LETTER

# Utility Maximization with Packet Collision Constraint in Cognitive Radio Networks*

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**SUMMARY** We study joint rate control and resource allocation with a packet collision constraint that maximizes the total utility of secondary users in cognitive radio networks. We formulate and decouple the original optimization problem into separable subproblems and then develop an algorithm that converges to optimal rate control and resource allocation. The proposed algorithm can operate on different time-scales to reduce the amortized time complexity.

***key words:*** *utility maximization, rate control and resource allocation, cognitive radio networks*

## 1. Introduction

Cognitive radio networks can help alleviate the conventional spectrum scarcity and improve the utilization of the existing spectrum [5]–[7]. The key point of cognitive networks is to allow the secondary users (SUs) to access to the spectrum of legacy primary users (PUs) opportunistically without interfering with the PU. Hence, the standard spectrum access strategy in cognitive networks is to maximize the total utility of SUs while still guaranteeing the protection requirement of PUs.

Resource allocation for maximizing utility has been widely investigated for wireless networks [3] yet rarely for cognitive networks. In this letter, we propose a utility maximization framework that takes into account the PU protection metric for cognitive networks. Here we choose packet collision probability as the metric for PU protection; it is widely used in the research community [5], [7]. We decouple the utility optimization into joint rate control and resource allocation subproblems, where SUs can solve the rate control problem distributively while resource allocation is solved by the base station (BS) in a centralized manner. The resource in this context is the spectrum that would be allocated to SUs. We first provide the optimal resource allocation algorithm which entails high computational complexity. Next, we propose a near-optimal algorithm that offers significantly less complexity than the optimal algorithm, which makes it more practical and robust in dynamic environments.

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## 2. System Model and Problem Definition

We consider a multi-channel spectrum sharing cognitive radio networks comprising a set of SUs' node pairs $\mathcal{M} = \{1, 2, \ldots, M\}$. Each SU's node pair consists of one dedicated transmitter and its intended receiver. SUs share a common set of $\mathcal{K} = \{1, 2, \ldots, K\}$ orthogonal channels with PUs and the PUs can send their data over their own licensed channels to the BS simultaneously. Each SU is assumed to have a utility function $U_m(x_m)$, a function of the flow rate $x_m$, which can be interpreted as the level of satisfaction attained by SU $m$ [3]. A large class of user fairness can be characterized by the following general utility function [4] parameterized by $\kappa$

$$U_m^{\kappa}(x_m) = \begin{cases} (1-\kappa)^{-1} x_m^{1-\kappa}, & \text{if } \kappa \geq 0, \kappa \neq 1 \\ \log x_m, & \text{if } \kappa = 1. \end{cases} \quad (1)$$

For example, it provides proportional fairness with $\kappa = 1$, harmonic mean fairness with $\kappa = 2$ and max-min fairness with $\kappa \to \infty$.

Fixed link capacities of SU's and PU's are denoted by $c_m$ and $c_k$, respectively. $\gamma_k$ is the maximum probability that a PU $k$'s packet can have a collision, which is set at the BS *a priori*. Hence the maximum packet collision rate that a PU $k$ can tolerate is $\gamma_k c_k$. We denote the probability that channels are idle (i.e. channels are not occupied by PUs) by the vector $\boldsymbol{\pi} = (\pi_1, \pi_2, \ldots, \pi_K)$, which is achieved by SUs through the knowledge of traffic statistics and/or channel probing [7].

### 2.1 Primal Problem

We formulate the utility maximization problem for SUs in a cognitive radio network as the followings:

(**P**):

$$\underset{\boldsymbol{x},\boldsymbol{\phi},\boldsymbol{e}}{\text{maximize}} \quad \sum_m U_m(x_m) \quad (2)$$

$$\text{subject to} \quad x_m \leq \sum_k c_m \pi_k \phi_{mk}, \quad \forall m \quad (3)$$

$$e_k \leq \gamma_k c_k, \quad \forall k \quad (4)$$

$$\sum_m \phi_{mk} = 1, \ \sum_k \phi_{mk} = 1 \quad \forall m, k, \quad (5)$$

$$0 \leq x_m \leq x_m^{max}, \quad \forall m \quad (6)$$

where $x_m^{max}$ is the maximum data rate of SU $m$, $e_k$ represents the collision rate of a PU and $\phi_{mk}$ is the probability that a given channel $k$ is allocated to SU $m$. Define an allocation function at any time instant $t$ as follows:

$$I_{mk}(t) = \begin{cases} 1 & \text{if channel } k \text{ is allocated to } m \text{ at } t \\ 0 & \text{otherwise.} \end{cases} \quad (7)$$

