\tilde{U}_s(x_s;\theta_s)-\sum_{l\in\mathcal{L}}\lambda_l\left(\sum_{s:l\in L(s)}x_s-\tilde{c}_l(\tilde{\boldsymbol{P}};\boldsymbol{\eta}^l)\right)\right) \\ &= \max_{\boldsymbol{x}}\left(\sum_{s\in\mathcal{N}}\left(\tilde{U}_s(x_s;\theta_s)-\left(\sum_{l\in L(s)}\lambda_l\right)x_s\right)\right) \\ &\quad +\max_{\tilde{\boldsymbol{P}}}\left(\sum_{l\in\mathcal{L}}\lambda_l\tilde{c}_l(\tilde{\boldsymbol{P}};\boldsymbol{\eta}^l)\right) \\ &= \max_{\boldsymbol{x}} L_x(\boldsymbol{x},\boldsymbol{\lambda})+\max_{\tilde{\boldsymbol{P}}} L_P(\tilde{\boldsymbol{P}},\boldsymbol{\lambda}) \end{aligned} \tag{17}$$

where $L_x(\boldsymbol{x}, \lambda)$ is defined by (5) and $L_P(\tilde{\boldsymbol{P}}, \boldsymbol{\lambda}) \triangleq \sum_{l\in\mathcal{L}}\lambda_l\tilde{c}_l(\tilde{\boldsymbol{P}}; \boldsymbol{\eta}^l)$. The dual problem is

$$\min_{\boldsymbol{\lambda}\geq\mathbf{0}} D_2(\boldsymbol{\lambda}) \tag{18}$$

Also, the dual problem (18) is solved using the gradient projection algorithm. The gradient update is given by

$$\lambda_l^{(\tau)}(t+1)=\left[\lambda_l^{(\tau)}(t)+\kappa\left(\sum_{s:l\in L(s)}x_s^{(\tau)}(t)-\tilde{c}_l\left(\tilde{\boldsymbol{P}}^{(\tau)}(t);\boldsymbol{\eta}^{l(\tau)}\right)\right)\right]^+ \tag{19}$$

for all $l\in\mathcal{L}$, where $\boldsymbol{x}^{(\tau)}(t)$ and $\tilde{\boldsymbol{P}}^{(\tau)}(t)$ are the optimal solutions to the subproblems $\max_{\boldsymbol{x}} L_x(\boldsymbol{x}, \boldsymbol{\lambda})$ $\max_{\tilde{\boldsymbol{P}}} L_P(\tilde{\boldsymbol{P}}, \boldsymbol{\lambda})$ given by (17) at time instant t.

We next derive the solution to the subproblems. $L_x(\boldsymbol{x}, \lambda)$, $L_P(\tilde{\boldsymbol{P}}, \boldsymbol{\lambda})$ are all concave functions in terms of $\boldsymbol{x}$ and $\tilde{\boldsymbol{P}}$, respectively. Hence, the optimal points of the subproblems also satisfy the KKT conditions. The first subproblem is exactly the same as (5), thus, the rate update is also the update (8). The first derivative condition of the second subproblem is

$$\frac{\partial L_P(\tilde{\boldsymbol{P}},\boldsymbol{\lambda})}{\partial\tilde{P}_l}=\lambda_l\eta_l^l-\sum_{k\neq l}\frac{\lambda_k G_{kl}\mathrm{e}^{\tilde{P}_l}}{\sum_{j\neq k}G_{kj}\mathrm{e}^{\tilde{P}_j}+n_k}=0, \quad \forall l\in\mathcal{L} \tag{20}$$

Transforming (20) back into $\boldsymbol{P}$ space, we obtain the power update as follows

$$P_l^{(\tau)}(t)=\left[\frac{\lambda_l^{(\tau)}(t)\eta_l^{l(\tau)}}{\sum_{k\neq l}G_{kl}m_k^{(\tau)}(t)}\right]_{P_l^{\min}}^{P_l^{\max}}, \quad \forall l\in\mathcal{L} \tag{21}$$

where

$$m_k^{(\tau)}(t)\triangleq\frac{\lambda_k^{(\tau)}(t)\mathrm{SIR}_k^{(\tau)}(t)}{P_k^{(\tau)}(t)G_{kk}} \tag{22}$$

4.3 Successive approximation algorithm for the joint NUM

Similar to the main framework in Section 3, we propose the following Algorithm 2 (see Fig. 3) for the joint NUM.

