Pricing Mechanisms and Equilibrium Behaviors of Noncooperative Users in Cognitive Radio Networks

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***Abstract*—We study the pricing mechanisms and their effects on equilibrium behaviors of self-optimizing secondary users (SUs) sharing a single channel of primary users (PUs) operated by a service provider (SP) in cognitive radio networks. From SUs' point of view, a spectrum access decision on whether to join a queue or not is characterized through an individual optimal strategy. With this strategy, we show that there exists a unique equilibrium in terms of SUs' joining probability. This strategy also requires each SU to know its average queueing delay, which is a non-trivial problem because of multiple SUs service's interruptions from the returns of PUs; we, however, can analyze this queueing delay based on the general distribution of SUs's service time and PUs' traffic model by using renewal theory. We also provide a sufficient condition and iterative algorithms for the convergence of equilibrium points. From the SP's point of view, two pricing mechanisms are proposed with different goals: revenue maximization and social welfare maximization. And the optimal price can be solved efficiently using numerical methods.**

I. Introduction

Cognitive radio (CR) is expected to mitigate spectrum shortage by enabling the spectrum sharing for secondary users (SUs) who opportunistically capture the temporal and spatial "spectrum holes" in spectral white spaces of primary users (PUs). Among many spectrum access control methods, pricing has long been considered an important approach due to its simplicity yet effectivity.

By considering delay-sensitive SUs wishing to share a PU's single channel operated by an SP, we examine the SP's pricing effect on the equilibrium behaviors of non-cooperative SUs. This behavior is represented by a SU's spectrum access decision it has to make upon arrival: joining the list of other SUs who also want to share the same channel, or balking. When all arriving SUs are considered as price-takers, two important questions are raised: (1) Given an admission price charged by the SP, what is the equilibrium point of SUs' individual optimal strategies and how to achieve it? (2) What are the pricing policies of the SP to maximize its revenue and the network social welfare?

Considering the first question, we introduce an individual optimal strategy employed by each SU in order to make its spectrum access decision based on a utility function that captures the delay-sensitivity heterogeneity of SUs. We next show that there exists a unique equilibrium of the SUs' behaviors. Each SU has to evaluate its average queueing delay of a virtual queue that is used to model a congestion effect happening when many SUs intend to share the same PUs' channel. By using renewal theory, we provide a queueing delay analysis based only on the statistical information of PUs' activities and SUs' service time since SUs cannot observe exactly how many others SUs being in the CR networks. Finally, we examine the equilibrium dynamics through iterative algorithms and provide a sufficient condition for the equilibrium convergence.

Considering the second question, we devise two pricing mechanisms for different SP's perspectives. If the SP is operated by a commercial planner, we propose a revenue-optimal pricing policy to maximize the SP's revenue. Otherwise, if the SP is a social planner, we propose a socially optimal pricing policy to maximize the network social welfare. Both can be solved efficiently using numerical methods with single variable.

II. Related Work

The pricing mechanisms and its impact on the equilibrium strategies of users in a queueing system can be traced back from the original work of [1] and surveyed in the monograph [2]. In the context of CR networks, even though pricing-based methods have been employed extensively for spectrum access control [3]–[6], few papers are in the category of pricing effect on equilibrium behavior of queueing systems [7]–[10]. Similarly to [7], we use the unobservable queue system, which appropriately models the non-cooperative and distributed nature of SUs, whereas [8], [10] employ the observable queue system, which requires either a centralized control server or a feedback mechanism with time overhead. We apply the renewal theory for the queueing analysis on general distributions of PUs' channel and SUs' service, whereas the others with Markov chain analysis either restricted their models to Bernoulli [8], or exponential [7], [9] distributions. Furthermore, these authors consider only either socially-optimal pricing [8], [9] or revenue-optimal pricing [7] in the shared-use model with homogeneous SUs, whereas we provide not only both social and revenue maximization pricing mechanisms but also the price competition between the shared-use and exclusive-use models with heterogeneous SUs.

III. System Model

We consider a CR network with a single licensed channel operated by a SP. This licensed band is used exclusively by

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PUs, and shared opportunistically by multiple SUs based on an admission price charged by the SP.

A. Primary Users

The traffic patterns of PUs on the licensed band can be modelled as an ON-OFF renewal process alternating between ON (busy) and OFF (idle) periods. We model the sojourn times of ON and OFF periods as i.i.d random variables Y and Z, with the probability density function (pdf) $f_Y(y)$ and $f_Z(z)$, respectively. We assume that the ON and OFF periods are independent.