Then we have $\phi_{mk} = \lim_{t\to\infty} \frac{1}{t} \sum_{\tau=0}^{t-1} I_{mk}(\tau)$. The utility function of each SU is assumed to be increasing and strictly concave. Constraint (3) ensures that the source rate on a SU link cannot exceed its attainable link rate with channel-occupancy information. (4) is precisely the collision constraint. Constraint (5) allows at most one SU to be allocated to channel $k$ and at most one channel $k$ to be allocated to one SU at any time instant. It is straightforward that (**P**) is a convex optimization problem.

### 2.2 Dual Problem

In order to use the duality approach for solving problem (**P**), we first form the partial Lagrangian:

$$L(\boldsymbol{x}, \boldsymbol{e}, \boldsymbol{\phi}, \boldsymbol{\lambda}, \boldsymbol{\mu}) = \sum_m U_m(x_m) + \sum_k \mu_k(\gamma_k c_k - e_k) + \sum_m \lambda_m \left( \sum_k c_m \pi_k \phi_{mk} - x_m \right), \quad (8)$$

where $\boldsymbol{\lambda} = (\lambda_m, m \in \mathcal{M}) \geq 0$ and $\boldsymbol{\mu} = (\mu_k, k \in \mathcal{K}) \geq 0$, the Lagrange multipliers of constraints (3) and (4), are considered as the congestion price and collision price respectively. The dual objective function is:

$$D(\boldsymbol{\lambda}, \boldsymbol{\mu}) = \max_{\boldsymbol{x},\boldsymbol{e},\boldsymbol{\phi}} \quad L(\boldsymbol{x}, \boldsymbol{e}, \boldsymbol{\phi}, \boldsymbol{\lambda}, \boldsymbol{\mu}) \quad \text{subject to} \quad (5), (6) \quad (9)$$

Then, the dual optimization problem is:

$$(\mathbf{D}): \quad \underset{\boldsymbol{\lambda}\geq \mathbf{0}, \boldsymbol{\mu} \geq \mathbf{0}}{\text{minimize}} \quad D(\boldsymbol{\lambda}, \boldsymbol{\mu})$$

Given the assumptions on utility function, it is not difficult to see that Slater condition is satisfied, and strong duality holds [1]. This allows us to solve the primal via the dual.

## 3. Joint Rate Control and Resource Allocation with Packet Collision Constraint

In this section, we present the optimal algorithm and near-optimal algorithm. We first show the decomposition structure of the current issue.

### 3.1 Decomposition Structure

Note that by the definition of $e_k$, we have a relationship:

$$e_k = \sum_m \phi_{mk}(1 - \pi_k)c_k. \quad (10)$$

By substituting (10) into (8) and rearranging the order of summation, we can decompose (9) into the following two subproblems (partial dual functions):

$$D_x(\boldsymbol{\lambda}) = \max_{0 \leq \boldsymbol{x} \leq \boldsymbol{x}^{max}} \quad \sum_m [U_m(x_m) - \lambda_m x_m] \quad (11)$$

and

$$D_\phi(\boldsymbol{\mu}) = \max \sum_m \sum_k \phi_{mk} \left[ \lambda_m \pi_k c_m - \mu_k (1 - \pi_k) c_k \right] \quad \text{subject to} \quad \sum_m \phi_{mk} = 1, \ \sum_k \phi_{mk} = 1 \quad \forall m, k. \quad (12)$$

The maximization problem (11) can be conducted in parallel and in a distributed fashion by SUs. In contrast, if we consider (12) at an arbitrary time instant $t$, we have the equivalent problem:

$$\max \quad \sum_m \sum_k I_{mk}(t)[\lambda_m(t) \pi_k c_m - \mu_k(t)(1 - \pi_k)c_k] \quad \text{subject to} \ \sum_m I_{mk}(t) = 1, \ \sum_k I_{mk}(t) = 1, \quad \forall m, k, \quad (13)$$

which is a combinatorial optimization problem that needs to be solved in a centralized fashion by the BS. This problem is the Maximum Weighted Bipartite Matching problem on an $M \times K$ bipartite graph between $M$ secondary users and $K$ channels where the weight of the edge between SU $m$ and channel $k$ is $\lambda_m(t) c_m \pi_k - \mu_k(t)(1 - \pi_k)c_k$.

