Spectrum Handoff Model Based on Hidden Markov Model in Cognitive Radio Networks

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***Abstract*—Cognitive Radio Network (CRN) is one of technologies to enhance the spectrum utilization by allowing unlicensed users to exploit the spectrum in an opportunistic manner. In CRN, the spectrum handoff function is a necessary component to provide a resilient service for the unlicensed users. This function is used to discover spectrum holes in a licensed network and avoid interference between unlicensed users and licensed users. Due to the randomness of the appearance of Primary users, disruptions to communications of Secondary users are often difficult to prevent and lead to low throughput of CRN. In our paper, we analyze the status of channels and propose the spectrum handoff model based on Hidden Markov model (HMM) to optimize the spectrum handoff scheme for CRN. Moreover, we compare our method with the random channel selection in the simulation.**

***Keywords*—Cognitive radio network, Cognitive Radio, Hidden Markov Models, Spectrum sensing, Viterbi Algorithm, Forward-backward procedure**

I. INTRODUCTION

The main concept of Cognitive radio network is introduced by Joseph Mitola in [1]. Cognitive Radio (CR) refers to the potential wireless systems in that the utilization spectrum hole is based on the context-awareness and capable of reconguration based on the surrounding environments. A cognitive radio network (CRN) is built on the following principle: a network of secondary users (users without license) continuously senses the usage of a spectrum band by primary users and opportunistically utilizes the band when primary users are absent. Any secondary user (SU) in a CRN performs two main functions: (1) sensing spectrum usage to identify the presence of a primary user (PU), and (2) transmitting at appropriate power when the channel is idle. PUs of the licensed system have higher priority to use the frequency bands, while SUs of the cognitive radio system may access frequency bands which have already been licensed but not occupied fully by PUs. When a PU appears, SUs have to vacate the frequency channel to avoid interference to PUs and claim to another channel.

One critical challenge is that SUs should avoid causing harmful interference to primary users (PUs), and support seamless communications regardless of the appearance of PUs. Therefore, the most important function of CRN is spectrum mobility which refers to the capability of SUs to switch idle channels. Spectrum mobility gives rise to a new type of handoff called spectrum handoff which refers to the process that when a SU is interrupted by the occupancy of PUs, the SU must determine switching to a new idle channel to continuously transmit data. However, there is no negotiation between PUs and CRN, the CRs just gather information from spectrum sensing. Various models have been proposed to analyze the function of spectrum handoff with two categories: reactive approach and proactive approach [2]. Reactive approach bases on the result of sensing channels then select the channel to switch. Another approach, SUs predict the status of PUs behavior in the future and perform spectrum handoff before the disruptions by PU transmissions, namely the proactive approach [2].

However, there are still limitations in the analysis of proactive approach. That is the reason why proactive spectrum handoff is still a promising approach to increase efficiently the exploration spectrum holes. Our objective concentrates on modeling a system with all components from sensing channels to transmitting data in CRN. In this system, we propose an heuristic algorithm to decrease sensing overhead in proactive spectrum sensing, and analyze the status of channels in spectrum handoff. From these analyses, we regard deeply about the status on each channel and use Hidden Markov Model (HMM) in correcting spectrum sensing sequence and predicting which can enhance the spectrum-opportunities of SUs.

The rest of the paper is organized as follows. The related works are reviewed in Section II. The nework model and necessary preliminaries are presented in Section III. Section IV presents our proposed model based on Hidden Markov model. The simulation results are reported in Section V and the conclusion of the paper is given in Section VI.

II. RELATED WORKS

Spectrum handoff plays a critical role in enhancing the efficiency of wireless resource utilization. Therefore, analytical models in this area often focus on the reactive approach and especially the proactive approach. In [3], the authors were given a proactive sensing model which was performed on Orthogonal Frequency Division Multiplexing systems (OFDM). They considered a consistent period sensing on each channel and optimized this. However, the arrival rate of PU is different from other channels. Hence, the opportunities for transmitting of SUs will be affected by this important parameter. From the recent research [4] in 2012, a spectrum handoff framework was proposed for cognitive radio ad hoc networks. The authors represented a model with the PU traffic model is known and the channel statistics (e.g, PU packet arrival rate, PU packet length) are obtained from the scanning radio [5], [6]. More specifically, we also take into account the period sensing parameter and use their assumptions and results to develop and simulate for our prediction model.