In Algorithm 2 (see Fig. 3), the initial value of a new iteration is the stationary value of the previous iteration. In step 6, each link l calculates the value m_l locally according to (22) and pass this information to all the other links in the network for the power update in (21). In step 5, η_l^l, $\forall l\in\mathcal{L}$ is updated locally by (15). Each source s calculates its utility and passes this information to all other sources in the network to update θ according to (4).

Theorem 2: Algorithm 2 (see Fig. 3) converges to a stationary point satisfying the KKT conditions of $\mathbb{P}6$.

Proof: Beside the notations $f_u(\boldsymbol{x})$ and $\tilde{f}_u(\boldsymbol{x})$ from the proof of Theorem 1, we denote $f_c(\boldsymbol{x}, \boldsymbol{P})\triangleq\left(\sum_{s:l\in L(s)}x_s\right)/\left(c_l(\boldsymbol{P})\right)$ and $\tilde{f}_c(\boldsymbol{x}, \boldsymbol{P})\triangleq\left(\sum_{s:l\in L(s)}x_s\right)/\hat{c}_l\left(\boldsymbol{P};\ \eta^{(\tau)}\right)$. Similar to the proof of Theorem 1, we need to verify the following three conditions for the convergence to the KKT point of the algorithm:

1. $f_u(\boldsymbol{x})\leq\tilde{f}_u(\boldsymbol{x})$ and $f_c(\boldsymbol{x}, \boldsymbol{P})\leq\tilde{f}_c(\boldsymbol{x}, \boldsymbol{P})$,
2. $f_u\left(\boldsymbol{x}^{(\tau)*}\right)=\tilde{f}_u\left(\boldsymbol{x}^{(\tau)*}\right)$ and $f_c\left(\boldsymbol{x}^{(\tau)*}, \boldsymbol{P}^{(\tau)*}\right)=\tilde{f}_c\left(\boldsymbol{x}^{(\tau)*}, \boldsymbol{P}^{(\tau)*}\right)$,
3. $\nabla f(\boldsymbol{x})|_{\boldsymbol{x}=\boldsymbol{x}^{(\tau)*}}=\nabla\tilde{f}(\boldsymbol{x})|_{\boldsymbol{x}=\boldsymbol{x}^{(\tau)*}}$ and $\nabla f_c(\boldsymbol{x}, \boldsymbol{P})|_{\boldsymbol{x}=\boldsymbol{x}^{(\tau)*};\boldsymbol{P}=\boldsymbol{P}^{(\tau)*}}=\nabla\tilde{f}_c(\boldsymbol{x}, \boldsymbol{P})|_{\boldsymbol{x}=\boldsymbol{x}^{(\tau)*};\boldsymbol{P}=\boldsymbol{P}^{(\tau)*}}$, where $\left(\boldsymbol{x}^{(\tau)*}, \boldsymbol{P}^{(\tau)*}\right)$

is the optimal solution of the τth iteration.

Condition (1) and (2) are satisfied by (3) and (13). It is straightforward to verify condition (3) by comparing the corresponding partial derivatives. Therefore the theorem is proved. □

5 Numerical results

In the experiments, all the utilities in this section are normalised or have the closed values at $\boldsymbol{x}^{\max}=10$ Mbps (see Fig. 1). The flow rates are updated according to Table 1. $\boldsymbol{x}^{\min}$ is 0 Mbps. The network is considered converged as all the flow rates converge. Specifically, the stationary conditions for inner iterations (step 6) in Algorithm 1 (see Fig. 2) and Algorithm 2 (see Fig. 3) are $(|\mathbf{x}(t+1)-\boldsymbol{x}(t)|/\boldsymbol{x}(t))<\varepsilon$ and $(|\boldsymbol{P}(t+1)-\boldsymbol{P}(t)|/\boldsymbol{P}(t))<\varepsilon$. The stationary conditions of the outer iterations are $(|\boldsymbol{x}^{(\tau+1)}-\boldsymbol{x}^{(\tau)}|/\boldsymbol{x}^{(\tau)})<\varepsilon$ and $(|\boldsymbol{P}^{(\tau+1)}-\boldsymbol{P}^{(\tau)}|/\boldsymbol{P}^{(\tau)})<\varepsilon$. The constant step-size $\kappa=0.1$ and the error bound $\varepsilon=10^{-4}$, unless specified otherwise.