### 3.2 Optimal Algorithm

For $\boldsymbol{\lambda}$ fixed, (11) can be decomposed into $m$ independent maximization problems, and each of them is solved by taking the derivative as follows

$$\frac{\partial D_x(\boldsymbol{\lambda})}{\partial x_m} = U_m'(x_m) - \lambda_m = 0, \ 0 \leq x_m \leq x^{max}. \quad (14)$$

Solving (14), we have the optimal solution of (11)

$$x_m^* = \min\left\{ [U_m'^{-1}(\lambda_m)]^+, \ x_m^{max} \right\}, \quad \forall m. \quad (15)$$

where $[z]^+ = \max\{z, 0\}$ and $U_m'^{-1}$ is the inverse of the first derivative of utility function.

Similarly for $\boldsymbol{\mu}$ fixed, the optimal solution $\phi_{mk}^*$ of maximization (13) can be found using Hungarian method [2].

Now we can solve the dual problem (2.2) by using a subgradient projection method [1]. Since $D(\boldsymbol{\lambda}, \boldsymbol{\mu})$ is affine with respect to $(\lambda_m(t), \mu_k(t))$, the subgradient of it at $(\lambda_m(t), \mu_k(t))$ is

$$\frac{\partial D}{\partial \lambda_m(t)} = \sum_k c_m \pi_k I_{mk}(t) - x_m(t) \quad (16)$$

$$\frac{\partial D}{\partial \mu_k(t)} = \gamma_k c_k - \sum_m I_{mk}(t)(1 - \pi_k)c_k, \quad (17)$$

and the updates of dual variables are

$$\lambda_m(t+1) = \left[\lambda_m(t) - \alpha(t)\left(\frac{\partial D}{\partial \lambda_m(t)}\right)\right]^+ \quad (18)$$

$$\mu_k(t+1) = \left[\mu_k(t) - \alpha(t)\left(\frac{\partial D}{\partial \mu_k(t)}\right)\right]^+, \quad (19)$$

where $\alpha(t) > 0$ is the appropriate step-size that leads to the convergence of the optimal dual values [1].

### 3.3 Near-Optimal Algorithm

Next we present the near-optimal scheme. Variables are initialized at 0 and the algorithm stops if the convergence reached.

**At the BS level**

1. For every iteration $t$, each BS updates the new and average collision prices on each channel $k$:

$$\mu_k(t+1) = [\mu_k(t) - \alpha(t)\,\partial D/\partial \mu_k(t)]^+, \quad (20)$$

$$\overline{\mu}_k(t+1) = \beta\overline{\mu}_k(t) + (1-\beta)\mu_k(t+1), \quad (21)$$

   where $\alpha(t)$ is the step size and $0 < \beta < 1$.

2. For every $T \geq t$, the BS solves the following problem then broadcasts new $I_{mk}(T), \forall m, k$ on all channels.

$$\max \quad \sum_m \sum_k I_{mk}(T)[\overline{\lambda}_m(T)\pi_k c_m - \overline{\mu}_k(T)(1-\pi_k)c_k]$$
$$\text{subject to} \sum_m I_{mk}(T) = 1, \ \sum_k I_{mk}(T) = 1, \quad \forall m, k, \quad (22)$$

**At the SU level**

1. For every iteration $t$, each SU:
   - adjusts its source rate by solving (11)

$$x_m(t+1) = \min\left\{[U_m^{'-1}(\lambda_m(t))]^+, \ x_m^{max}\right\}, \quad (23)$$

   where $U_m^{'-1}(.)$ is the inverse of derivative of $U_m$.