This ON-OFF process of the PUs traffics, which is illustrated in Fig. 1, can be considered as a channel model for SUs services. This model captures the idle time period in which SUs can utilize the channel without causing harmful interference to PUs.

B. Secondary Users

1) Arrival Rates and Service Time: We assume that SUs arrive to the network according to a Poisson process with rate λ. Each SU is associated with a distinct job (e.g., a packet, session or connection) that it carries upon arrival. The service time to complete a job is represented by random variable X, with its pdf $f_X(x)$. This service time is assumed to be independent of the arrival process and ON-OFF process.

2) Delay-Sensitive User Types: Since SUs are assumed to carry delay-sensitive application traffics, each job is attached to a specific application type characterized by a parameter θ. This parameter represents for an individual preference that reflects how much delay-sensitivity of a SU's applications. The value of θ varies across the job types capturing *SUs heterogeneity*. We also assume that this parameter follows a uniform distribution on $[0, \theta_{\mathsf{up}}]$, which is common in the literature [11]–[13]. The relationship between θ and application types is presented through some examples: many multimedia traffics with stringent delay requirements will have high values of θ; on the other hand, applications with θ equal to zero are insensitive to delay.

3) Steady-State Virtual Queue: Since there are many SUs attempting to share the same licensed channel, the congestion occurs, which clearly affects the delay of each SU job. Therefore, when a SU job arrives, it will evaluate its job's delay in a queue containing other SUs jobs that also attempt to use this licensed channel. This queue is only a *virtual queue* because each SU cannot observe how many other jobs waiting before its job since naturally each SU cannot know how many other SUs trying to share the licensed channel. Therefore, each SU forms a virtual queue based on statistical information of λ, $f_X(x)$, $f_Y(y)$ and $f_Z(z)$, which are assumed to be estimated by existing methods [14], to assess the average queueing delay. Henceforth, we simply use "queue" for this virtual queue. We denote $T(\lambda)$ and $\mathbf{E}[T(\lambda)]$ as random and average steady-state queueing delay (i.e., waiting time + service time), respectively, induced by arrival rate λ.

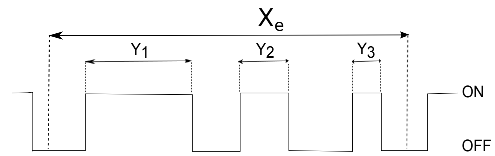

Fig. 1. An ON-OFF process with a realization of an effective service time X_e where its original SU service time is increased because of three interruptions from ON periods Y_1, Y_2 and Y_3.

4) Individual Utilities: The value θ of a SU is realized at the instant it arrives. Then this so-called type-θ SU has to make a decision: either join the queueing system or balk. The utility of any balking SU is set to zero. For a type-θ SU who joins the system, its utility function is given by

$$U(\theta) = V - \theta\,\mathbf{E}[T(\lambda)] - c. \tag{1}$$

The reward V is considered as the maximum utility of a SU for entering the system. The total cost consists of two elements: the admission fee c charged by the SP, and the waiting cost $\theta\,\mathbf{E}[T(\lambda)]$ of a job that spends an average queueing delay $\mathbf{E}[T(\lambda)]$. In this waiting cost, the parameter θ can be interpreted as a waiting cost per unit time, an interpretation that still follows the delay-sensitivity mindset of θ: higher waiting cost per unit time induces more negative effect of queueing delay, which shows more sensitivity to delay.

IV. Individual Optimal Strategy

In this section, we first introduce an individual optimal strategy and study an equilibrium of the system. Then we derive the average queueing delay, which is required by all SUs in order to employ their strategies. We finally examine the convergence property of the equilibrium dynamics.

A. SUs' Equilibrium

We consider a stream of potential arriving SUs that are rational and self-optimizing. Specifically, upon arrival each potential type-θ SU has to make an individual decision whether to join the system with probability $p(\theta)$ or balk with probability $1 - p(\theta)$ to maximize its expected utility $p(\theta)U(\theta) + (1 - p(\theta))0$ by choosing $p(\theta) \in [0, 1]$. Therefore, an individual optimal strategy is defined first.

Definition 1. All individually optimizing type-θ SUs will follow the same strategy: join with probability $p(\theta) = 1$ if its $U(\theta) > 0$; and with probability $p(\theta) = 0$ otherwise.