5.1 Convergence of the algorithm

We want to verify the convergence of Algorithm 1 (see Fig. 2) in this experiment. Consider a network with a single link and two flows (see Fig. 4a), one flow is inelastic, one flow is elastic. Their utilities are $1/\left(1+\mathrm{e}^{-2(x_1-5)}\right)$ and $\log(x_2+1)/$

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log(11), respectively. With the link capacity of 6 Mpbs, which does not satisfied the link capacity condition according to [8], we could not find any step-size for the convergence of the standard dual-based algorithm even with the diminishing step-size (see Fig. 5). Using Algorithm 2 (see Fig. 3), the solution sequence of the series of the approximation problems converges at $\boldsymbol{x}^* = [5.69, 0.31]$ Mbps and $U^* = 0.91$ (see Fig. 6). In this case, the local optimum is also the global optimal solution. Our proposed algorithm results to a higher aggregate utility in comparison to the one of the utility-proportional fair algorithm in [14] (see Fig. 7).

5.2 Two-link example

In this experiment, the network has two-links and three-inelastic flows as described in Fig. 4*b*. Utility functions are all $1/\left(1 + e^{-(x_i - 5)}\right)$, $i = 1, 2, 3$. In case $\boldsymbol{c} = [4, 8]$ Mbps, Algorithm 1 (see Fig. 2) results to $\boldsymbol{x}^* = [0, 4, 8]$ Mbps and $U^* = 1.23$. In the case of $\boldsymbol{c} = [7, 20]$ Mbps, $\boldsymbol{x}^* = [0, 9, 10]$ Mbps and $U^* = 1.98$, which are also the optimal values calculated from sum-of-square method in [10, Example 3].

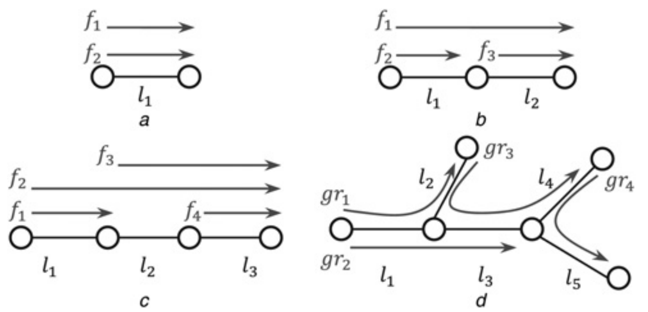

Fig. 4 *Network topologies*

a one-link topology (experiments in subsection 5.1)
b two-link topology (experiments in subsections 5.2 and 5.5)
c three-link topology (experiments in subsection 5.3)
d medium-size network (experiments in subsection 5.4)

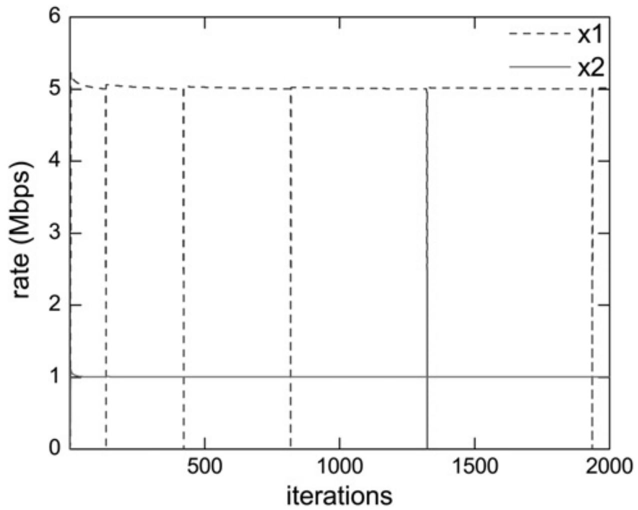

Fig. 5 *Non-convergence of the standard dual-based algorithm in [7, 8]*

One-link topology (Fig. 4a), C = 6 Mbps

5.3 Varying the initial point

Given θ, the approximation problem has a unique optimal solution due to its strict convexity. Hence, the result of Algorithm 1 (see Fig. 2) will depend on choosing the initial $\theta^{(0)}$. In this experiment, we verify the result of the algorithm when varying the initial point. Consider the network with three indirect links and four flows as described in Fig. 4*c*. The utility functions are $U_1(x_1) = 1/\left(1 + e^{-(x_1-2)}\right)$, $U_2(x_2) = 0.1x_2$, $U_3(x_3) = \log(x_3 + 1)/\log(11)$ and $U_4(x_4) = 1/\left(1 + e^{-2(x_4-4)}\right)$, respectively. The link capacities are all 10 Mbps. We conduct 100 experiments from 100 uniformly random initial $\theta^{(0)}$. The stationary aggregate utilities from Algorithm 1 (see Fig. 2) are shown in Fig. 8. A 92% of the experiments achieve the global optimal solution which has $\boldsymbol{x}^* = [8.67, 1.33, 3.22, 5.45]$ Mbps and $U^* = 2.68$.