Furthermore, to enhance the discoverable spectrum-opportunities, many studies have focused on the channel quality prediction for CRN, which is called the proactive approach. These models are usually based on the past channel usage for forecasting the dynamic spectrum access or spectrum handoff. In [2], the authors considered the retransmissions of the collided packets in discrete-time Markov chain. In [7], a preemptive resume priority queueing model is analyzed for the total service time of SU, but the authors just considered only one pair of SUs in a network. In [8], [9], a dynamic model for CR network base on stochastic queue analysis is represented the steady-state queue length of SUs. In [10], [11], [12], the authors showed probabilistically model the errors and then formulated a spectrum sensing paradigm as a Hidden Markov model and Bayesian model that predicts the true states of a channel. From these analysis, we regard deeply about the status on each channel and use HMM in prediction which can increase the spectrum-opportunities of SUs.

III. PRELIMINARIES

A. Network topology

In this paper, without loss of generality, we assume that a network scenario has a group of SUs which is controlled by a base station (BS). And, there is no other secondary networks (SNs) collision or cooperating with the the secondary network. We also suppose that every SU in SN is equipped with a unique antenna that can be tuned to any combination of N consecutive licensed channels. Moreover, in our model, all SUs in an SN should participate in sensing a channel at the same time for each scheduled measurement period to enhance the detection of PU signal even. Sensing information will be collected and coordinated by a BS. So that, all cognitive functions are centralized and maintained by a BS. Our prediction model focuses on the selected channel scheme of SN which avoids the interference from PUs, and gets more opportunities to transmit data.

B. Channel-usage model

Spectrum sensing mainly focuses on checking a channel's availability. For channel i $(i = 1, 2, 3..., N)$, we model a channel as an ON-OFF source alternating between ON (busy) and OFF (idle) period [5], [13], [14], [15]. Whenever sensing is performed on a channel and an opportunity on the channel is discovered, the channel is merged into a pool of available channels. This can be done by using the OFDM technique with selective allocation of sub-carriers to the channels to be utilized.

In proactive sensing method [3], each channel should be sensed periodically with its own sensing period T_P^i. An example about sensing period is shown in Fig. 1. Although the periodic sensing is performed on every channel independently, the concurrent sensing of N channels must be scheduled in such a way that there would be no other scheduled sensing while a measurement on channel i is being performed. On each channel, T_P^i is a constant and depends on the average fraction of busy time of channel i. In this paper, we proposed an algorithm for sensing spectrum with the dynamic sensing spectrum period and optimized the number of sensing times to reduce the overhead on each channel.

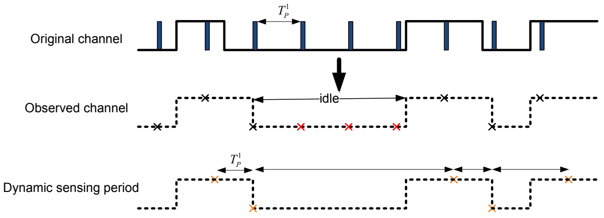

Fig. 1. The observed channel-usage pattern model.

C. Proactive spectrum handoff

In this part, we review the analysis in [4] and use this model in our paper. According to the formula $(6), (7)$ in [4], the policy about switching of SUs to a new channel is based on the probability that channel i is idle at time t $\Pr(N_i(t) = 0) < \tau_L$, where τ_L is the probability threshold below which a channel is considered to be busy and the SU needs to carry out a spectrum handoff. And the policies that a channel j becomes a candidate channel at time t are

$$\begin{cases} \Pr(Nj(t) = 0) \geq \tau_H, \\ \Pr(t_{j,off} > \eta | N_j(t) = 0) \geq 0, \end{cases}$$

where $t_{j,off}$ represents the duration from t to the beginning of the next PU packet on channel i, and $\Pr(t_{j,off} > \eta | N_j(t) = 0)$ is the probability that the duration of idleness is longer than η given that the channel is idle at t. If the PU traffic model is known and the channel statistics (e.g., PU packet arrival rate, PU packet length) are obtained from the scanning radio, based on this analysis, we proposed a policy that chooses a list of candidate channels which SUs select for switching.

IV. SENSING SPECTRUM ALGORITHM AND SPECTRUM HANDOFF MODEL

In this section, we concentrate on two problems: proposing the heuristic algorithm in sensing spectrum and representing the spectrum handoff model.