Given the joining rule $\{p(\theta), \theta \geq 0\}$, the unconditional probability that a potential SU joins the SP is $p = \int_0^\infty p(\theta) dF_\Theta(\theta)$, and the actual arrival rate to the system is λp. We have the following result

Theorem 1. *For a given admission price c, there exists a unique equilibrium of the SUs' joining probability p^* as follows*

$$p^* = \int_0^\infty p(\theta) dF_\Theta(\theta) = \int_0^{\frac{V-c}{\mathbf{E}[T(\lambda p^*)]}} dF_\Theta(\theta)$$
$$= F_\Theta\left(\frac{V-c}{\mathbf{E}[T(\lambda p^*)]}\right). \tag{2}$$

Proof: Define $\Phi(p) = F_\Theta\left(\frac{V-c}{\mathbf{E}[T(\lambda p)]}\right) - p$ for $p \in [0,1]$. We can see that $\Phi(p)$ is a strictly decreasing function because $F_\Theta(.)$ is an increasing function and $\mathbf{E}[T(\lambda p)]$ is strictly increasing. Then, it suffices to show that $\Phi(p)$ has a unique root on its domain. We see that

$$\Phi(0) = F_\Theta\left(\frac{V-c}{\mathbf{E}[X_e]}\right) \geq 0$$
$$\Phi(1) = F_\Theta\left(\frac{V-c}{\mathbf{E}[T(\lambda)]}\right) - 1 < 0. \quad (3)$$

Since $\Phi(p)$ is a continuous function, there exists a unique root $p^* \in [0,1]$. ■

This theorem shows that once reaching an equilibrium point, the proportion of joining SUs remains the same at this point hereafter.

B. Queueing Delay Analysis

We assume that a SU can use its spectrum sensing to inform the SU whether the channel is busy or idle. When the channel is sensed idle, the SU job can be in service. When the channel is sensed busy, the spectrum handoff procedure is performed to return the channel to PUs. During the service time of a SU job, it is likely that this SU has to perform multiple spectrum handoffs due to multiple interruptions from the returns of PUs represented by ON periods. Clearly, due to multiple spectrum handoffs, the original service time X of the SU job is increased and this increased service time is called *effective service time*, denoted by a random variable X_e and illustrated in Fig. 1.

We start the analysis by denoting $\mathbf{E}[W(\lambda p^*)]$ as the average waiting time in a queue induced by arrival rate λp^*, $p^* \in [0,1]$. This queueing system can be considered as a M/G/1 queue with its average service time $\mathbf{E}[X_e]$; moreover, this queue is in steady-state when the condition $\lambda p^* < 1/\mathbf{E}[X_e]$ is satisfied. Applying the mean value analysis [15], we have

$$\mathbf{E}[T(\lambda p^*)] = \mathbf{E}[W(\lambda p^*)] + \mathbf{E}[X_e]. \quad (4)$$

According to the Pollaczek-Khinchin formula [15], the average waiting time is calculated as follows:

$$\mathbf{E}[W(\lambda p^*)] = \frac{\lambda p^* \, \mathbf{E}\left[X_e^2\right]}{2(1 - \lambda p^* \, \mathbf{E}[X_e])}. \quad (5)$$

Then, the problem boils down to how to derive $\mathbf{E}[X_e]$ and $\mathbf{E}\left[X_e^2\right]$, the first and second moments of effective service time respectively, in order to estimate average queueing delay.

1) The First Moment of Effective Service Time: Defining a random variable, $N(X)$, as the number of renewals (i.e. ON periods) occurring in the interval $(0, X)$, we have

$$\mathbf{E}[X_e] = \mathbf{E}\left[X + \sum_{i=1}^{N(X)} Y_i\right] = \mathbf{E}[X] + \mathbf{E}\left[\sum_{i=1}^{N(X)} Y_i\right]$$
$$= \mathbf{E}[X] + \mathbf{E}[Y]\,\mathbf{E}[N(X)], \quad (6)$$

where the final equality occurs because Y is independent of X. From [16, pp. 45], we have

$$\mathbf{E}\left[N(X) \mid X = x\right] = \frac{x}{\mathbf{E}[Z]}. \quad (7)$$

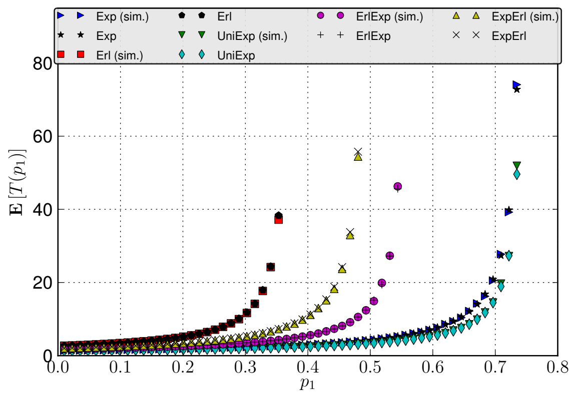

Fig. 2. Average queueing delay performance comparison.