   - updates the new and average congestion prices:

$$\lambda_m(t+1) = [\lambda_m(t) - \alpha(t)\,\partial D/\partial \lambda_m(t)]^+ \quad (24)$$

$$\overline{\lambda}_m(t+1) = \beta\overline{\lambda}_m(t) + (1-\beta)\lambda_m(t+1) \quad (25)$$

2. For every $T \geq t$, each SU sends $\overline{\lambda}_m(T)$ to the BS, then receives the new value of $I_{mk}(T)$ from the BS.

The algorithm operates on two levels with different time-scale as follows: At the smaller time-scale $t$, each SU adjusts its source rate (23) using the current congestion price $\lambda_m(t)$, which is updated (24) using $I_{mk}(T)$ broadcast by BS at a periodic time $T \geq t$ (i.e. The update (24) uses the same old $I_{mk}(T)$ for consecutive $T$ iterations). At a larger timesscale $T$, it sends $\overline{\lambda}_m(T)$, which is updated gradually at timescale $t$ (25), to the BS. At time-scale $T$, the BS periodically makes use of $\overline{\lambda}_m(T)$ received from SUs and its $\overline{\mu}_k(T)$ to compute $I_{mk}(T)$ (22) and broadcasts $I_{mk}(T)$ on all channels. Its periodic $\overline{\mu}_k(T)$ is updated gradually at smaller time-scale $t$ with (20) and (21). The closed-loop in Fig. 1 shows the relationship between variables of BS and SU. The interaction between two levels with different time-scale implies that the design of our algorithm allows the BS to track just the *average* congestion price and collision price. The reason behind it is to reduce the computation burden on the BS in terms of amortized analysis, which makes our algorithm much more implementable. For example, if the BS solves (22) by using Hungarian algorithm [2] with the complexity $O(V^3)$ for a bipartite graph $G(V, E)$ and chooses $T = V^2$, then the amortized complexity per operation is only $O(V^3)/V^2 = O(V)$. Finally, an appropriate choice of stepsize $\alpha(t) > 0$ leads to the convergence of the optimal dual values [1].

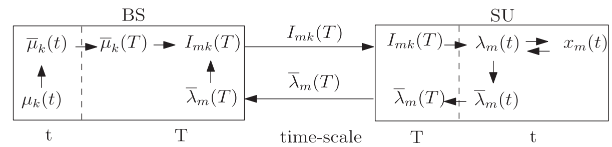


**Fig. 1** Closed-loop structure between BS and SU.

## 4. Simulation Results

We consider the system of 5 SUs opportunistically accessing to 9 orthogonal channels. We use Hungarian algorithm [2] solve (22), and choose $\gamma_k = 0.02$. For scenarios of using fixed $\kappa$, we choose proportional fairness ($\kappa = 1$) as the main fairness criteria between SUs. The criterion for the convergence is $\max_m \frac{|x_m(t)-x_m(t-1)|}{x_m(t-1)} < \epsilon$ where $\epsilon$ is set to $10^{-4}$.

### 4.1 Comparison between Optimal and Near-Optimal Algorithm

In this scenario link capacities are distributed uniformly on [10, 15] Mbps. We set $\kappa = 1$ and vary $\alpha(t)$ and $\beta$ values to validate their impact on the objective value ($\sum U_m(x_m)$). Table 1 shows that $\beta$ has very little impact on near-optimal algorithm, which shows that our method is robust to the dynamic environment. The objective is almost identical for various $\beta$ and we easily observe that the objective values of near-optimal scheme is very close to optimal scheme. Different step sizes give slightly different objective values because diminishing step size ($\alpha(t) = 1/t$) can attain the optimal points while constant step size ($\alpha(t) = 10^{-3}$) only converge to the neighborhood of optimal points [1].

Next we fix $\beta$=0.8 and vary $\kappa$ to employ different kinds of utility functions (1) combining with different types of step size $\alpha(t)$. Table 2 shows that the objective values of near-optimal algorithm are also close to the optimal ones. The objective values are slightly varying between two step size selections as discussed above. These results strongly confirm that the proposal is very efficient and practical.