A. Heuristic algorithm in sensing spectrum

In proactive sensing, we take into account the times of sensing in the idle duration. Because, in the sensing period, SU cannot transmit data. In proactive sensing, the sensing still occurs again and again although the channel is still idle. It wastes more time for sensing and increases sensing overhead. Fig.1 illustrates the observed channel which SUs use to control transmitting data. The unexplored opportunity depends on the parameter T_P^i. Therefore, we state that when SU discovers an idle channel, the transmission will be started and stopped the sensing demand. This way can improve the transmission time and not interrupt SU. When the channel is reclaimed by PU, the timer of sensing will be started again.

In addition, from the analysis in [10], we use the history of traffic of channels and collect the channel statistics from the scanning radio. On each channel, a sensing period T_P^i is assigned suitably with the average of PU packet arrival rate and PU packet length ℓ. During the sensing and transmitting, the parameter T_P^i will be updated continuously (increase T_P^i if the busy duration is greater than T_P^i and decrease T_P^i if the

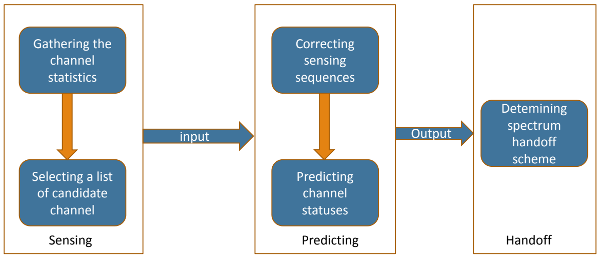

Fig. 2. The observed channel-usage pattern model.

idle duration is least than). Fig. 1 shows the sensing overhead and the result of heuristic algorithms in detecting spectrum.