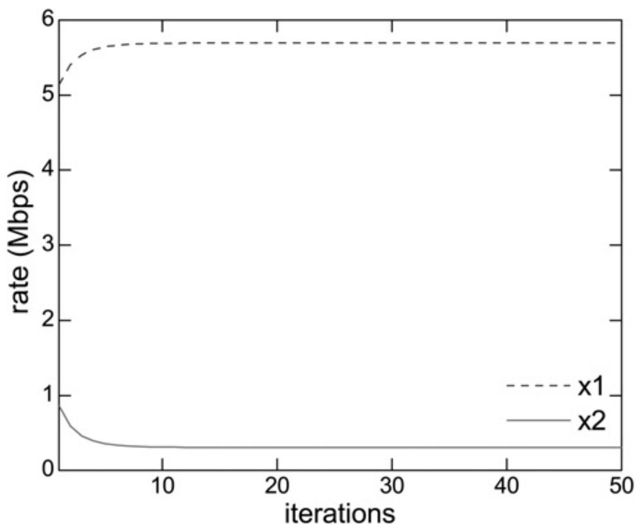

Fig. 6 *Convergence of Fig. 2*

One-link topology (Fig. 4a), C = 6 Mbps

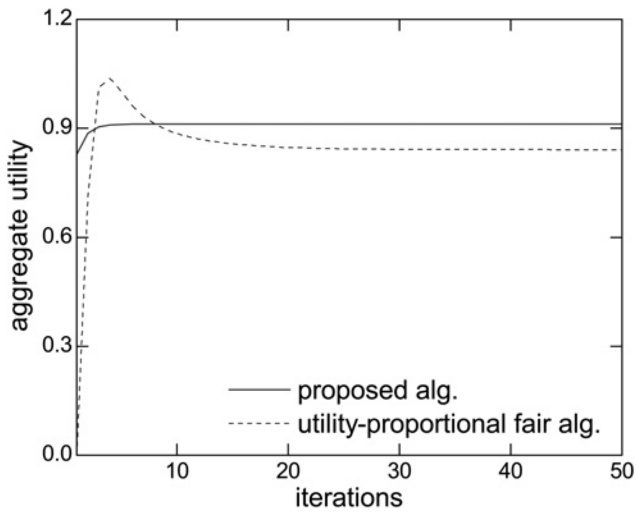

Fig. 7 *Aggregate utility of Fig. 2 compares to the utility-proportional fair algorithm in [14]*

One-link topology (Fig. 4a), C = 6 Mbps

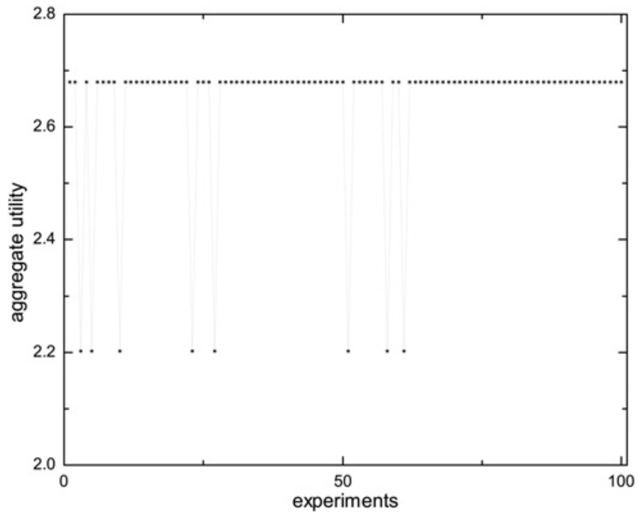

Fig. 8 *Aggregate utility as randomising the initial $\theta^{(0)}$*

Table 2 Average convergence time (in seconds) with different network sizes and stopping conditions $((\|x^{(\tau+1)} - x^{(\tau)}\|/x^{(\tau)}) \leq \varepsilon)$. The inner iteration updates every 10 ms and each outer iteration includes 50 inner iterations

Number of flows	$\varepsilon = 1\times10^{-2}$	$\varepsilon = 1\times10^{-3}$	$\varepsilon = 1\times10^{-4}$
16	0.50	0.50	0.50
24	2.00	3.40	4.40
32	3.05	4.75	5.75
40	3.45	5.10	6.45
48	4.15	6.15	7.70
56	4.60	7.15	8.95
64	6.2	9.35	11.95
72	6.85	10.85	14.50
80	8.20	12.40	16.15

5.4 Varying the network size

In this experiment, we want to monitor the convergence time of the proposed algorithm with different network sizes. The heuristic implementation is used with 50 inner iterations per outer iteration. We utilise topology in [11, Fig. 7] for the experiment. There are four groups of flows in a network with five links as shown in Fig. 4*d*. In each group, half of the flows are elastic and the other half are inelastic. The step-size is 2×10^{-5}. We monitor the convergence time of the algorithm as increasing the number of flows in the groups gradually. In each case, we conduct ten experiments from ten random initial points. The convergence time of each case is averaged over ten results. The number of flows in four groups vary from 24 to 80 flows. We add one elastic flow and one inelastic flow in every group with each increment. The link capacities are all 100 Mbps.