As a consequence, $\mathbf{E}[N(X)] = \frac{\mathbf{E}[X]}{\mathbf{E}[Z]}$, which is then substituted into (6) so as to obtain

$$\mathbf{E}[X_e] = \mathbf{E}[X]\left(1 + \frac{\mathbf{E}[Y]}{\mathbf{E}[Z]}\right). \quad (8)$$

2) The Second Moment of Effective Service Time: We continue with We continue with

$$\mathbf{E}\left[X_e^2\right] = \mathbf{E}\left[\left(X + \sum_{i=1}^{N(X)} Y_i\right)^2\right] =$$
$$\mathbf{E}\left[X^2\right] + 2\,\mathbf{E}\left[X \sum_{i=1}^{N(X)} Y_i\right] + \mathbf{E}\left[\left(\sum_{i=1}^{N(X)} Y_i\right)^2\right]. \quad (9)$$

Using the law of iterated expectations, the second term of the right side of (9) can be shown to be

$$\mathbf{E}\left[X \sum_{i=1}^{N(X)} Y_i \mid X = x\right] =$$
$$x\,\mathbf{E}[N(x)Y] = x\,\mathbf{E}[N(x)]\,\mathbf{E}[Y] = x^2 \frac{\mathbf{E}[Y]}{\mathbf{E}[Z]}, \quad (10)$$

where we have the second equality because of the independence between Y and $N(x)$. Hence, we obtain

$$\mathbf{E}\left[X \sum_{i=1}^{N(X)} Y_i\right] = \mathbf{E}\left[X^2\right] \frac{\mathbf{E}[Y]}{\mathbf{E}[Z]}. \quad (11)$$

Next, we derive the third term on the right side of (9) as follows

$$\mathbf{E}\left[\left(\sum_{i=1}^{N(X)} Y_i\right)^2\right] = \mathbf{E}\left[\sum_{i=1}^{N(X)} Y_i^2\right] + \mathbf{E}\left[\sum_{\{(i,j)|i \neq j\}} Y_i Y_j\right]$$
$$= \mathbf{E}[N(X)]\,\mathbf{E}\left[Y^2\right] + \mathbf{E}[Y]^2\,\mathbf{E}\left[\big(N(X) - 1\big)N(X)\right]. \quad (12)$$

We define $g(X \mid X = x) \triangleq \mathbf{E}\left[\big(N(X) - 1\big)N(X) \mid X = x\right]$ and denote the Laplace transform of an arbitrary function $f(x)$ by $f^*(s)$. Using the similar technique of deriving the variance of the number of renewals in [16, pp. 55], we can easily obtain the following result

$$g^*(s) = \frac{2}{s^2\,\mathbf{E}[Z]} \frac{f_Z^*(s)}{1 - f_Z^*(s)}. \quad (13)$$

An inverse Laplace transform can then be applied to $g^*(s)$ so as to obtain $g(x)$. Therefore, $g(X)$ can be found correspondingly. From (9), (11), and (12), we can see that $\mathbf{E}\left[X_e^2\right]$ is completely derived.

3) Examples with Analysis and Simulation Comparisons: We consider five examples. The first is termed Exp where X, Y and Z all have the exponential distributions with $f_X(x) = \mu_X e^{-\mu_X x}$, $f_Y(y) = \mu_{\mathsf{on}} e^{-\mu_{\mathsf{on}} y}$ and $f_Z(z) = \mu_{\mathsf{off}} e^{-\mu_{\mathsf{off}} z}$. The second is termed Erl where X, Y and Z all have the Erlang distributions with $f_X(x) = \mu_X^2 x e^{-\mu_X x}$, $f_Y(y) = \mu_{\mathsf{on}}^2 y e^{-\mu_{\mathsf{on}} y}$ and $f_Z(z) = \mu_{\mathsf{off}}^2 z e^{-\mu_{\mathsf{off}} z}$. The third is termed UniExp where X is uniformly distributed on $[x_{\mathsf{lo}}, x_{\mathsf{up}}]$, whereas Y and Z have the exponential distributions with $f_Y(y) = \mu_{\mathsf{on}} e^{-\mu_{\mathsf{on}} y}$ and $f_Z(z) = \mu_{\mathsf{off}} e^{-\mu_{\mathsf{off}} z}$. The fourth is termed ErlExp where X has the Erlang distribution with $f_X(x) = \mu_X^2 x e^{-\mu_X x}$, whereas Y and Z have the exponential distributions with $f_Y(y) = \mu_{\mathsf{on}} e^{-\mu_{\mathsf{on}} y}$ and $f_Z(z) = \mu_{\mathsf{off}} e^{-\mu_{\mathsf{off}} z}$. The fifth is termed ExpErl where X has the exponential distribution with $f_X(x) = \mu_X e^{-\mu_X x}$, whereas Y and Z have the Erlang distributions with $f_Y(y) = \mu_{\mathsf{on}}^2 y e^{-\mu_{\mathsf{on}} y}$ and $f_Z(z) = \mu_{\mathsf{off}}^2 z e^{-\mu_{\mathsf{off}} z}$. Due to the limited space, we only provide two sample results of ErlExp and UniExp as follows. In case of ErlExp, we have