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Input: on channel i, assign $T_P^i$ with the threshold $\ell$.
    Suppose that, in the first sensing, the channel is
    busy. If the channel is idle, waiting for a random
    time to the next busy period, then starting the
    sensing
Result: Updating the sensing period
initialization: in the first sensing $T_P^i = \ell$ ;
while not at end of sensing do
    In sensing times $k^{th}$: Set timer t=$T_P^i$;
    if the channel is idle then
        $T_P^i = T_P^i + 1$;
        Stop countdown the timer;
        Wait for the channel be busy to countdown the
        timer t to 0;
        Start sensing;
    end
    if the channel is busy then
        if $T_P^i > \ell$ then
            $T_P^i = T_P^i - 1$;
        end
        Countdown the timer t to 0;
        Start sensing;
    end
end
```

Algorithm 1: Updating the sensing period

B. The proposed Hidden Markov Model

In traditional methods, the authors often assume that SUs can sense all the channels simultaneously and make the correct decisions. In reality, the true states (occupancy by PUs) of each channel are never known to the SUs. Moreover, there are some errors in detecting the using of PU such as mis-detect and false alarm. Fig. 3 shows an example of mis-detect and false alarm in channel i and $i+2$. Reducing these errors and predicting behaviors of PU are the main purposes of this section. Based on capacity of correcting and predicting of HMM, in our method, we propose a new HMM for spectrum handoff to exploit a novel idea defining the emission probabilities. This HMM is considered deeply the true states of each channel, which is occupied by PUs or SUs.

Our model includes three components which are showed in Fig. 2. The first component is used to sensing channels with two functions: gathering the historical spectrum usage in the past and sensing. Then, we apply the algorithm, proposed in [4], to select a list of candidate channels which are used in spectrum handoff. All candidates are the input of the second component. As show in Fig. 2, this component uses HMM to correct sensing sequences then perform a prediction function. The output of predicting component is used to decide the spectrum handoff in SN, operated in third component. Such, difference from other papers, our model use the historical spectrum usage in the past to filter in handoff. Only the candidate channels, which are satisfy the threshold (i.e. more opportunity), are input into the prediction component. The number of channels which will be used in spectrum handoff of SUs in the network are reduced, so that the calculation in HMM does not increase exponentially with the number of channels. Furthermore, the combination of two methods makes the prediction of HHM more accuracy. To describe our model clearly, in this part, we represent two functions: correcting sensing sequences and predicting channel status.

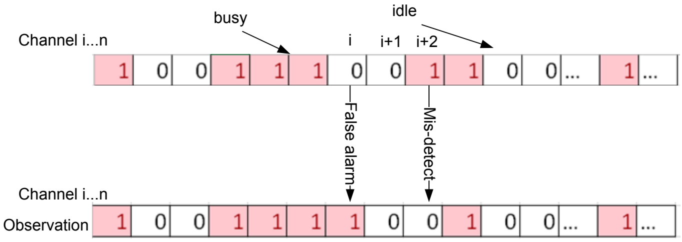

Fig. 3. Description of mis-detect and false alarm.

As you may know about HMM which can solve three basic problems: model evaluation, most probable path decoding, and model training. In this paper, we are inspired by the work in [4], [10] to find the ”correct” state sequence (i.e correct the mis-detect and false alarm) and proposed a new HMM to compute the probability of observed sequence. Hence for practical situation, we used Viterbi algorithm to correct the sensing sequences, and forward-backward procedure to predict statuses.

1) Correcting sensing sequences: Fig. 3 illustrates an example about mis-detect and false alarm. Whereas the channel i is idle, the sensing result is busy, and in otherwise, the same error is called mis-detection on channel $i+2$. To enhance accuracy of the detecting process, we apply Viterbi algorithm to find the correct state sequence [4].

In the above derivation, just K candidates channels are input into the second component. Hence, the number of channel occupancy states 2^K, the number of distinct observation symbols per state 2^K. That is the reason of the computation in this component, which is enhanced more efficiently than [4].