**Table 1** Impact of step size $\alpha(t)$ and $\beta$ on objective values.

| $\beta$ | optimal alg. | | near-optimal alg. | |
|---|---|---|---|---|
| | $\alpha(t) = 10^{-3}$ | $\alpha(t) = 1/t$ | $\alpha(t) = 10^{-3}$ | $\alpha(t) = 1/t$ |
| 0.2 | 12.695 | 12.237 | 12.525 | 12.131 |
| 0.4 | 12.695 | 12.237 | 12.487 | 12.138 |
| 0.6 | 12.695 | 12.237 | 12.487 | 12.144 |
| 0.8 | 12.695 | 12.237 | 12.488 | 12.142 |

**Table 2** Objective values comparison.

| $\kappa$ | optimal alg. | | near-optimal alg. | |
|---|---|---|---|---|
| | $\alpha(t) = 10^{-3}$ | $\alpha(t) = 1/t$ | $\alpha(t) = 10^{-3}$ | $\alpha(t) = 1/t$ |
| 1 | 12.695 | 12.237 | 12.488 | 12.142 |
| 1.2 | −15.251 | −15.326 | −15.092 | −15.496 |
| 1.4 | −4.492 | −4.698 | −4.508 | −4.793 |
| 1.6 | −1.825 | −1.921 | −1.807 | −1.921 |
| 1.8 | −0.799 | −0.884 | −0.828 | −0.880 |
| 2 | −0.434 | −0.434 | −0.398 | −0.429 |

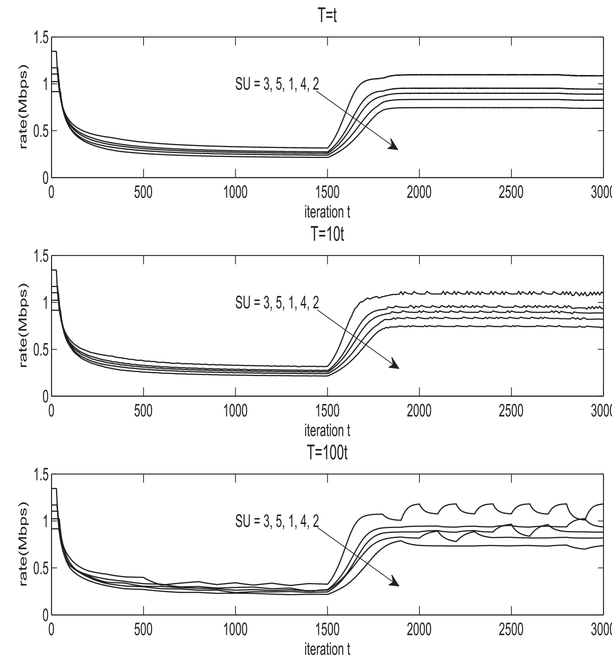


**Fig. 2** SUs flow rates with different values of T.

### 4.2 Impact of T on the Near-Optimal Algorithm

In this scenario we set $\alpha(t) = 0.02$, $\beta = 0.8$, $\kappa = 1$, and link capacities are distributed uniformly on [0.4, 1.6] Mbps. To deal with varying traffic, we consider two cases: high and low channel-occupancy of PUs, corresponding to uniform distribution on [0.1, 0.3] and [0.7, 0.9] of $\boldsymbol{\pi}$.

In Fig. 2, from iteration 0 to 1500, the system works under the high channel-occupancy condition. All 5 SUs flow rates converge to the optimal values [0.261, 0.217, 0.318, 0.242, 0.276] according to the convergence criterion. With high channel-occupancy, the value of $T$ has little impact on system performance. While we cannot see the difference between $T = t$ and $T = 10t$, there is a very small oscillation of SUs flow rates with $T = 100t$. At iteration 1500, we switch to low channel-occupancy condition leading to the increase of SUs flow rates. And after that we also see the convergence of SUs flow rates to optimal values [0.898, 0.745, 1.094, 0.832, 0.952]. However with low channel-occupancy, while the difference between $T = t$ and $T = 10t$ is very little, the SUs flow rates strongly oscillate with $T = 100t$ due to the long delay in acquiring the information needed for updating the prices. Consequently, our algorithm is more robust at high channel-occupancy than low channel-occupancy condition.

## 5. Conclusion

We proposed a joint rate control and resource allocation scheme with packet collision constraint for cognitive radio networks. Our algorithm operates on different time-scales, which reduces significantly the computational burden on the BS.

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