$$\mathbf{E}\left[X_e\right] = \frac{2}{\mu_X}\left(1 + \frac{\mu_{\mathsf{off}}}{\mu_{\mathsf{on}}}\right), \quad (14)$$

$$\mathbf{E}\left[X_e^2\right] = \frac{6\mu_{\mathsf{off}}^2}{\mu_{\mathsf{on}}^2 \mu_X^2} + \frac{4\mu_{\mathsf{off}}}{\mu_{\mathsf{on}}^2 \mu_X} + \frac{12\mu_{\mathsf{off}}}{\mu_{\mathsf{on}} \mu_X^2} + \frac{6}{\mu_X^2}. \quad (15)$$

In case of UniExp, we have

$$\mathbf{E}\left[X_e\right] = \frac{x_{\mathsf{lo}} + x_{\mathsf{up}}}{2}\left(1 + \frac{\mu_{\mathsf{off}}}{\mu_{\mathsf{on}}}\right), \quad (16)$$

$$\mathbf{E}\left[X_e^2\right] = \frac{N}{48\mu_{\mathsf{off}}\mu_{\mathsf{on}}^2 (x_{\mathsf{up}} - x_{\mathsf{lo}})}, \quad (17)$$

where $N = (x_{\mathsf{up}}^3 - x_{\mathsf{lo}}^3)(4\mu_{\mathsf{off}}^3 + 16\mu_{\mathsf{off}}^2\mu_{\mathsf{on}} + 16\mu_{\mathsf{off}}\mu_{\mathsf{on}}^2) + 42\mu_{\mathsf{off}}^2(x_{\mathsf{up}}^2 - x_{\mathsf{lo}}^2) + 6\mu_{\mathsf{off}}(x_{\mathsf{up}} - x_{\mathsf{lo}}) + 3(e^{-2\mu_{\mathsf{off}} x_{\mathsf{lo}}} - e^{-2\mu_{\mathsf{off}} x_{\mathsf{up}}})$.

In order to validate our queueing analysis, we simulate a single-server queue with service interruptions for the performance comparison. In all five examples, we fix $\lambda = 1$ and vary p to adjust the traffic load into the queue. We set $\mu_X = 1$ for both Exp and Erl, $\mu_X = 1.5$ for ErlExp, $\mu_X = 2/3$ for ExpErl and $[x_{\mathsf{lo}}, x_{\mathsf{up}}] = [0.1, 1.9]$ for UniExp, $\mu_{\mathsf{on}} = 1.5$ and $\mu_{\mathsf{off}} = 0.5$. The comparison between the analysis and the simulation is presented in Fig. 2. Despite the variation in numerical settings, Fig. 2 shows that the analysis results are very similar to the simulation results.

C. Equilibrium Convergence

The network system is assumed to operate over a succession of time periods, labeled $t = 0, 1, \ldots$. During the time period t the arrival rate is fixed at a particular value λ_t and each time period is assumed to last long enough for the system to attain the steady state. An arbitrary SU is supposed to hold a prediction on the queueing delay, denoted by $\hat{T}_{t+1}$, and make a joining decision to maximize its utility in the next time instant $t+1$. Hence, this SU will join the network at period $t+1$ if and only if $V > \theta \hat{T}_{t+1} + c$. One of possible prediction techniques is every SU expects that the queueing delay in the next period is equal to that in the current period: $\hat{T}^{(t+1)} = \mathbf{E}\left[T(\lambda p^{(t)})\right]$. By defining $q(p^{(t)}) \triangleq F_\Theta\left(\frac{V-c}{\mathbf{E}\left[T(\lambda p^{(t)})\right]}\right)$, we will have two iterative algorithms, namely *static expectations* and *dynamic expectations* [17], of which the joining probability evolves respectively as follows