2) Prediction channel status: The next step in our model is to predict channel statuses for each sensing times. This is the most important to determine the spectrum handoff of SUs. Given the sensing sequences, we use forward-backward algorithm to find the most probable path. However, to make more accuracy in prediction, we consider four hidden statuses which are more complex than [11]. Based on the time slotted channels, we define parameters for our model:

a) An observation period $\tau = \{1, 2, ..., T\}$, where i represents the i^{th} slot time. For each time slot, the observed status on each channel is busy or idle.

b) In many previous researches about spectrum handoff, SUs often switch into another channel when PU reclaims the channel. However, if this channel has more opportunities to transmit data and the probability of idle state is higher than others, SUs maybe still stay on this channel to wait a new chance which can transmit again without handoff. Therefore, we define four states of the channel which will be described below.
A sequence $Y = \{\mathrm{y}_1, \mathrm{y}_2, ..., y_T\}$, which represents the hidden states in the corresponding time periods. At the i^{th} time instant, each channel has four states:

- $y(0,0)$ if the channel is idle and SU does not occupy on this channel.
- $y(0,1)$ if the channel is idle and SU is occupying on this channel.
- $y(1,0)$ if the channel is busy (PU claims the channel) and SU does not occupy on this channel,
- and $y(1,1)$ if the channel is busy (PU claims the channel) and SU is occupying on this channel

As we considered above, in spectrum handoff, SU may still stays on the current channel to wait for new chance of transmitting or switch into the new busy channel but has more opportunities to transmit data. Hence, the probability of spectrum handoff can be represent in some special probability values:
$\Pr(y(0,1)|y(0,0)), \Pr(y(0,1)|y(1,0))$: *probability of event that SU occupies the new idle channel.*
$\Pr(y(0,1)|y(0,1)), \Pr(y(0,1)|y(1,1))$: *probability of event that SU still stays on the idle channel, given the previous state is idle or busy.*
$\Pr(y(1,1)|y(0,0)), \Pr(y(1,1)|y(1,0))$: *probability of event that SU occupies a new busy channel, given the previous state is idle or busy.*
$\Pr(y(1,1)|y(0,1)), \Pr(y(1,1)|y(1,1))$: *probability of event that SU still stays on the busy channel to wait for a new chance.*
$\Pr(y(0,0)|y(0,1)), \Pr(y(0,0)|y(1,1))$: *probability of event that SU releases the channel and switch into another.*
$\Pr(y(1,0)|y(0,1)), \Pr(y(1,0)|y(1,1))$: *probability of event that SU releases the channel and switch into another, given the previous state is idle or busy.*

c) The CR sensing output is represented by a sequence $X = \{x_1, x_2, ..., x_T\}$ of sensed states in the corresponding time periods. We also has four states of sensing at the ith sensing slot: the entity $x(0,0)$ if the state of the channel is sensed to be free and the SU is not in it; $x(0,1)$ when the SU is occupying and sensing on the free; entity $x(1,0)$ is the state of busy channel is sensed by the CR and it is not occupied by any SU and entity $x(1,1)$ is the state of sensing is the busy channel is occupied by the SU.

d) In this paper, we assume that all information are controlled by a BS so that the SN can have information about the channel which is occupied by SUs.

e) The state transition probability distribution $P = p_{ij}$ where

$$\Pr(y_n = j|\mathrm{y}_1 = \mathrm{i}_1, ..., y_{n-2} = i_{n-2}, y_{n-1} = i) \\ = \Pr(y_n = j|y_{n-1} = j) = p_{ij} \tag{1}$$

for every $\mathrm{i}_1.\mathrm{i}_2...., i_{n-2}, i, j \notin S$ and $2 \leq n \leq T$ with state space $S = \{S_i\}$ (in our model, we consider four states of y_i)

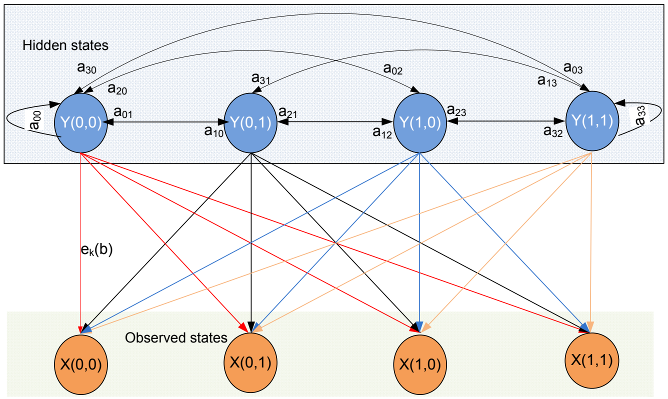

Fig. 4. Description of mis-detect and false alarm.

f) The observation symbol probability distribution in state k, $E = \{e_k(b)\}$: *emission probability*, where

$$e_k(b) = \Pr(X_n = b|Y_n = k) \tag{2}$$

g) When the SUs sense the channels, they can be prone to errors. The probability of predicting a state to be free when it is reality occupied is the probability of mis-detection (PMD) denote by e_m. In addition, probability of predicting a state to be occupied when it is free is known as probability of false-alarm (PFA) denoted by e_f. As mentioned earlier, the BS controls spectrum handoff so that the SN knows clearly about the information of occupancy by SUs. Therefore, the emission probability is specified $k = 1,2,3,4$: *the hidden state of system* and $b = 1,2,3,4$: *is the observation state*