Assume that the link prices update in every 10 ms. Then it takes 500 ms per outer iteration. Table 2 shows the convergence time of the algorithm as we increase the number of flows in the network gradually. According to Table 2, as the number of flows in the network increases, the convergence time of the algorithm increases. Also in the same network condition, when the stopping condition is more stringent, the convergence time is longer.

5.5 Jointly rate and power control

This experiment verifies Algorithm 2 (see Fig. 3) which jointly allocates the rate and power for multiclass traffic. We consider the network topology with three-flows and two-directed links (Fig. 4*b*). Two link l_1 and l_2 have the respective transmitting nodes 1 and 2. The parameters of the simulation are $W = 1$ MHz; $K = -1.5/\log(5\ \text{BER})$ with $\text{BER} = 10^{-3}$ for multi-quadrate amplitude modulation (MQAM) modulation. The channel gain is calculated by $h(d) = h_o(d/15)^{-4}$, where h_o is a reference channel gain at a distance 15 m. The maximum and minimum power are $\boldsymbol{P}^{\max} = 100$ mW and $\boldsymbol{P}^{\min} = 5$ mW, respectively. The utility functions of the flows are

$$U_1(x_1) = \frac{1}{1 + e^{-10(x_1-2)}}, \quad U_2(x_2) = \frac{\log(x_2+1)}{\log(11)} \quad \text{and}$$
$$U_3(x_3) = \frac{1}{1 + e^{-2(x_3-4)}}$$

The constant step-size in this experiment is 0.05. Fig 9 shows the convergence of the rate and power allocation for flows and transmitting nodes to the values $\boldsymbol{x}^* = [2.41, 1.99, 4.72]$ Mbps and $\boldsymbol{P}^* = [0.1, 0.086]$ W.

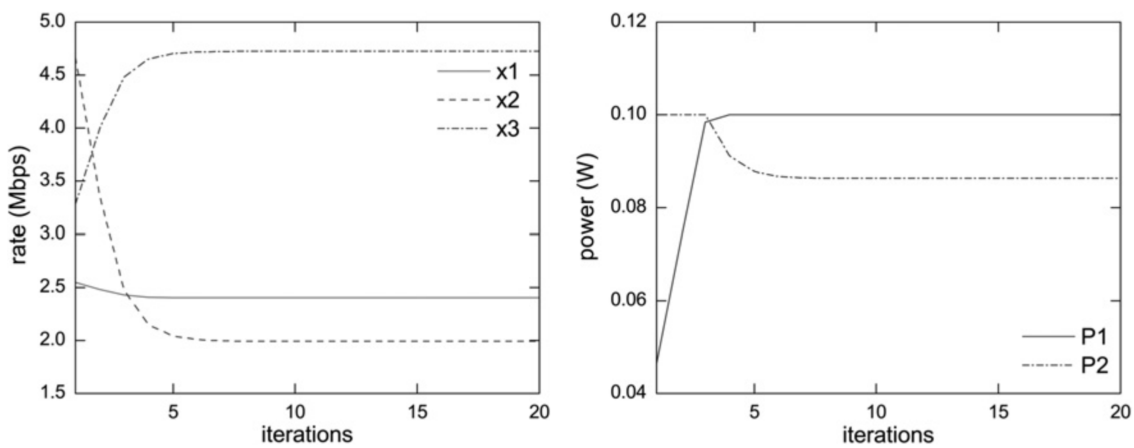

Fig. 9 *Rate and the power generated by Algorithm 2 (see Fig. 3)*

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6 Conclusions

Based on the successive approximation method, we have proposed an algorithm converging to a local optimal solution which also satisfies the KKT conditions of the non-convex NUM with elastic and inelastic utilities. We also extend the framework to jointly allocate the rate and the power for two kinds of flows in a multihop wireless networks. It is shown that any log-concave utilities can be applied to our framework. The numerical results show that our proposed algorithm converges in case of scarce resources whereas the standard dual-based algorithm does not. The global solution can be achieved from many initial points.

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