$$p^{(t+1)} = q(p^{(t)}) \quad (18)$$

and

$$p^{(t+1)} = (1-\alpha)p^{(t)} + \alpha q(p^{(t)}), \quad (19)$$

where $\alpha \in (0, 1]$. Adaptive expectations method reduces to static expectations when $\alpha = 1$. In order to alleviate these shortcomings of static expectations, the adaptive expectation model – with the intuition that only α fraction of SUs makes a changing decision at a time instant – allows SUs to learn from and correct for past errors. We have the following result which is proved in Appendix A.

Theorem 2. *With any starting point $p^{(0)} \in [0, 1]$, the sufficient condition for the equilibrium convergence of joining probability dynamics* (19) *is*

$$\mathbf{E}\left[X_e^2\right] < \frac{2\,\mathbf{E}\left[X_e\right](\lambda p\,\mathbf{E}\left[X_e\right] - 1)^2}{\lambda\big(\alpha + p(\lambda p\,\mathbf{E}\left[X_e\right] - 1)\big)}, \quad \forall p \in [0, 1]. \quad (20)$$

The insight of this theorem is that if the effective service time's variability is bounded by a function of its expectation and joining probability, then the convergence of equilibrium points is always guaranteed.

V. Optimal Pricing Mechanisms

The main focus of this section is on the SP's pricing mechanisms to maximize revenue and social welfare.

A. Revenue-Optimal Pricing

When charging a price c, the SP can attain an equilibrium revenue $R(c) \triangleq c\,p^*(c)$, where $p^*(c)$ is the equilibrium proportion of SUs who join the system at price c. Then, the problem of finding an *revenue-optimal* price c_{R} of the SP that maximizes its equilibrium revenue is expressed as

$$\max_{c \in [c_{\mathsf{lo}}, c_{\mathsf{up}}]} R(c). \quad (21)$$

Based on the fixed-point equation of SUs' equilibrium joining probability (2), queueing delay (4) and waiting time (5), we have

$$p^*(c) = \begin{cases} 0 & \text{, if } c \geq c_{\mathsf{up}} \\ 1 & \text{, if } c \leq c_{\mathsf{lo}} \\ \frac{-\sqrt{S} + \mathbf{E}[X_e](\lambda(V-c) + \theta_{\mathsf{up}})}{\lambda\theta_{\mathsf{up}}\left(2\,\mathbf{E}[X_e]^2 - \mathbf{E}[X_e^2]\right)} & \text{, otherwise} \end{cases} \quad (22)$$

where

$$c_{\mathsf{up}} = V,$$
$$c_{\mathsf{lo}} = \max\{0, V - \theta_{\mathsf{up}}\,\mathbf{E}\left[T(\lambda)\right]\},$$
$$S = 2\lambda\theta_{\mathsf{up}}(V-c)\,\mathbf{E}\left[X_e^2\right] + \mathbf{E}\left[X_e\right]^2\Big(\lambda(c-V) + \theta_{\mathsf{up}}\Big)^2.$$

The revenue-optimal solution c_{R} of the problem (21) can be solved efficiently using numerical methods for a single variable [18]. When the SP uses this c_{R} for admission pricing and all SUs employ the individual optimal strategy, the corresponding revenue-optimal equilibrium joining probability will be $p_{\mathsf{R}} \triangleq p^*(c_{\mathsf{R}})$.

B. Socially Optimal Pricing

When the SP charge a price c, an equilibrium social welfare, which consists of the aggregate utility obtained by the SUs can be described as

$$S(c) = \int_0^{\tilde{\theta}(c)} \big(V - \theta\, \mathbf{E}\left[T(\lambda p^*(c))\right]\big)\, dF_\Theta(\theta), \tag{23}$$

where

$$\tilde{\theta}(c) = \frac{V - c}{\mathbf{E}\left[T(p^*(c))\right]}, \tag{24}$$

which can be seen as a cut-off SU at price c, meaning that only the SU who has its θ less than $\tilde{\theta}(c)$ decides to join the system. Then the problem of optimal social welfare pricing can be cast as

$$\max_{c \in [c_{\mathsf{lo}}, c_{\mathsf{up}}]} S(c). \tag{25}$$

However, solving this problem is difficult because of the complex functions $\tilde{\theta}(c)$ and $p^*(c)$. Observing that