h) The initial state distribution $\pi = \pi_i$ where $\pi_i = P[y_1 = S_i],\ 1 \leq i \leq 4$

In this component, we apply the Forward-backward procedure [16]. Consider the forward variable

$$\alpha_t(i) = P(X_1, X_2, ...X_t, y_t = S_i|\lambda) \tag{3}$$

The forward probability calculates the

$$P(X|\lambda) = \sum_{i=1}^{N} \alpha_T(i) \tag{4}$$

from the initialization $\alpha_1(i) = \pi_i e_i(X_1)$.
In backward, we also consider a backward variable

$$\beta_t(i) = P(X_{t+1}X_{t+2}...X_T|y_t = S_i, \lambda) \tag{5}$$

From the initialization $\beta_T(i) = 1$, we can solve for $\beta_t(i) = 1$ inductively:

$$\beta_t(i) \sum_{j=1}^{N} p_{ij} e_j(y_{t+1}) \beta_{t+1}(j) \tag{6}$$

$t = T-1, T-2, .., 1, 1 \leq i \leq N$

TABLE I. PROBABILITY OF IDLE IN THE CHANNEL

Channel	1	2	3	4	5	6	7	8	9	10
Probability	0.56	0.63	0.45	0.76	0.48	0.46	0.55	0.65	0.55	0.36

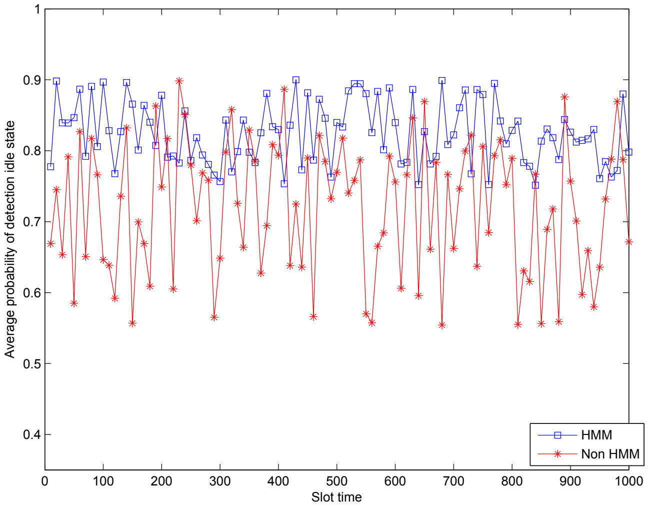

Fig. 5. Average probability of detection.

Following the algorithm, applied into our model, we computed the probability of the sensing sequence from all candidate channels. Furthermore, to make the prediction of HMM more accuracy, we use the analysis in gathering the channel statistics component to assign suitable values to probabilities transition matrix on each channel which represent the capability of idle.

V. SIMULATION AND NUMERICAL

To evaluate our model, we simulate the system with 10 license channels and 1000 times of sensing. About sensing of SUs, we define $T_P^i = 0.5s$, PMD e_m=0.05 and PFA e_f=0.05. In each channel, we consider the status ON/OFF based on the Poisson distribution probability.And, the random selection scheme in spectrum handoff is simulated to compare with our model. When performing, a random channel is selected for SUs switch into.

With different frequencies of 10 channels (showed in Table.1), we calculate the average probability detection in HMM and non using HMM (i.e. using the random selection scheme). Based on the probability in Table. 1, we assign the values in the transition matrix which depend on the probability of idle of each channel. From Fig. 5, it shows that the capacity for detecting idle state in HMM is higher than non using HMM. Moreover, the fluctuation in 1000 times of sensing is lower than using random selecting scheme. It indicates that the our model detects idle statuses and corrects errors more accurately. In addition, we measure the ability of correcting mistakes in the proposed model by the probability of mis-detect and false alarm. To have this report, we simulate our system with 1000 times of sensing and compare the true channels with the observation channels. Fig. 6 shows the decreasing of mis-detect and false alarm in our model when using HMM. This reports the effect of the correcting sensing sequence of all channels.

The result also deems that HMM can provide more transmission opportunities to SUs or SUs can detect these opportunities more accurately. However, the capacity of correcting mistakes in sensing is not still stable (showed in Fig. 6 with the large fluctuation of the probability of mis-detect and false alarm), and our model just simulates with 10 channels. This is weakened with an increase in the number of channels.

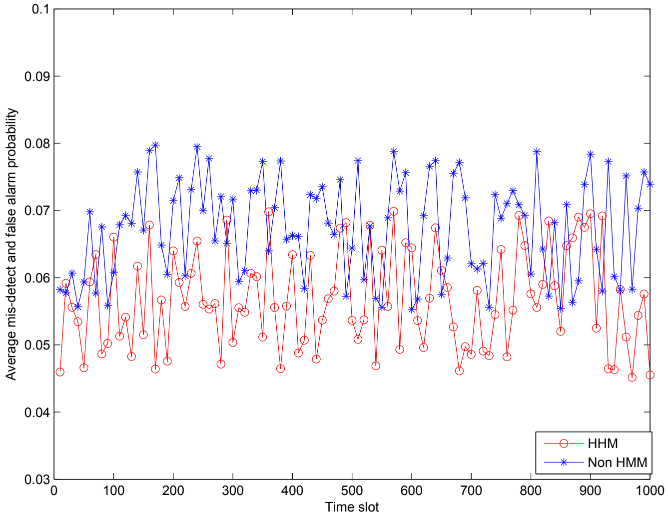

Fig. 6. Average probability of mis-detect and false alarm.

VI. CONCLUSION

In this paper, we propose a model spectrum handoff in CRN based on HMM to analyze the state of channel in each slot time. The proposed approach adaptively infers the efficient of HMM in correcting the sensing sequence and prediction the channel status. In addition, we also perform a heuristic sensing algorithm in proactive sensing algorithm and use filtering method to improve the performance of our system. The analysis and simulation show that our proposed system is adaptable and can be applied to the spectrum mobility function of CRN. Finally, although we focus on the spectrum handoff scenario in CRNs, our model is regardless the collision between PUs and SUs, SUs and SUs which is one of the problems in spectrum handoff.

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