$$p^*(c) = F_\Theta(\tilde{\theta}(c)) = \frac{\tilde{\theta}(c)}{\theta_{\mathsf{up}}}, \tag{26}$$

and the condition $\lambda p^*(c) < \frac{1}{\mathbf{E}[X_e]}$ must be satisfied, we formulate an equivalent maximization problem of (25) as follows after changing the optimization problem's variable from c to the cut-off parameter $\tilde{\theta}$

$$\max_{\tilde{\theta} \in \left[0, \frac{\theta_{\mathsf{up}}}{\lambda\, \mathbf{E}[X_e]}\right)} S(\tilde{\theta}), \tag{27}$$

where

$$\begin{aligned} S(\tilde{\theta}) &= \int_0^{\tilde{\theta}} \left(V - \theta\, \mathbf{E}\left[T\left(\lambda F_\Theta(\tilde{\theta})\right)\right]\right) dF_\Theta(\theta) \\ &= V F_\Theta(\tilde{\theta}) - \frac{\tilde{\theta}^2}{2\theta_{\mathsf{up}}}\, \mathbf{E}\left[T\left(\lambda F_\Theta(\tilde{\theta})\right)\right]. \end{aligned} \tag{28}$$

The optimal solution of the problem (27), denoted by θ_{S}, can be solved efficiently using numerical methods for a single variable [18]. From (24), the socially optimal price c_{S} can be calculated as follows

$$c_{\mathsf{S}} = V - \theta_{\mathsf{S}}\, \mathbf{E}\left[T(\lambda F_\Theta(\theta_{\mathsf{S}}))\right], \tag{29}$$

VI. Numerical Results

In this section, we apply the analysis results to numerically illustrate the SUs' equilibrium behaviors and the optimal pricing mechanisms. To facilitate the illustration, the parameter settings adhere to the following order of Exp, Erl, UniExp, ErlExp and ExpErl examples. Furthermore, we set $V = 1$, $\theta_{\mathsf{up}} = 1$ and $\alpha = 0.3$.

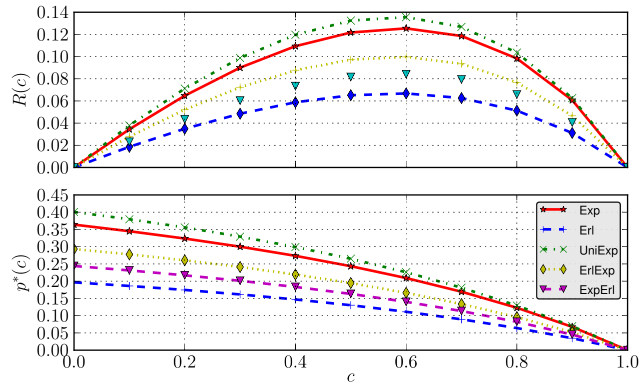

Fig. 3. Revenue and equilibrium joining probability as functions of admission price.

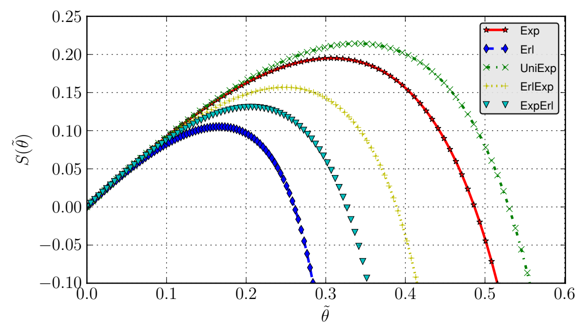

Fig. 4. Social welfare as a function of cut-off user.

A. Revenue Optimization

In the top part of Fig. 3, we show graphs of the SP's revenue where the choice variable is taken as the price c. We can see that the revenue functions have their maximum values are achieved nearly at the same price (0.58) with the corresponding revenues $0.13, 0.07, 0.14, 0.1$ and 0.08. The equilibrium joining probability $p^*(c)$ is plotted in the bottom part of Fig. 3. At the price $c_{\mathsf{R}} = 0.58$, we can see that the corresponding $p^*(c_{\mathsf{R}})$ are $0.21, 0.11, 0.23, 0.17$ and 0.14 with respect to the order of example settings. This plot also demonstrates that when c is increased, $p^*(c)$ is decreased.

B. Social Optimization

The network social welfare as a function of cut-off user $\tilde{\theta}$ is shown in Fig. 4. It can be seen that the socially-optimal cut-off SUs θ_{S} are $0.3, 0.16, 0.33, 0.25$ and 0.21, and the corresponding socially-optimal values $S(\theta_{\mathsf{S}})$ are $0.19, 0.1, 0.21, 0.16$ and 0.13 with respect to the order of the example settings. The respective socially-optimal prices c_{S} can be calculated according to (29). Compared with the bottom plot of Fig. 3, these prices map correctly with the corresponding values of $p^*(c_{\mathsf{S}})$, which is equal to θ_{S} since $\theta_{\mathsf{up}} = 1$.

C. Equilibrium Convergence Dynamics

With the starting point p^0 set to zero, the equilibrium convergence of all settings using static and adaptive expectations

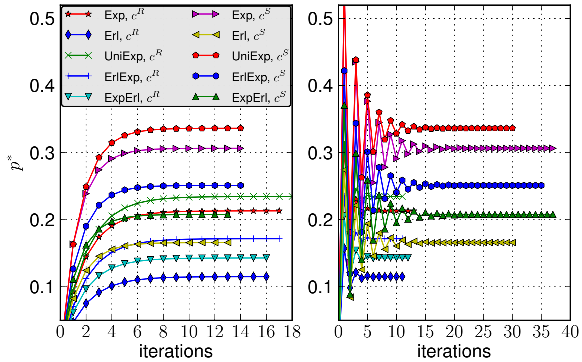

Fig. 5. The convergence of equilibrium joining probabilities $p^*(c_\mathsf{R})$ and $p^*(c_\mathsf{S})$ with static expectations (left plot) and adaptive expectations (right plot).

are illustrated in Fig. 5. Although the condition in (20) is violated in all five settings (i.e. $\lambda > 1/\mathbf{E}[X_e]$), it can still be seen that all joining probabilities converge to the expected equilibrium points presented previously as Theorem 2 gives a sufficient but not necessary condition.

VII. Conclusion

This paper studied the pricing effect on equilibrium behaviors of non-cooperative SUs who share a single PUs' channel. With SUs' individual optimal strategy, first, the existence and unique of a equilibrium was first examined. Second, the average queueing delay was derived using renewal theory. And third, a sufficient condition for the equilibrium convergence is provided. The SP's pricing were considered for two problems: revenue maximization and social welfare maximization. An inherent future work will focus on the multiple licensed channels analysis. Another interesting direction is the study of price competition between two or multiple SPs.

Appendix A
Proof of Theorem 2

In adaptive expectations, the equilibrium point can be achieved and is stable for any initial $p^{(0)}$ if the following condition is satisfied

$$\alpha\left|q'(p)\right| < 1, \quad \forall p \in [0,1], \tag{30}$$

which implies that $\alpha q(p)$ is a contraction mapping [18]. According to the definition $q(p^{(t)}) = F_\Theta\left(\frac{V-c}{\mathbf{E}\left[T(\lambda p^{(t)})\right]}\right)$, we have

$$\begin{aligned}\left|q'(p)\right| &= \left|f_\Theta\left(\frac{V-c}{\mathbf{E}\left[T(\lambda p)\right]}\right)\left(\frac{V-c}{\mathbf{E}\left[T(\lambda p)\right]}\right)\frac{\frac{d}{dp}\mathbf{E}\left[T(\lambda p)\right]}{\mathbf{E}\left[T(\lambda p)\right]}\right| \\ &\le \left|\max_{\theta\in[0,\theta_{\mathsf{up}}]} f_\Theta(\theta)\theta\right|\left|\frac{\frac{d}{dp}\mathbf{E}\left[T(\lambda p)\right]}{\mathbf{E}\left[T(\lambda p)\right]}\right| = \frac{\frac{d}{dp}\mathbf{E}\left[T(\lambda p)\right]}{\mathbf{E}\left[T(\lambda p)\right]} \\ &= \frac{\lambda\,\mathbf{E}\left[X_e^2\right]}{\left(\lambda p\,\mathbf{E}\left[X_e\right]-1\right)\left(2\lambda p\,\mathbf{E}\left[X_e\right]^2 - \lambda p\,\mathbf{E}\left[X_e^2\right] - 2\,\mathbf{E}\left[X_e\right]\right)}\end{aligned} \tag{31}$$

where the third equality happens because $\mathbf{E}\left[T(\lambda p)\right]$ is a positive and increasing function, and $\max_{\theta\in[0,\theta_{\mathsf{up}}]} f_\Theta(\theta)\theta = 1$. From (30) and (31), we have the result.

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